Homework #9
Economics 411, Fall 2005
Due Friday, November 25.
Christiano

1. Consider a model in which a final good is produced using intermediate goods. The final good, y, is produced by a competitive, representative firm using the following homogeneous technology:

$$y = \exp \int_0^1 [\log y_j]\, dj.$$

The firm maximizes profits:

$$y - \int_0^1 p_j y_j dj,$$

taking p_j as given. Here, the price of the final good has been normalized at unity. The j^{th} intermediate good is produced by a monopolist using the following technology:

$$\begin{aligned} y_j &= \begin{cases} f(k_j, l_j) - \phi & f(k_j, l_j) \geq \phi \\ 0 & \text{otherwise} \end{cases}, \qquad (1) \\ f(k_j, l_j) &= k_j^{\alpha} l_j^{1-\alpha},\ 0 < \alpha < 1. \end{aligned}$$

Thus, if the monopolist is to sell y_j units of goods, they must produce the fixed quantity, ϕ, first. The monopolist is competitive in the market for labor and capital and takes the rental rate on capital, r, and the wage rate, w, as given.

   (a) Derive the demand curve for the j^{th} intermediate good. Consider the profit maximization problem of the j^{th} intermediate good firm. Show that it has no solution. That is, for any finite price-quantity pair on the demand curve, profits are always increased by increasing the price level.

   (b) Suppose that there are other potential entrants into the production of the j^{th} intermediate good, and that they have access to the same technology, (1). Explain why this implies that the profits of the intermediate good producer must be zero.

(c) Show that cost minimization by the j^{th} intermediate good producer, linear homogeneity of f, and the zero profit condition imply that output can be written

$$y_j = \frac{1}{\mu_j} f(k_j, l_j),$$

where μ_j is the firm markup, the ratio of price to marginal cost, λ_j:

$$\mu_j = \frac{p_j}{\lambda_j}.$$

(d) Show that the zero profit condition implies the markup must fall when the firm produces more output. Provide the intuition for this result.

(e) Explain why it is that in equilibrium, final output has the following representation:

$$y = \frac{1}{\mu} f(k, l),$$

where l is household labor supply, k is the supply of capital by households, and μ is the markup. Conclude that this constitutes a 'theory of TFP'.

2. Consider an economy in which households seek to maximize utility of consumption, C_t :

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t), \ \frac{C^{1-\sigma}}{1-\sigma}. \tag{2}$$

The resource constraint is:

$$C_t + I_t \leq Y_t, \tag{3}$$

and the production function is:

$$Y_t = \varepsilon_t \left(u_t k_t\right)^{\alpha}, \tag{4}$$

where ε_t is a technology shock. Also, u_t denotes the rate at which capital is utilized. Investment is composed of two parts:

$$I_t = I_t^k + I_t^u \tag{5}$$

The component, I_t^k, is used to increase the quantity of physical capital:

$$k_{t+1} = (1-\delta)k_t + I_t^k, \tag{6}$$

where $\delta \in (0,1)$ is the rate of depreciation on physical capital. The component of investment, I_t^u, reflects maintenance expenditures that arise as capital is utilized more intensely:

$$I_t^u = a\left(u_t\right)k_t. \tag{7}$$

Here, the level of the function, a, is zero in steady state, and a', a'' are both positive. It is convenient to restrict the functional form of a so that in a steady state, $u_t = 1$. The following functional form is useful:

$$a(u) = 0.5b\sigma_a u^2 + b(1-\sigma_a)u + b((\sigma_a/2) - 1), \tag{8}$$

where σ_a is a parameter that controls the curvature of a at steady state, $u_t = 1$ (curvature is defined as $a''(u)u/a'(u)$). Also, b, is a function of the other parameters of the model, and it must be chosen so that $u = 1$ in a steady state.

To complete the description of the model, we have to say something about the stochastic process governing ε_t. Let ε denote the unconditional mean of ε_t, so that $\varepsilon = E\varepsilon_t$. Define $\hat{\varepsilon}_t = (\varepsilon_t - \varepsilon)/\varepsilon$. Then, we suppose:

$$\hat{\varepsilon}_t = \rho\hat{\varepsilon}_{t-1} + e_t,$$

where e_t is a white noise process. Note that $E\hat{\varepsilon}_t = 0$, which is consistent with our assumption, $\varepsilon = E\varepsilon_t$.

(a) A baseline set of parameter values for the model is, $\sigma = 1$ (i.e., the log utility case), $\delta = 0.02$, $\alpha = 0.36$, $\rho = 0.95$, $\varepsilon = 1$, $\sigma_a = 0.1$, $\beta = 1.03^{-0.25}$. Compute the nonstochastic steady state stock of capital. What is the value of b?

(b) Show that the steady state is invariant to the value of σ_a.

(c) Consider the first order condition associated with the choice of u_t. Express it in the form of "the marginal benefit of a marginal increase in u_t equals the associated marginal cost". Show that as $u_t \to 0$ the marginal benefit of capital utilization goes to infinity, as $u_t \to \infty$ the marginal benefit goes to zero, and marginal benefits are strictly decreasing for all $u > 0$. Note that the marginal cost is strictly increasing, finite for $u = 0$ and positive for u sufficiently large. From this, you can conclude that for each k_t, ε_t there is a unique u_t where marginal benefit and marginal cost intersect. Show that as ε_t increases the optimal choice of u_t increases. As k_t increases, the optimal choice of u_t *decreases*. Can you provide intuition for this result?

(d) The rate of return on capital, R_t^k, is the marginal product of capital, plus what is left over next period after depreciation and maintenance expenses:

$$R_t^k \equiv MP_{k,t} + 1 - \delta - a(u_t).$$

Here, $MP_{k,t}$ is the marginal product of capital. Given our specification of technology, this is $\alpha \varepsilon_t u_t^\alpha k_t^{\alpha-1}$. The intertemporal Euler equation is:

$$u'(C_t) = E_t \beta u'(C_{t+1}) R_{t+1}^k.$$

Show that a higher value of u_t leads to a *fall* in the rate of return on investment, $MP_{k,t} + 1 - \delta - a(u_t)$. Conclude that if something causes utilization to rise, the rise in utilization per se would reduce the incentive to invest.

(e) Consider the linearized policy rule for capital in this economy. Show that the policy rule is identical to the linearized policy rule in our standard neoclassical growth model in which $u_t = 1$ always, in which α is greater than 0.36. Provide intuition for this result.

3. Consider an economy in which final output is produced by a perfectly competitive firm, which uses intermediate inputs, Y_{it}, $i \in (0, 1)$:

$$Y_t = \left[\int_0^1 Y_{it}^\rho di \right]^{\frac{1}{\rho}}, \ 0 < \rho \leq 1.$$

The price of the i^{th} input is p_{it}, and the output price is p_t. The firm's problem is to maximize profits:

$$p_t Y_t - \int_0^1 p_{it} Y_{it} di,$$

taking all prices parametrically. This leads to the following first order condition:

$$Y_{it} = Y_t \left(\frac{p_t}{p_{it}} \right)^{\frac{1}{1-\rho}}, \ i \in (0,1).$$

Substituting this back into the final goods production function:

$$p_t = \left[\int_0^1 p_{it}^{\frac{\rho}{\rho-1}} di \right]^{\frac{\rho-1}{\rho}}$$

Each intermediate good is produced by a single producer, who sets price equal to marginal cost because of the existence of a competitive fringe. Any intermediate good firm that attempted to set a higher price would be bumped out of the market. Each intermediate good firm has a linear production function in labor, with marginal productivity equal to unity. What differentiates the intermediate good firms is that those with $i \in (0, \alpha)$ must borrow the wage bill in advance at gross rate of interest, R_t, while the rest can finance the wage bill out of receipts. Those firms have no financing requirements. As a result, the marginal cost of a unit of labor for firms, $i \in (0, \alpha)$ is $w_t R_t$ and the marginal cost of a unit of labor is w_t for the rest.

You should take the aggregate supply of labor by households, L, as a given number.

(a) Derive an expression for the output of final goods that has the following form:

$$Y = \phi(R)L,$$

provide a simple, closed form expression for $\phi(R)$. Show that $\phi(1) = 1$, $\phi'(1) = 0$. Evidently, the heterogeneous borrowing requirements of different agents has the potential to supply a theory of TFP.

(b) Consider a jump in the interest rate from $R = 1.05$ to 1.10. Is there a value of α or ρ that will associate this jump in R with something like a 10 percent drop in efficiency?

4. Consider a model in which utility is a function not just of market consumption, c, and market labor effort, l, but also of consumption

of home produced goods or services, c_n, and home labor effort, l_n. Specifically,

$$\log(c + c_n) - \gamma \log\left(\frac{l^{1+\psi}}{1+\psi} + l_n\right),$$

where $\gamma, \psi > 0$. The home labor effort yields services via the home production function, $c_n = \psi_0 l_n$. Show that this formulation implies a utility function in terms of market goods and labor having the following form:

$$\text{constant} + a \log\left(c - \psi_0 \frac{l^{1+\psi}}{1+\psi}\right),$$

where ‘constant’ and a are parameters. (Hint: recall how we got $F(k, k')$ for the version of the growth model in which utility is a function of labor effort, in addition to consumption.)