1. Consider the endogenous growth model discussed in class. One sector produces a homogeneous output good, which is transformed one-for-one into consumption and investment using a Cobb-Douglas production function:

\[ c_t + k_{t+1} - (1 - \delta) k_t = k_t^\alpha n_t^{1-\alpha}. \]

Another sector produces human capital according to the following accumulation equation:

\[ h_{t+1} = h_t + \lambda (h_t - n_t), \]

where \( \lambda > 0, c_t \geq 0, k_{t+1} \geq (1 - \delta) k_t, 0 \leq n_t \leq h_t, \) and \( h_0, k_0 \) are given. Preferences are:

\[ \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}, \]

\( \gamma > 0. \) To ensure boundedness, we require \( \beta (1 + \lambda)^{1-\gamma} < 1. \) In class, the problem was reformulated in recursive form. It was shown that there are policy rules of the form, \( x_{t+1} = f(x_t), y_t = g(x_t), \) where \( x_t = k_t/h_t \) and \( y_t = h_{t+1}/h_t. \)

- Set \( \alpha = 1/3, \delta = 0.10, \beta = 0.97, \lambda = 0.04, \gamma = 1.1. \) Compute steady state values of \( x, y. \) How do these values change with \( \alpha \) and with \( \lambda? \) Provide intuition.

- Develop a formula for the date \( t \) price (in consumption units) of a unit of human capital, \( h_{t+1}. \) Develop a formula for the period \( t+1 \) payoff associated with an extra unit of \( h_{t+1} \) (hint: the payoff is the maximal increase in consumption that is possible in period \( t+1, \) while leaving the consumption opportunities unchanged in periods \( t+2 \) and later).

- Suppose \( x_t \) is below its steady state, either because some physical capital has been destroyed, or because the general level of human
capital rose (say, because of immigration of high-human capital people). Is the price of human capital, \( h_{t+1} \), low or high? What about the period \( t+1 \) payoff of \( h_{t+1} \)? Provide economic intuition. Recall that the one-period rate of return on an asset is the period \( t+1 \) payoff divided by the period \( t \) price. Can you say what the rate of return on human capital is when \( x_t \) is low? Explain.

- Write down a set of functional equations that define the equilibrium policy functions, \( x_{t+1} = f(x_t) \), and \( y_t = g(x_t) \). Explain how you would use the perturbation method to develop Taylor series approximations to \( f \) and \( g \).

2. (Problem with incentive constraints). Consider the two-period version of the Atkeson model, which was discussed in recitation and appears in the first section of a handout on the website (see ‘Sudden Stop’). Atkeson shows that it is possible to have an equilibrium in which a country’s current account is positive when output is low, and negative when it is high. Adopt functional forms for the utility (constant elasticity seems like a good bet) and probability functions (the handout supplies a candidate). Also, you need to adopt numerical values for \( Y_H \) and \( Y_L \). Can you find a parameterization having the property that the current account is negative for \( Y = Y_H \) and negative for \( Y = Y_L \)?

3. Suppose the representative household has the following preferences:

\[
\sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \quad u(c, l) = \frac{[c(1-l)^{\eta}]^{1-\gamma}}{1-\gamma}, \quad 0 < \beta < 1.
\]

The technology for producing consumption goods is:

\[ c_t \leq A k_{ct}^{\alpha} l_{tt}^{1-\alpha}, \quad 0 < \alpha < 1 \]

The technology for producing investment goods is:

\[ I_t = bk_{it}, \quad b > 0, \]

while the technology for increasing capital is:

\[ k_{t+1} = (1-\delta)k_t + I_t, \quad 0 < \delta < 1. \]
The allocation of capital must satisfy:

\[ k_t = k_{ct} + k_{it}, \]

and \( k_0 \) is given. Also,

\[ b > \delta, \quad \beta(1 - \delta + b)^{\alpha(1 - \gamma)} < 1. \]

(a) Describe a decentralized, sequence of markets equilibrium for this economy and show that the allocations in that equilibrium coincide with the efficient allocations. Provide an explicit formula for the price of investment goods, \( P_{t,t} \), in the equilibrium. Here, \( P_{t,t} \) is the price in units of period \( t \) consumption goods, of \( I_t \).

(b) Provide a formula for the equilibrium growth rate of capital, \( \lambda_k = k_{t+1}/k_t \), in terms of the model parameter values.

(c) Let \( y_t \) denote aggregate output for this economy, measured in consumption units. Provide a formula, in terms of model parameters, for the equilibrium growth rate of output, \( \lambda_y = y_t/y_{t-1} \). Provide a formula for the growth rate of consumption, \( \lambda_c = c_t/c_{t-1} \) and the growth rate of the price of new capital goods, \( \lambda_{P'k'} = P'_{k',t}/P'_{k',t-1} \).

(d) Can you find values for the parameters that imply: the share of output paid to labor, \( w_t l_t / y_t \), is \( 2/3 \); the growth rate of output is \( 1.5 \) percent (i.e., \( \lambda_y = 1.015 \)); and \( \lambda_{P'k'} = 1 - .03 \) (i.e., \( P'_{k'} \) is falling at the rate of \( 3\% \) per year)?