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411-1, Fall 2003

MIDTERM EXAM

There are three questions. The number of points available for each question is indicated. If a question seems ambiguous, state why, sharpen it up and answer the revised question. You have 1 hour and 55 minutes. Good luck!

1. (40) Consider the canonical model,

$$\max_{\{x_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t F(x_t, x_{t+1}), \ x_{t+1} \in \Gamma(x_t),$$

$t = 0, 1, 2, \ldots$. Here, $\Gamma : X \to X$, $F : A \to R$, where

$$A = \{x, y : x \in X, \ y \in \Gamma(x)\}.$$

Suppose that (i) X is a convex subset of R^l, Γ is non-empty, compact, convex and continuous, (ii) F is bounded, continuous, $0 \leq \beta \leq 1$, (iii) F is strictly increasing in its first argument and (iv) Γ is monotone. Under (i) and (ii), the above sequence problem is 'equivalent' to the following problem. Find g and v, where

$$\begin{aligned} v(x) &= \max_{x' \in \Gamma(x)} F(x, x') + \beta v(x'), \\ g(x) &= \arg \max_{x' \in \Gamma(x)} F(x, x') + \beta v(x'). \end{aligned}$$

   (a) (5) Explain the meaning of 'equivalence' here.

   (b) (5) What does it mean that Γ is 'monotone'?

   (c) (10) Prove that v is strictly increasing. (Be clear on exactly where each assumption is used.)

   (d) (20) Add the assumptions, (v) Γ is strictly concave and (vi) Γ is convex. State clearly the definition of convexity of Γ and prove that under (i)-(vi), v is not just strictly increasing but also strictly concave.

2. (40) Consider a model in which a final good is produced using intermediate goods. The final good, y, is produced by a competitive, representative firm using the following homogeneous technology:

$$y = \exp \int_0^1 [\log y_j]\, dj.$$

The firm maximizes profits:

$$y - \int_0^1 p_j y_j dj,$$

taking p_j as given. Here, the price of the final good has been normalized at unity. The j^{th} intermediate good is produced by a monopolist using the following technology:

$$\begin{aligned} y_j &= \begin{cases} f(k_j, l_j) - \phi & f(k_j, l_j) \geq \phi \\ 0 & \text{otherwise} \end{cases}, \qquad (1) \\ f(k_j, l_j) &= k_j^a l_j^{1-\alpha},\ 0 < \alpha < 1. \end{aligned}$$

Thus, if the monopolist is to sell y_j units of goods, they must produce the fixed quantity, ϕ, first. The monopolist is competitive in the market for labor and capital and takes the rental rate on capital, r, and the wage rate, w, as given.

(a) (8) Derive the demand curve for the j^{th} intermediate good. Consider the profit maximization problem of the j^{th} intermediate good firm. Show that it has no solution. That is, for any finite price-quantity pair on the demand curve, profits are always increased by increasing the price level.

(b) (2) Suppose that there are other potential entrants into the production of the j^{th} intermediate good, and that they have access to the same technology, (1). Explain why this implies that the profits of the intermediate good producer must be zero.

(c) (14) Show that cost minimization by the j^{th} intermediate good producer, linear homogeneity of f, and the zero profit condition imply that output can be written

$$y_j = \frac{1}{\mu_j} f(k_j, l_j),$$

where μ_j is the firm markup, the ratio of price to marginal cost, λ_j:

$$\mu_j = \frac{p_j}{\lambda_j}.$$

(d) (8) Show that the zero profit condition implies the markup must fall when the firm produces more output. Provide the intuition for this result.

(e) (8) Explain why it is that in equilibrium, final output has the following representation:

$$y = \frac{1}{\mu} f(k, l),$$

where l is household labor supply, k is the supply of capital by households, and μ is the markup.

3. (30) Consider the two-sector economy, in which consumption and new capital are produced according to the following technologies,

$$c_t = k_{ct}^a n_{ct}^{1-\alpha}, \; k_{t+1} - (1-\delta)k_t = z_t k_{it}^\alpha n_{it}^{1-\alpha},$$

respectively. The price, in units of date t consumption goods, of new capital goods is p_t. Firms in the two sectors are competitive in output, capital and labor markets and take the rental rate on capital, r_t, and the wage rate, w_t, as given. The investment good firm takes p_t as given. All these prices are denominated in units of the date t consumption good. Profits of the consumption good firms, denominated in consumption units, are $k_{ct}^a n_{ct}^{1-\alpha} - r_t k_{ct} - w_t n_{ct}$, and profits of the investment good firms is $p_t z_t k_{it}^a n_{it}^{1-\alpha} - r_t k_{it} - w_t n_{it}$.

(a) (10) What is the restriction across r_t and w_t that must be satisfied in equilibrium? Assume from here on that that restriction is satisfied.

(b) (20) Show that the capital-labor ratios in the two sectors are the same, i.e., $k_{ct}/n_{ct} = k_{it}/n_{it}$.