

Homework #5  
Economics 411  
Due Thursday, October 27  
Christiano

1. Consider the neoclassical growth model studied in class, with  $\beta = 1/1.03$ ,  $\alpha = 1/3$ ,  $\delta = 0.10$ ,  $\gamma = 1$ , where preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \quad u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

and the aggregate resource constraint is given by:

$$c_t + k_{t+1} - (1 - \delta)k_t \leq k_t^\alpha.$$

- (a) What is the steady state value of  $k$ ?
- (b) Let  $k' = g(k)$  denote the policy rule in the recursive formulation of the model. Compute  $g'$ , the derivative of  $g$  at the steady state value of  $k$ .
- (c) According to the first order Taylor expansion of  $g$  about steady state,

$$\frac{k_t - k}{k_0 - k} = (g')^t,$$

where  $k$  denotes the steady state value of  $k$ ,  $k_0$  is the initial value of the capital stock, and  $k_t$  is the value of the capital stock in period  $t \geq 0$ . From the above expression, one can compute how much time,  $t$ , it takes to close, say, 95 percent of an initial gap,  $k_0 - k$ , between the initial capital stock and its steady state. That is, one can compute the value of  $t$  required for the value of  $k_t - k$  to be 5 percent of the value of  $k_0 - k$ . Compute how much time (i.e., the value of  $t$ ) is required to close 95 percent of a gap between an initial value of the capital stock,  $k_0$ , and the steady state value.

- (d) Recompute the time needed to close 95 percent of the gap when the value of  $\delta$  is changed to 0.99. Then, set  $\delta$  back to 0.10 and instead change the value of  $\gamma$  to 4. In each case, provide the economic intuition behind the change in the time needed to close 95 percent of the gap.

- (e) Consider the Solow model, which uses the resource constraint described above, but which assumes that people save and invest a fixed fraction,  $s$ , of gross output,  $k^\alpha$  :

$$k_{t+1} - (1 - \delta)k_t = sk_t^\alpha$$

What value of  $s$  is required in order for the steady states of the neoclassical and Solow models to coincide? How much time does it take for 95 percent of the gap between  $k_0$  and steady state capital to be closed in the Solow model? Provide economic intuition behind the different amounts of time required, in the Solow and neoclassical growth models, to close 95 percent of the gap.

2. Consider the handout on the course website, ‘Recursive Methods and Participation Constraints’. Do questions 1-3 at the end.