

Financial Frictions

Many economic activities require financing, because they involve the outlay of resources for a period of time, before output occurs. At the same time, the people with resources to spare and the people with ideas about how to use those resources productively are often not the same. That is, economic activities that require financing typically involve cooperation between the suppliers of funds and the users of funds. Conflicts can arise in this relationship because of fundamental asymmetries between the two parties. In particular, the outcome of almost all economic activities involves some uncertainty, and the people actually undertaking those activities know best what the nature of that uncertainty is. This can create conflict because the user of funds has an incentive to report that things did not go well, in order to justify paying little to the supplier of funds, and keeping a larger share for themselves. In principle, this can take a variety of forms, including applying low effort to the project, or simply taking a big share of the output.

In the standard neoclassical model, the conflicts associated with financing are absent. In part, this reflects the assumption that households are homogeneous. Consider, for example, the process by which physical capital is produced in the model. First, homogeneous output is produced by firms. Then, households purchase that output and use it as input into a technology that converts it one-for-one into consumption goods and new capital goods. Although one can imagine that financing is used here, that financing involves no conflict because the people applying the resources (the output goods used to produce new capital) and the people supplying the resources are the same. Recently, Carlstrom and Fuerst (1997) have shown how to modify the model so that the finance needed to produce capital involves different people and there is a conflict with them. They introduce this friction into an otherwise standard neoclassical growth model, and show how the friction winds up changing the dynamic properties of the model: thinking about financing frictions leads to ideas for new sources of shocks, and it changes the propagation mechanism for shocks that originate outside the financing relationship.

To create the financing friction, Carlstrom and Fuerst introduce a new type of household, an ‘entrepreneur’. This agent is in possession of the technology for converting output into new capital goods. To some extent, the entrepreneur uses his own resources to build new capital. However, he can increase his return even more by borrowing additional resources from a representative, competitive bank so that he can produce more capital than his own resources permit. The source of funds to the bank is another set of households. This borrowing relationship involves frictions. The reason is that not all entrepreneurs are equally productive. Some are more successful and others, less. In principle, the bank’s lending arrangement (‘contract’) with the entrepreneur should be one in which the bank simply takes a fraction of whatever the entrepreneur’s return is. That is, the bank should take an equity stake in the entrepreneur. However, this is not a good arrangement in the Carlstrom-Fuerst model. The reason is that whether the entrepreneur does well or poorly is private information to the entrepreneur. An outside party can observe what happened to the entrepreneur only by expending resources in monitoring. If banks had equity relationships with entrepreneurs,

then even successful entrepreneurs would be tempted to claim that they had been unsuccessful, in order to minimize their payments to the bank and maximize their own return.¹ To avoid this, banks could in principle monitor each entrepreneur. However, this would be prohibitively expensive, as it would involve a great deal of monitoring.

This environment was considered long ago by Gale and Hellwig (1985), Townsend (1979), and Williamson (1986). These authors show that a simple ‘standard debt contract’ works better than the equity arrangement in this environment. In particular, the bank offers the entrepreneur a particular loan amount and interest rate. Entrepreneurs who are successful simply pay the agreed-upon interest rate charge, and the relationship with the bank is over. Entrepreneurs who are unsuccessful and cannot pay back the interest charge (i.e., who go bankrupt), must pay the bank whatever they have. Banks monitor the bankrupt entrepreneurs. An advantage of this arrangement is that banks do not have to monitor all entrepreneurs.

There is a very large number of entrepreneurs, so that the bank does not have any risk. It knows how many of its entrepreneurs will declare bankruptcy, although it does not know which entrepreneurs will do so (ex ante, not even the entrepreneurs know who will succeed and who will fail). The purpose of these notes is to describe the Carlstrom-Fuerst model in detail. A similar model with financial frictions, where the frictions lie with the buyers of capital - instead of the producers of capital, as it is here - may be found in Bernanke, Gertler and Gilchrist (1999).

1. Entrepreneurial Debt Contracts

The state of an entrepreneur at the time that he presents himself at the bank for a loan is his ‘net worth’, a_t . The entrepreneur’s net worth represents claims on a_t units of the final output good. Later, we will discuss how the entrepreneur obtains a_t . For each possible value of a_t , there are many entrepreneurs with that level of net worth. So, there is a large number of entrepreneurs. The entrepreneur seeks a loan to help him invest in a technology for producing capital goods. The entrepreneur who invests i_t goods into the capital technology draws an idiosyncratic shock, ω , and produces new capital goods in the amount, $i_t\omega$. These goods are sold at market price, q_t , so that the consumption good value of $i_t\omega$ is $q_t i_t\omega$. The cumulative distribution function of ω is

$$\Phi(x) \equiv \text{prob}[\omega \leq x].$$

The random variable, ω , is realized after the entrepreneur invests i_t . After ω is realized, only the entrepreneur knows its value. For an outsider to observe ω they must pay a monitoring cost discussed below.

An entrepreneur receives a contract from the bank which specifies a loan amount and well as an interest rate. Both features of the contract are in principle a function of the

¹Considerations like reputation do not appear in the Carlstrom-Fuerst model, because it is assumed either that entrepreneurs and banks are randomly matched in each period, or there is simply no capacity for keeping records. For analyses of debt contracts when there are long-term relationships between banks and entrepreneurs, see Albuquerque and Hopenhayn (2004), Cooley, Marimon and Quadrini, (2003), Cooley and Quadrini, (2006) and Monge (2001) and the extensive literature that they cite.

entrepreneur's net worth, a_t . An entrepreneur with net worth a_t , who invests i_t^a must borrow

$$i_t^a - a_t$$

from the bank. Under the contract, the entrepreneur must pay back $R_t^a (i_t^a - a_t)$ units of consumption goods (R_t^a is a gross interest rate). According to the contract, if an entrepreneur's ω is so low that it is infeasible to repay $R_t^a (i_t^a - a_t)$, then they must repay whatever they have, namely, $i_t^a \omega$.² When an entrepreneur declares bankruptcy, the bank verifies this by monitoring the entrepreneur. If the bank did no monitoring, the entrepreneur would have an incentive to under-report the value of ω , and repay only a small amount to the bank. But, monitoring is expensive. The bank must expend μi_t^a units of capital goods to monitor an entrepreneur.³

It is useful to develop several variables relating to the debt contract. The average revenues, across all entrepreneurs with net worth a_t is:

$$q_t i_t^a \int \omega d\Phi(\omega) = q_t i_t^a$$

There is a value of ω , $\bar{\omega}_t^a$, such that for all $\omega < \bar{\omega}_t^a$ it is infeasible for the entrepreneur with net worth a_t to repay his loan. It satisfies

$$R_t^a (i_t^a - a_t) - q_t \bar{\omega}_t^a i_t^a = 0. \quad (1.1)$$

The debt contract can be specified in terms of (R_t^a, i_t^a) , or, equivalently, in terms of $(\bar{\omega}_t^a, i_t^a)$. The average net profits (i.e., after all costs) across all entrepreneurs with net worth a_t who invest i_t^a is:

$$\begin{aligned} & \underbrace{q_t i_t^a \int \omega d\Phi(\omega)}_{\text{average revenues}} - \underbrace{\int_{\bar{\omega}_t^a}^{\infty} R_t^a (i_t^a - a_t) d\Phi(\omega)}_{\text{average costs of non-bankrupt entrepreneurs}} - \underbrace{\int_0^{\bar{\omega}_t^a} q_t i_t^a \omega d\Phi(\omega)}_{\text{average costs of bankrupt entrepreneurs}} \\ &= q_t i_t^a \int \omega d\Phi(\omega) - \int_{\bar{\omega}_t^a}^{\infty} q_t \bar{\omega}_t^a i_t^a d\Phi(\omega) - \int_0^{\bar{\omega}_t^a} q_t i_t^a \omega d\Phi(\omega) \\ &= q_t i_t^a \int_{\bar{\omega}_t^a}^{\infty} (\omega - \bar{\omega}_t^a) d\Phi(\omega) \\ &= q_t i_t^a \left[\int_{\bar{\omega}_t^a}^{\infty} \omega d\Phi(\omega) - \bar{\omega}_t^a (1 - \Phi(\bar{\omega}_t^a)) \right] \\ &= q_t i_t^a f(\bar{\omega}_t^a), \end{aligned}$$

say. The entrepreneur's expected rate of return in the capital producing technology must be no less than zero, because he can always earn a zero return by simply holding onto a_t and

²Below, we explain that an entrepreneur who is bankrupt in period t must consume zero in period t . If entrepreneurs had utility functions that satisfied the Inada conditions, then things would be different. Either entrepreneurs, out of caution, would not invest all of a_t in the capital-producing technology. Or, the nature of the debt contract would allow bankrupt entrepreneurs to keep some resources.

³We rule out the possibility that the bank adopts a random monitoring strategy.

not producing capital goods.⁴ So, the participation constraint of the entrepreneur is:

$$q_t i_t^a f(\bar{\omega}^a) \geq a_t.$$

That is, the goods obtained by operating the capital-producing technology must be no less than the the goods, a_t , the entrepreneur puts into the technology.

The earnings of the financial intermediary, integrating across all entrepreneurs with net worth, a_t , is:

$$\underbrace{q_t i_t^a \left[\int_0^{\bar{\omega}_t^a} \omega d\Phi(\omega) - \mu \Phi(\bar{\omega}_t^a) \right]}_{\text{income, after monitoring costs, from bankrupt entrepreneurs}} + \underbrace{R_t^a (i_t^a - a_t) [1 - \Phi(\bar{\omega}_t^a)]}_{\text{income from entrepreneurs who are not bankrupt}}$$

or,

$$\begin{aligned} & q_t i_t^a \left[\int_0^{\bar{\omega}_t^a} \omega d\Phi(\omega) - \mu \Phi(\bar{\omega}_t^a) \right] + q_t \bar{\omega}_t^a i_t^a [1 - \Phi(\bar{\omega}_t^a)] \\ &= q_t i_t^a \left[\int_0^{\bar{\omega}_t^a} \omega d\Phi(\omega) - \mu \Phi(\bar{\omega}_t^a) + \bar{\omega}_t^a (1 - \Phi(\bar{\omega}_t^a)) \right] \\ &= q_t i_t^a g(\bar{\omega}_t^a), \end{aligned}$$

say. The sum of these the two fractions is:

$$\begin{aligned} & g(\bar{\omega}_t^a) + f(\bar{\omega}_t^a) \tag{1.2} \\ &= \int_0^{\bar{\omega}_t^a} \omega d\Phi(\omega) - \mu \Phi(\bar{\omega}_t^a) + \bar{\omega}_t^a (1 - \Phi(\bar{\omega}_t^a)) + \int_{\bar{\omega}_t^a}^{\infty} \omega d\Phi(\omega) - \bar{\omega}_t^a (1 - \Phi(\bar{\omega}_t^a)) \\ &= 1 - \mu \Phi(\bar{\omega}_t^a) \end{aligned}$$

so that on average $\mu \Phi(\bar{\omega}_t^a)$ of produced capital is destroyed in monitoring.

The source of funds for the bank is the household. However, the bank needs to only pay the household a zero rate of return because the household has no alternative use of its funds over the (very brief!) period of time when the entrepreneur requires financing. In particular, if the bank obtains $i_t^a - a_t$ units of output goods from the household, it must return the same amount in period t . Thus, banks face the following non-negativity constraint with debt contracts issued to entrepreneurs with net worth, a_t :

$$q_t i_t^a g(\bar{\omega}_t^a) \geq i_t^a - a_t.$$

Competition ensures that, in equilibrium, the debt contract maximizes entrepreneurial welfare subject to the previous profit condition. Consider an entrepreneur with net worth, a_t , at the beginning of the period. The expected state of such an entrepreneur at the end of the loan contract is $q_t i_t^a f(\bar{\omega}^a)$. Because the entrepreneur is assumed to have linear utility in consumption (see below), his value function is a linear increasing function of $q_t i_t^a f(\bar{\omega}^a)$.

⁴Here, we implicitly use the assumption of risk neutrality of entrepreneur, made explicit below. Risk neutrality is the reason the entrepreneur compares the expected return on investing in capital with the sure, zero return obtained by simply holding a_t .

Thus, the problem of maximizing the entrepreneur's expected welfare subject to the profit condition has the following Lagrangian representation:

$$\max_{\bar{\omega}_t^a, i_t^a} q_t i_t^a f(\bar{\omega}_t^a) + \lambda^a [q_t i_t^a g(\bar{\omega}_t^a) - i^a + a_t]. \quad (1.3)$$

In this problem, q_t and a_t are treated as given constants, reflecting the assumption that banks are competitive (implicitly, we assume the participation constraint of the entrepreneur is non-binding; in any computations this would have to be verified). The first order conditions of this problem are:

$$\begin{aligned} q_t f(\bar{\omega}_t^a) + \lambda [q_t g(\bar{\omega}_t^a) - 1] &= 0 \\ q_t i_t^a f'(\bar{\omega}_t^a) + \lambda q_t i_t^a g'(\bar{\omega}_t^a) &= 0 \\ q_t i_t^a g(\bar{\omega}_t^a) - i^a + a_t &= 0. \end{aligned}$$

Combine the first two equations to substitute out λ^a :

$$q_t f(\bar{\omega}_t^a) = \frac{f'(\bar{\omega}_t^a)}{g'(\bar{\omega}_t^a)} [q_t g(\bar{\omega}_t^a) - 1] \quad (1.4)$$

$$q_t i_t^a g(\bar{\omega}_t^a) - i_t^a + a_t = 0. \quad (1.5)$$

Note from (1.4) that $\bar{\omega}_t^a$ is a function only of q_t , and not of the level of net worth of the entrepreneur. As a result, we drop the a superscript from $\bar{\omega}_t$ from here on. In addition, (1.5) shows that i_t/a_t is independent of the level of the entrepreneur's net worth. From (1.1), we deduce that the lending rate paid by non-bankrupt entrepreneurs is independent of the entrepreneur's net worth, a_t . So, entrepreneurs with every level of net worth, a_t , pay the same gross rate of interest:

$$R_t = \frac{q_t \bar{\omega}_t i_t^a}{i_t^a - a_t} = q_t \bar{\omega}_t \frac{1}{1 - a_t/i_t} = \frac{\bar{\omega}_t}{g(\bar{\omega}_t)}.$$

A loan to an individual entrepreneur is risky, in that it may not be repaid fully, and in the event that it is not the bank must incur monitoring costs. So, a natural measure of the risk premium is the excess of R_t over the sure rate of return, which in this case is unity. Thus, the risk premium is:

$$R_t - 1 = \frac{\bar{\omega}_t}{g(\bar{\omega}_t)} - 1.$$

Under the standard debt contract (i.e., the one that solves (1.3)), the level of investment that an entrepreneur can operate is proportional to his net worth. A consequence of this is that in working out the aggregate implications of the model, we do not have to keep track of the distribution of net worth across entrepreneurs. Although that distribution is non-trivial, we can simply work with i_t and a_t , which we interpret as the average, across all entrepreneurs, of investment and net worth, respectively.

Using Leibniz's rule to evaluate the derivatives in (1.4):

$$\begin{aligned} f'(\bar{\omega}) &= -\bar{\omega}\Phi'(\bar{\omega}) - (1 - \Phi(\bar{\omega})) + \bar{\omega}\Phi'(\bar{\omega}) = -(1 - \Phi(\bar{\omega})) \\ g'(\bar{\omega}) &= \bar{\omega}\Phi'(\bar{\omega}) - \mu\Phi'(\bar{\omega}) + (1 - \Phi(\bar{\omega})) - \bar{\omega}\Phi'(\bar{\omega}) = -\mu\Phi'(\bar{\omega}) + (1 - \Phi(\bar{\omega})), \end{aligned}$$

so that the first order conditions in (1.4)-(1.5) reduce to:

$$q_t f(\bar{\omega}_t) = \frac{1}{\mu \frac{\Phi'(\bar{\omega}_t)}{1-\Phi(\bar{\omega}_t)} - 1} [q_t g(\bar{\omega}_t) - 1] \quad (1.6)$$

$$i_t = \frac{1}{1 - q_t g(\bar{\omega}_t)} a_t. \quad (1.7)$$

From (1.7), we can see how much investment, i_t , an entrepreneur with net worth, a_t , can do. Since expected revenues, after costs, to an entrepreneur who invests i_t , is $q_t i_t f(\bar{\omega}_t)$, we conclude that an entrepreneur with assets a_t , can convert these into net revenues:

$$q_t f(\bar{\omega}_t) i_t = \frac{q_t f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} a_t. \quad (1.8)$$

The coefficient on a_t in (1.8) is the expected return on the entrepreneur's internal funds, a_t . The participation constraint of the entrepreneur implies that the entrepreneur's expected return on funds must be no less than unity. This is because the entrepreneur could receive a zero return simply by not constructing new capital. We show this formally as follows. The entrepreneur's participation constraint is $q_t i_t f(\bar{\omega}_t) > a_t$, so that, after using (1.7) we obtain:

$$q_t i_t g(\bar{\omega}_t) - i_t + q_t i_t f(\bar{\omega}_t) > 0,$$

or,

$$\frac{q_t f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} > 1. \quad (1.9)$$

When the entrepreneur's expected return is substantially larger than unity, this shows the benefits of 'leverage'. Leverage occurs when an entrepreneur borrows in order to invest at a level higher than his own resources permit. If the borrowing is very large, in comparison with net worth, then the entrepreneur is said to be 'highly levered'. From here on, I assume (1.9) is satisfied.

2. A Dynamic Economy

Carlstrom and Fuerst (1997) showed how to integrate the above debt arrangement into an otherwise standard version of the neoclassical growth model. The economy is composed of firms, a mass, η , of entrepreneurs and a mass, $1 - \eta$, of identical households. The household problem is

$$\max_{\{c_t, k_{ct+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \quad 0 < \beta < 1,$$

subject to the budget constraint:

$$c_t + q_t [k_{c,t+1} - (1 - \delta) k_{c,t}] \leq w_t l_t + r_t k_{ct}, \quad (2.1)$$

and to an initial level of capital, $k_{c,0}$. Here, c_t and $k_{c,t}$ denote household consumption and the household stock of capital, respectively. In addition, l_t denotes household employment.

The first order conditions of the household are (2.1) with a strict equality and:

$$u_{c,t}q_t = \beta u_{c,t+1} [r_{t+1} + q_{t+1} (1 - \delta)] \quad (2.2)$$

$$\frac{-u_{l,t}}{u_{c,t}} = w_t. \quad (2.3)$$

Entrepreneurs are assumed to be risk neutral and to have discounted utility:

$$E_0 \sum_{t=0}^{\infty} (\beta\gamma)^t c_{et}.$$

Note that entrepreneurs discount utility at a higher rate, $\gamma\beta$, than households ($0 < \gamma < 1$). The entrepreneurs are endowed with one unit of labor time, which they supply inelastically to the market.

After the period t shock, θ_t , is realized, households and entrepreneurs supply their labor, $(1 - \eta) l_t$ and η , respectively. They earn competitive wage rates, w_t and $w_{e,t}$, respectively. Also, households supply $k_{c,t}$ and entrepreneurs supply their average stock of capital, $k_{e,t}$. So, total beginning-of-period t capital, k_t , supplied to the capital-rental market is:

$$k_t = (1 - \eta) k_{c,t} + \eta k_{e,t}.$$

Households and entrepreneurs earn the competitive rental rate, r_t , on their capital supply.

Final output, Y_t , is produced by goods-producers using a technology that is linear homogeneous in capital, household labor and entrepreneurial labor:

$$Y_t = F(k_t, \theta_t, (1 - \eta) l_t, \eta).$$

Profit maximization implies:

$$F_{k,t} = r_t, \quad F_{l,t} = w_t, \quad F_{\eta,t} = w_{e,t}. \quad (2.4)$$

Households allocate their income to consumption and purchases of capital goods, $k_{c,t+1} - (1 - \delta) k_{c,t}$, as discussed above. In doing so, they supply

$$q_t [k_{c,t+1} - (1 - \delta) k_{c,t}]$$

goods to banks. They require only a zero return on these deposits because the household opportunity cost on these output goods is zero. (See Figure 1, for an illustration of the events during period t .)

After period t goods production, the average income of entrepreneurs is $w_{e,t} + r_t k_{e,t}$ in units of output. The average value of their undepreciated capital is $q_t (1 - \delta) k_{e,t}$, in output units. At this point, the average value, in consumption units, of the entrepreneurs' resources is:

$$a_t = w_{e,t} + [r_t + q_t (1 - \delta)] k_{e,t}.$$

Entrepreneurs invest an average of a_t consumption goods, together with a loan from the bank of $i_t - a_t$, into the production of capital goods. At the end of the period, after the debt contract with the bank is paid off, the entrepreneurs who do not go bankrupt in the

process of producing capital have income that can be used to buy consumption goods and new capital goods:

$$c_{et} + q_t k_{et+1} \leq \begin{cases} R_t(i_t - a_t) - \omega i_t & \omega \geq \bar{\omega}_t \\ 0 & \omega < \bar{\omega}_t \end{cases}. \quad (2.5)$$

An entrepreneur who is bankrupt in period t must set $c_{et} = 0$ and $k_{et+1} = 0$. In period $t + 1$, these entrepreneurs start with net worth $w_{e,t+1}$.⁵ Entrepreneurs who are not bankrupt in period t can purchase positive amounts of c_{et} and $k_{e,t+1}$ (except in the non-generic case, $\omega = \bar{\omega}_t$). For these entrepreneurs, the marginal cost of purchasing $k_{e,t+1}$ is q_t units of consumption. The expected marginal payoff from $k_{e,t+1}$ at the beginning of period $t + 1$ is $E_t[r_{t+1} + q_{t+1}(1 - \delta)]$. In particular, in each aggregate state in period $t + 1$, the entrepreneur expands his net worth by the value of $[r_{t+1} + q_{t+1}(1 - \delta)]$ in that state. This additional net worth permits a further expansion in the entrepreneur's payoff by investing in the capital production technology. The expected value (relative to date $t + 1$ idiosyncratic uncertainty) of that payoff is the coefficient on a_t in (1.8). So, the expected rate of return available to entrepreneurs who are not bankrupt in period t is:

$$E_t \left[\frac{F_{k,t+1} + q_{t+1}(1 - \delta)}{q_t} \times \frac{q_{t+1}f(\bar{\omega}_{t+1})}{1 - q_{t+1}g(\bar{\omega}_{t+1})} \right],$$

which they optimally equate to $1/(\beta\gamma)$. The expression to the left of '×' coincides with the rate of return enjoyed by households. As explained above, the term after '×' must be no less than unity. (The entrepreneur can always obtain unity, simply by not producing any capital.) In this expression, we see why it is assumed that entrepreneurs discount the future more heavily than households do. Entrepreneurs earn a higher intertemporal rate of return on saving than do households. As a result, entrepreneurs with the same discount rate as households would save at a higher rate, eventually accumulating enough capital (and, hence, net worth) so that they have no need to borrow from banks. The assumption, $\gamma < 1$, helps ensure that the financial frictions remain operative indefinitely in this economy.

3. Equilibrium

We now collect the equilibrium conditions for the economy. Substituting (2.4) into (2.2) and (2.3):

$$u_{c,t}q_t = \beta E_t u_{c,t+1} [F_{k,t+1} + q_{t+1}(1 - \delta)] \quad (3.1)$$

$$\frac{-u_{l,t}}{u_{c,t}} = F_{l,t}. \quad (3.2)$$

⁵Now, one can see why it is assumed that entrepreneurs earn wage income. An entrepreneur with no assets cannot borrow anything from the bank, and zero assets would become an absorbing state. Since all entrepreneurs at some point in their infinitely long lives go bankrupt, if they earned no wage income all entrepreneurs would eventually have zero net worth, and no further capital accumulation would be possible. This is not an interesting outcome.

Net production of new capital goods by all entrepreneurs is:

$$\begin{aligned} & \eta \left[i_t \int_0^\infty \omega d\Phi(\omega) - \mu i_t \int_0^{\bar{\omega}_t} d\Phi(\omega) \right] \\ &= I_t [1 - \mu \Phi(\bar{\omega}_t)], \end{aligned}$$

where I_t is total investment:

$$I_t \equiv \eta i_t.$$

Averaging the budget constraint across all entrepreneurs:

$$c_{et} + q_t k_{et+1} = q_t f(\bar{\omega}_t) \frac{I_t}{\eta}. \quad (3.3)$$

The definition of entrepreneurial wealth is:

$$a_t = F_{4,t} + [F_{k,t} + 1 - \delta] k_{e,t} \quad (3.4)$$

Total capital accumulation is:

$$k_{t+1} = (1 - \delta) k_t + I_t [1 - \mu \Phi(\bar{\omega}_t)]. \quad (3.5)$$

The resource constraint is:

$$(1 - \eta) c_t + \eta c_{e,t} + I_t = F(k_t, \theta_t, (1 - \eta) l_t, \eta). \quad (3.6)$$

The intertemporal Euler equation of non-bankrupt entrepreneurs is:

$$E_t \left[\frac{F_{k,t+1} + q_{t+1} (1 - \delta)}{q_t} \times \frac{q_{t+1} f(\bar{\omega}_{t+1})}{1 - q_{t+1} g(\bar{\omega}_{t+1})} \right] = \frac{1}{\gamma \beta} \quad (3.7)$$

Combining (1.6) with (1.2):

$$q_t = \frac{1}{1 - \mu \left[\Phi(\bar{\omega}_t) + \frac{\Phi'(\bar{\omega}_t)}{1 - \Phi(\bar{\omega}_t)} f(\bar{\omega}_t) \right]} \quad (3.8)$$

$$I_t = \frac{1}{1 - q_t g(\bar{\omega}_t)} \eta a_t. \quad (3.9)$$

The 9 variables to be determined with the 9 equations, (3.1)-(3.9) are: c_t , $c_{e,t}$, I_t , k_t , $k_{e,t}$, l_t , q_t , $\bar{\omega}_t$, a_t .

4. Calibration

Carlstrom and Fuerst assume a production function of the following form:

$$F(k_t, \theta_t, (1 - \eta) l_t, \eta) = k_t^\alpha [\theta_t (1 - \eta) l_t]^{1 - \alpha - \varsigma} \eta^\varsigma.$$

They assign a share of 0.36 to capital (i.e., $\alpha = 0.36$) and a share of 0.6399 and 0.0001 to household employment and entrepreneurial employment, respectively (i.e., $\varsigma = 0.0001$). The

small share of employment by entrepreneurs implies their wage rate is very small, though they still earn enough of a wage so that bankrupt entrepreneurs can finance at least some investment. Because the share of income going to entrepreneurial labor is so small, when $\mu = 0$ the economy essentially collapses to the real business cycle model (note from (3.8) that $q = 1$ in this case). Carlstrom and Fuerst set $\delta = 0.02$ and $\beta = 0.99$. They set $\mu = 0.25$. The utility function is assumed to be:

$$u(c, l) = \log(c) + \nu(1 - l),$$

where ν is chosen so that steady state $l = 0.3$. To obtain the parameters of the log-normal distribution, Carlstrom and Fuerst suppose, first, that $E\omega = 1$. There now remain two parameters to set: γ and σ . The latter is the standard deviation of the normal random variable, $\log \omega$. These two parameters were pinned down by specifying values for the bankruptcy rate in steady state, $\Phi(\bar{\omega})$, and the risk premium on loans to entrepreneurs, $R - 1 = \bar{\omega}/g(\bar{\omega}) - 1$. Carlstrom and Fuerst specify the annualized risk premium to be 187 basis points (i.e., 1.87 percentage points) and a quarterly bankruptcy rate of 0.974 percent. In addition, note from (3.7) that in steady state, the entrepreneur's intertemporal problem is:

$$\frac{F_k + q(1 - \delta)}{q} \times \frac{qf(\bar{\omega})}{1 - qg(\bar{\omega})} = \frac{1}{\beta\gamma}.$$

But, the first term to the left of the \times is $1/\beta$ by the household intertemporal Euler equation. Then,

$$\frac{qf(\bar{\omega})}{1 - qg(\bar{\omega})} = \frac{1}{\gamma}.$$

Given the target risk premium and the latter equation, Carlstrom and Fuerst report $\sigma = 0.207$ and $\gamma = 0.947$.

Carlstrom and Fuerst (1997) report other features of the state. The ratio of internal funds, a_t , to total investment, i_t , is $a/i = 0.38$. Also, $c^e/a = 0.067$, $1/\gamma = 1.056$. Also, $f(\bar{\omega}) = 0.39$ and $q = 1.024$.

5. Financial Frictions and Adjustment Costs

Carlstrom and Fuerst (1997) show that their model with financial frictions is similar to a version of the neoclassical growth model in which there is curvature ('adjustment costs') in the rate at which additional investment produces more capital goods. In particular, the capital accumulation equation in this model is:

$$k_{t+1} = (1 - \delta)k_t + \psi(I_t).$$

The resource constraint in this economy is:

$$c_t + I_t \leq k_t^\alpha (\theta_t l_t)^{1-\alpha}$$

and household preferences are:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

The equilibrium conditions for this economy may be found by solving the following Lagrangian problem:

$$\begin{aligned} \max_{k_{t+1}, l_t, I_t} E_0 \sum_{t=0}^{\infty} \beta^t \{ & u(c_t, l_t) + \mu_t [k_t^\alpha (\theta_t l_t)^{1-\alpha} - I_t - c_t] \\ & + \lambda_t [(1 - \delta) k_t + \psi(I_t) - k_{t+1}] \}. \end{aligned}$$

The first order conditions with respect to c_t , l_t , I_t and k_{t+1} are:

$$\begin{aligned} u_{c,t} &= \mu_t \\ u_{l,t} + \mu_t (1 - \alpha) \frac{k_t^\alpha (\theta_t l_t)^{1-\alpha}}{l_t} &= 0 \\ \mu_t &= \lambda_t \psi'(I_t) \\ \frac{\lambda_t}{\mu_t} &= \beta E_t \frac{\mu_{t+1}}{\mu_t} \left[\alpha k_{t+1}^{\alpha-1} (\theta_{t+1} l_{t+1})^{1-\alpha} + \frac{\lambda_{t+1}}{\mu_{t+1}} (1 - \delta) \right]. \end{aligned}$$

Note that from the capital accumulation equation,

$$dk_{t+1} = \psi'(I_t) dI_t,$$

and from the resource constraint

$$dc_t = -dI_t,$$

so that the price of k_{t+1} , in units of c_t , is

$$q_t \equiv \frac{1}{\psi'(I_t)} = \frac{\mu_t}{\lambda_t}.$$

Substituting this into the intertemporal equation, we conclude that the equilibrium conditions are:

$$q_t = \beta E_t \frac{u_{c,t+1}}{u_{c,t}} \left[\alpha k_{t+1}^{\alpha-1} (\theta_{t+1} l_{t+1})^{1-\alpha} + q_{t+1} (1 - \delta) \right] \quad (5.1)$$

$$u_{l,t} + u_{c,t} (1 - \alpha) \frac{k_t^\alpha (\theta_t l_t)^{1-\alpha}}{l_t} = 0 \quad (5.2)$$

$$q_t = \frac{1}{\psi'(I_t)} \quad (5.3)$$

$$k_{t+1} = (1 - \delta) k_t + \psi(I_t) \quad (5.4)$$

$$I_t + c_t = k_t^\alpha (\theta_t l_t)^{1-\alpha} \quad (5.5)$$

Note that (3.1) and (3.2) coincide with (5.1) and (5.2), respectively. Note, too, that (3.6) is very similar to (5.5). Finally, (5.3) and (5.4) are to be compared with (3.9) and (3.5), respectively. To see this, note that (5.3) implies:

$$I_t = s(q_t),$$

where $s(q_t)$ is increasing in q_t because ψ is assumed to be concave. Carlstrom and Fuerst show that (3.8) and (3.9) imply

$$I_t = S(q_t, a_t).$$

Carlstrom and Fuerst (1997) show that S is increasing in q_t and a_t .

Suppose $\psi(I_t)$ is a constant elasticity function of I_t , so that

$$\psi(I_t) = \frac{1}{1-\nu} I_t \psi'(I_t),$$

where $1-\nu > 0$. In this case, (5.4) can be written, after taking into account the previous expression, as well as (5.3),

$$k_{t+1} = (1-\delta)k_t + \frac{1}{1-\nu} I_t \frac{1}{q_t}. \quad (5.6)$$

According to Carlstrom and Fuerst (1997), (3.8) implies that $\bar{\omega}_t$ is increasing in q_t . As a result, the expression in square brackets in (3.5) is a decreasing function of q_t . Thus, apart from functional form, (3.5) is similar to (5.6). If we hold a_t fixed (this could be accomplished through a system of taxes and transfers between households and entrepreneurs), then the financial frictions model is equivalent to an investment adjustment cost model.

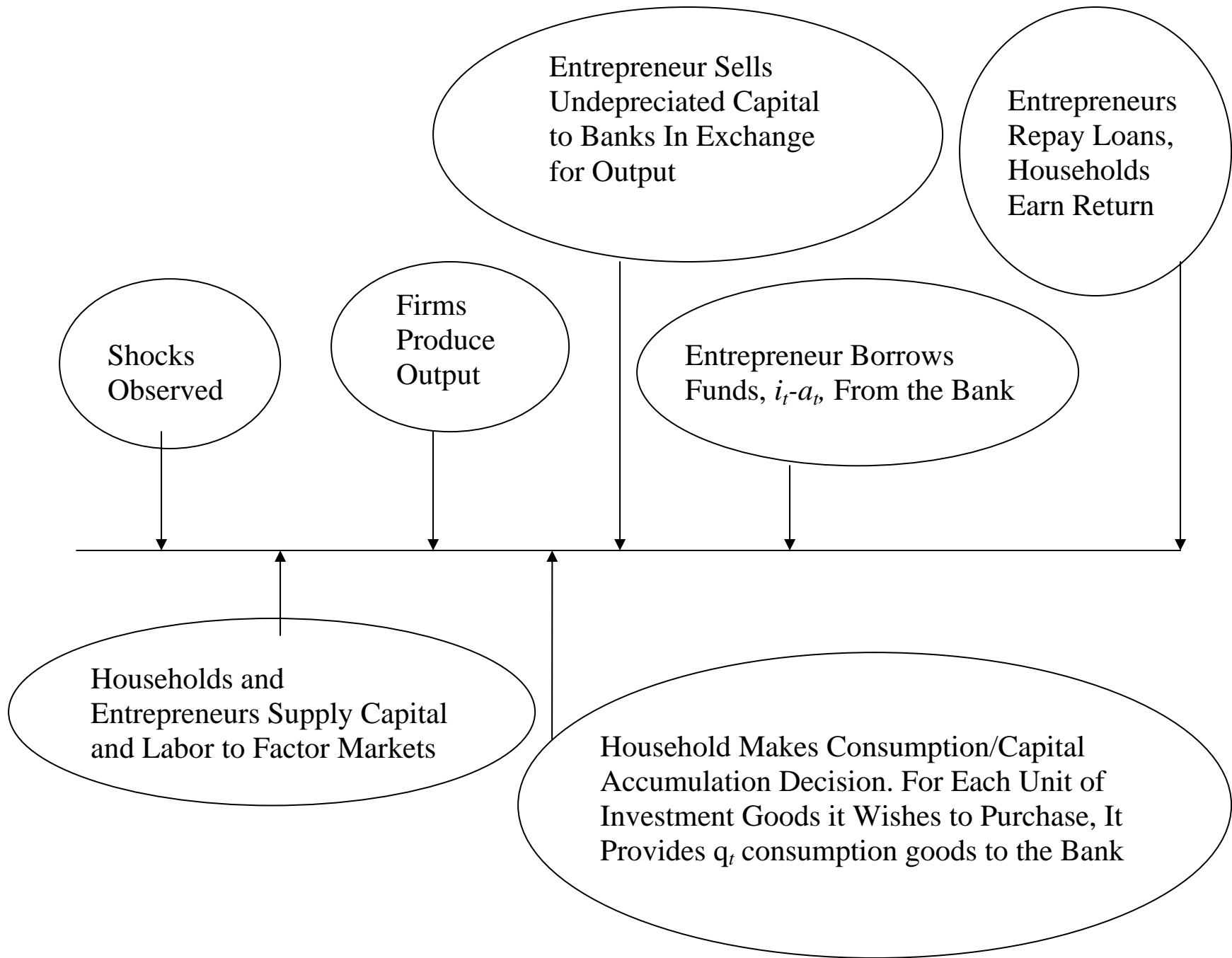
It is easy to obtain intuition about the relationship between the investment adjustment cost model and the financial frictions model. With a positive technology shock, the demand for capital by households increases, and we may suppose q_t rises as a result. This in turn makes $\bar{\omega}_t$ rise, so that $\Phi(\bar{\omega}_t)$, the bankruptcy rate, rises (presumably, this is an empirical weakness of the model). This all makes I_t rise. We may also suppose that with the positive technology shock, a_t increases, so that this gives an additional boost to I_t .

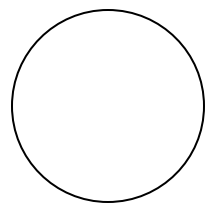
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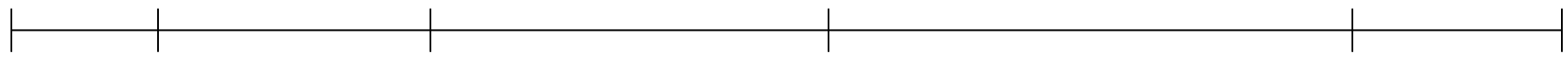
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Figure 1: Events During One Period In Carlstrom-Fuerst Model







t

$t+l$