Homework #7
Economics 411, Fall 2005
Due Thursday, November 10.
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1. Consider the endogenous growth model discussed in class. One sector produces a homogeneous output good, which is transformed one-for-one into consumption and investment using a Cobb-Douglas production function:
$$c_t + k_{t+1} - (1-\delta) k_t = k_t^{\alpha} n_t^{1-\alpha}.$$
Another sector produces human capital according to the following accumulation equation:
$$h_{t+1} = h_t + \lambda (h_t - n_t),$$
where $\lambda > 0$, $c_t \geq 0$, $k_{t+1} \geq (1-\delta) k_t$, $0 \leq n_t \leq h_t$, and h_0, k_0 are given. Preferences are:
$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma},$$
$\gamma > 0$. To ensure boundedness, we require $\beta (1+\lambda)^{1-\gamma} < 1$. In class, the problem was reformulated in recursive form. It was shown that there are policy rules of the form, $x_{t+1} = f(x_t)$, $y_t = g(x_t)$, where $x_t = k_t / h_t$ and $y_t = h_{t+1}/h_t$.

   - Set $\alpha = 1/3$, $\delta = 0.10$, $\beta = 0.97$, $\lambda = 0.04$, $\gamma = 1.1$. Compute steady state values of x, y. How do these values change with α and with λ? Provide intuition.
   - Develop a formula for the date t price (in consumption units) of a unit of human capital, h_{t+1}. Develop a formula for the period $t+1$ payoff associated with an extra unit of h_{t+1} (hint: the payoff is the maximal increase in consumption that is possible in period $t+1$, while leaving the consumption opportunities unchanged in periods $t+2$ and later).
   - Suppose x_t is below its steady state, either because some physical capital has been destroyed, or because the general level of human

capital rose (say, because of immigration of high-human capital people). Is the price of human capital, h_{t+1}, low or high? What about the period $t+1$ payoff of h_{t+1}? Provide economic intuition. Recall that the one-period rate of return on an asset is the period $t+1$ payoff divided by the period t price. Can you say what the rate of return on human capital is when x_t is low? Explain.

- Write down a set of functional equations that define the equilibrium policy functions, $x_{t+1} = f(x_t)$, and $y_t = g(x_t)$. Explain how you would use the perturbation method to develop Taylor series approximations to f and g.

2. (Problem with incentive constraints). Consider the two-period version of the Atkeson model, which was discussed in recitation and appears in the first section of a handout on the website (see 'Sudden Stop'). Atkeson shows that it is possible to have an equilibrium in which a country's current account is positive when output is low, and negative when it is high. Adopt functional forms for the utility (constant elasticity seems like a good bet) and probability functions (the handout supplies a candidate). Also, you need to adopt numerical values for Y^H and Y^L. Can you find a parameterization having the property that the current account is negative for $Y = Y^H$ and negative for $Y = Y^L$?

3. Suppose the representative household has the following preferences:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t),\ u(c,l) = \frac{\left[c(1-l)^\eta\right]^{1-\gamma}}{1-\gamma},\ 0 < \beta < 1.$$

The technology for producing consumption goods is:

$$c_t \leq A k_{ct}^{\alpha} l_t^{1-a},\ 0 < \alpha < 1$$

The technology for producing investment goods is:

$$I_t = b k_{it},\ b > 0,$$

while the technology for increasing capital is:

$$k_{t+1} = (1-\delta) k_t + I_t,\ 0 < \delta < 1.$$

The allocation of capital must satisfy:

$$k_t = k_{ct} + k_{it},$$

and k_0 is given. Also,

$$b > \delta, \;\; \beta(1 - \delta + b)^{\alpha(1-\gamma)} < 1.$$

(a) Describe a decentralized, sequence of markets equilibrium for this economy and show that the allocations in that equilibrium coincide with the efficient allocations. Provide an explicit formula for the price of investment goods, $P_{I,t}$, in the equilibrium. Here, $P_{I,t}$ is the price in units of period t consumption goods, of I_t.

(b) Provide a formula for the equilibrium growth rate of capital, $\lambda_k = k_{t+1}/k_t$, in terms of the model paramter values.

(c) Let y_t denote aggregate output for this economy, measured in consumption units. Provide a formula, in terms of model parameters, for the equilibrium growth rate of output, $\lambda_y = y_t/y_{t-1}$. Provide a formula for the growth rate of consumption, $\lambda_c = c_t/c_{t-1}$ and the growth rate of the price of new capital goods, $\lambda_{P_{k'}} = P_{k',t}/P_{k',t-1}$.

(d) Can you find values for the parameters that imply: the share of output paid to labor, $w_t l_t/y_t$, is 2/3; the growth rate of output is 1.5 percent (i.e., $\lambda_y = 1.015$); and $\lambda_{P_{k'}} = 1 - .03$ (i.e., $P_{k'}$ is falling at the rate of 3% per year)?