

Sudden Stop

In the past two decades several economies have experienced a ‘Sudden Stop’: over a brief period of time, output collapsed and the current account shifted sharply from negative to positive. The observation that these countries exported resources when they hit on hard times seems inconsistent with the notion that international financial markets are efficient. Efficiency, one might imagine, dictates that resources should flow *towards* countries in trouble, and away from countries that are doing fine. It is possible that Sudden Stop is a symptom of financial market inefficiencies. However, a 1991 *Econometrica* paper by Andrew Atkeson raises the possibility that the puzzling Sudden Stop observations may in fact be part of efficient mechanism after all.¹ This note explains the argument in a two period version of Atkeson’s model. A second model is presented which captures the same idea in a different way.

The basic argument is simple. In the model, countries borrow internationally in order to finance investment activities. Investment raises the probability that a high output state will occur in the future. A key feature of the environment is that investment is not observable to foreign lenders. Still, international financial arrangements work best if there is substantial investment, and so loan contracts must be structured to give countries a substantial incentive to invest. Since the effect of investment is to increase the likelihood of high output states, to give countries a strong incentive to invest requires that the level of consumption in the high output state is higher than what it is in the low output state. A point of Atkeson’s paper is that consumption might be so high in the high output state that it exceeds output itself in that state. Similarly, consumption may be so low in the low output state that it is less than output in that state. In this case, we have a current account surplus for countries in bad times (the low output state) and a current account deficit for countries in good times.

¹There is a version of a real business cycle model that may be able to capture the phenomenon. Suppose the drop in output is due to a bad shock to technology that is persistent. It makes sense for the citizens of that country to reduce domestic investment and generate a current account surplus, which corresponds to accumulating financial claims on investment projects in the rest of the world. A drop in domestic employment may be efficient under these circumstances since with the drop in investment there is less to do. Moreover, consumption might optimally fall too if the marginal utility of consumption falls with a rise in leisure.

The first model in these notes is a two period version of Atkeson's model (you may find it helpful to read about the model in Atkeson's paper, or in Chapter 15 of the Ljungqvist-Sargent textbook). A shortcoming of the two period version of the Atkeson model is that it seems impossible to make it dynamic without going to the full technical apparatus that he describes in his paper. For this reason, a second model is presented. That model will also be studied to determine if the optimal contract displays the Sudden Stop characteristics. An advantage of this model, is that it can be made dynamic simply by repeating it. The reason is that in the one-period contract version of the model, there are no physical state variables. The repeated version of the model presumably has more equilibria than simply the repetition of the equilibria in the one period version. But, that is not of interest at the moment.

Finally, a warning. These notes have been written up very quickly. They may not be error-free!

1. A one-period version of Atkeson's Model

Preferences of the typical borrower are:

$$u(c) + \beta P(I)u(c^H) + \beta(1 - P(I))u(c^L),$$

where H is the high state and L is the low state. Also, $P(I)$ is the probability of the high state, and I denotes the level of investment by the agent. This function is assumed to have the properties, $P(0) = 0$, $P'(I) > 0$, $P(\infty) = 1$, and $P''(I) < 0$. An example of this is:

$$P(I) = \frac{2}{1 + \exp(-aI)} - 1, \quad a > 0.$$

The budget constraint in period 0 is:

$$c + I \leq b,$$

where b denotes period 0 borrowing. The budget constraint in the next period, assuming the high state, is:

$$c^H \leq Y^H - d^H,$$

where d^H denotes the payment in the high state, and Y^H denotes output in the high state. Similarly,

$$c^L \leq Y^L - d^L.$$

There is a lender who has access to funds at the rate of interest, $1 + r$. The zero profit condition for the lender is:

$$b(1 + r) = P(I)d^H + (1 - P(I))d^L. \quad (1.1)$$

An optimal borrowing contract is a (b, d^H, d^L, I) that solves:

$$\begin{aligned} \max_{b, d^H, d^L, I} & u(b - I) + \beta P(I)u(Y^H - d^H) + \beta(1 - P(I))u(Y^L - d^L) \\ & + \lambda [P(I)d^H + (1 - P(I))d^L - b(1 + r)], \end{aligned} \quad (1.2)$$

where λ is the Lagrange multiplier.

Four equations for determining the five variables (including λ) are:

$$\begin{aligned} b & : & u'(b - I) &= \lambda(1 + r) \\ I & : & u'(b - I) &= \beta P'(I) [u(Y^H - d^H) - u(Y^L - d^L)] + \lambda P'(I) [d^H - d^L] \\ d^H & : & \beta u'(Y^H - d^H) &= \lambda \\ d^L & : & \beta u'(Y^L - d^L) &= \lambda \\ P(I)d^H + (1 - P(I))d^L &= & b(1 + r) \end{aligned}$$

From the d^H and d^L equations, we conclude $c^H = c^L = c^*$, which implies:

$$Y^H - Y^L = d^H - d^L. \quad (1.3)$$

Substituting into the I equation:

$$1 + r = P'(I) [d^H - d^L] = P'(I) [Y^H - Y^L],$$

so that

$$P'(I) = \frac{1 + r}{Y^H - Y^L},$$

which pins down I . So, it remains to find b , d^H and d^L . For this, we use (1.3), the zero profit condition and

$$u'(b - I) = \beta u'(Y^H - d^H)P'(I) [d^H - d^L].$$

Note that with the efficient, full information contract, we get perfect consumption smoothing. So, the current account, $Y - c^*$, is positive when Y is high, and negative when it is low.

Suppose now that I and c are not observed in the country. In this case, the full information contract is not feasible. Everyone would consume the full amount of their loan and set investment to zero. With investment equal to zero, in the second period output is low with probability one. Everyone would then collect the payment that would allow them to consume c^* . With everyone collecting and no one paying, the bank would not be able to meet its budget constraint, (1.1). A loan contract now has to be structured to give the household a sufficient incentive to invest.

The problem now is the same as (1.2), with the additional incentive requirement. The incentive constraint is that whatever I is envisioned in the contract should be a level of investment that the agent willingly chooses. That is, given b , I must satisfy:

$$u'(b - I) = \beta P'(I) [u(Y^H - d^H) - u(Y^L - d^L)]. \quad (1.4)$$

From this expression it is evident for investment to be positive, it is necessary to have $Y^H - d^H > Y^L - d^L$. That is, consumption in the high state must be lower than consumption in the low state.

The b , I , d^H and d^L that constitute the optimal contract must optimize household utility subject to (1.1) and (1.4). The Lagrangian formulation of the problem is:

$$\begin{aligned} \max_{b, d^H, d^L, I} & u(b - I) + \beta P(I)u(Y^H - d^H) + \beta(1 - P(I))u(Y^L - d^L) \quad (1.5) \\ & + \lambda [P(I)d^H + (1 - P(I))d^L - b(1 + r)] \\ & + \mu [u'(b - I) - \beta u'(Y^H - d^H)P'(I)(d^H - d^L)] \end{aligned}$$

There are four first order conditions associated with this problem, plus (1.1) and (1.4). These six equations can be used to determine the unknowns, b , d^H , d^L , I , as well as the two multipliers, λ and μ . Another computational strategy would fix b and d^H , and then compute I and d^L to satisfy (1.1) and (1.4). This makes I and d^L functions of b and d^H . Given this function, the utility of the household may be viewed as being a function only of b and d^H . It should be possible to maximize household utility as a function of b and d^H .

By inspecting this two-period problem, it is possible to see why computing an equilibrium for a multiperiod version of the model is hard. In the incentive compatibility constraint, one needs the utility of the household in the next period. In the present example, this utility is just next period's utility function with next

period's debt set to zero. In a multiperiod version of the model, the utility function would have to be replaced by the value function.

As noted after (1.4), to provide strong incentives to invest requires that $Y^H - d^H$ be substantially larger than $Y^L - d^L$. This could be achieved by setting $d^H < 0$, and $d^L > 0$. But, d^H is the current account surplus in the high output state and d^L is the current account surplus in the low output state (note, $Y^H - c^H = d^H$).

2. Another Model

Here is another model of international borrowing. Let the utility function of domestic residents be:

$$u(c, l) = c - \frac{1}{2}l^2,$$

with budget constraint:

$$c \leq (1 - \tau)wl + T,$$

where w denotes the wage rate, T denotes a transfer from the government, τ denotes the labor tax rate, l denotes labor effort, and c denotes consumption. For given T and τ , the household's optimum problem is:

$$\max_l (1 - \tau)wl + T - \frac{1}{2}l^2,$$

which has the following solution:

$$\begin{aligned} (1 - \tau)w &= l \\ c &= [(1 - \tau)w]^2 + T. \end{aligned}$$

The production function is:

$$y = zl,$$

where $k = 1$ is fixed. Then, $w = z$ where l is aggregate employment. The government is benevolent, meaning that its utility corresponds to the utility of the private agents in the economy. It is convenient to express the government's 'utility function' as a function of the government's own actions. We do this by replacing consumption and labor by the private sector allocation rules that characterize a private sector equilibrium:

$$\begin{aligned} u(\tau, T) &= [(1 - \tau)z]^2 + T - \frac{1}{2}[(1 - \tau)z]^2 \\ &= \frac{1}{2}[(1 - \tau)z]^2 + T. \end{aligned}$$

Also, tax revenues to the government are given by:

$$\tau wl = \tau(1 - \tau)z^2.$$

Note that this is a unimodal Laffer curve, with maximum at $\tau = 1/2$, where revenues are $1/4z^2$.

The timing in the model works like this. At the beginning of the period, the government borrows b on the international financial market. It splits b between g and T , subject to:

$$g + T \leq b.$$

Then, the value of z is realized. The variable, z , is random with $z \in (z^l, z^h)$, where $z = z^h$ with probability $p(g)$ and $z = z^l$ with the complementary probability, $1 - p(g)$. Suppose that p is concave, with $p \rightarrow 1$ as $g \rightarrow \infty$ and $p \rightarrow 0$ as $g \rightarrow 0$. A parametric form with this property is:

$$p(g) = \frac{2}{1 + \exp(-g)} - 1.$$

In this setup, g is a form of investment that does not directly give rise to utility. It only gives rise to utility indirectly by increasing the likelihood that a high value of domestic productivity will occur. At the same time, T does directly raise utility.

At this point, the government chooses the labor tax rate, and after this a private sector equilibrium occurs. Finally, at the end of the period, the government pays off its international debt. This gives rise to another restriction:

$$d \leq \tau(1 - \tau)z^2.$$

When g, T are not separately observed, there is a moral hazard problem, just like in the previous example. The government may have an incentive to borrow b , put all the proceeds into T , so that $z = z^l$ with probability one. The government could then claim that $z = z^l$ because of ‘bad luck’, and collect a high value of d^l . Whether these incentives are in fact present in some way in the model requires a closer analysis of the model.

A useful benchmark case occurs when z, g and T are all separately observed to everyone. Again, imagine a large economy in which $p(g)$ countries experience z^h and $1 - p(g)$ countries experience z^l . The lender’s zero profit condition is:

$$b(1 + r) = p(g)d^H + (1 - p(g))d^L, \tag{2.1}$$

where d^h is the amount that countries which experience z^h pay back, and d^l is the amount that countries experiencing z^l pay back.

The optimal contract, b, d^h, d^l, g maximizes expected utility of a country, subject to the zero profit condition. To compute this, we first solve the problem of a government, conditional on given values of b, d^h, d^l, g . Thus, the problem is:

$$\max_{\tau^h, \tau^l, T} \left\{ p(g) \left[\frac{1}{2} [(1 - \tau^H)z^H]^2 \right] + (1 - p(g)) \left[\frac{1}{2} [(1 - \tau^l)z^l]^2 \right] + T \right\},$$

subject to

$$g + T = b,$$

and

$$d^h = \tau^h(1 - \tau^h)(z^h)^2, \quad d^l = \tau^l(1 - \tau^l)(z^l)^2.$$

In Lagrangian form:

$$\begin{aligned} & \max_{\tau^h, \tau^l} \left\{ p(g) \left[\frac{1}{2} [(1 - \tau^H)z^H]^2 \right] + (1 - p(g)) \left[\frac{1}{2} [(1 - \tau^l)z^l]^2 \right] + b - g \right\} \\ & + \lambda^h \left[\tau^h(1 - \tau^h)(z^h)^2 - d^h \right] + \lambda^l \left[\tau^l(1 - \tau^l)(z^l)^2 - d^l \right]. \end{aligned}$$

The optimum will involve setting the constraints equal to zero (no point in raising more revenues than needed!) and will be the lowest tax rate that does this. Thus, τ^h and τ^l are the smaller of the two zeros of each of the two relevant second order polynomials. Denote the solutions by

$$\tau^h = \tau(z^h, d^h), \quad \tau^l = \tau(z^l, d^l).$$

For the problem to be interesting, we require:

$$d^h \leq \frac{1}{4}(z^h)^2, \quad d^l \leq \frac{1}{4}(z^l)^2.$$

This can be substituted into the objective of the government, to yield:

$$p(g) \left[\frac{1}{2} [(1 - \tau(z^h, d^h))z^H]^2 \right] + (1 - p(g)) \left[\frac{1}{2} [(1 - \tau(z^l, d^l))z^l]^2 \right] + b - g. \quad (2.2)$$

The optimal contract is found by maximizing (2.2) subject to (2.1), with respect to b, d^h, d^l, g .

We next ask whether this is in fact an equilibrium, when g, T, z are *not* separately observable. This requires that the values of g and T that emerge

from the solution to the above full information problem also solve the limited information problem. In particular, for this to be the case, we require that the optimal choice of g , denoted \tilde{g} , by the government coincide with the value of g implied by the optimal contract. Using the same reasoning applied in the model of the previous section, this corresponds to the requirement that the g in the optimal contract satisfy:

$$p'(g) \left[\frac{1}{2} \left[(1 - \tau(z^h, d^h)) z^H \right]^2 - \frac{1}{2} \left[(1 - \tau(z^l, d^l)) z^l \right]^2 \right] = 1. \quad (2.3)$$

This cannot occur, because when the government chooses g , it ignores the zero profit condition. So, the limited information, incentive-compatible contract is computed as the solution to the problem solved in the full information model, with (2.3) added as an extra (binding) restriction.