Homework #7
Economics 411, Winter 2005
Due Thursday, February 24.
Christiano

1. Suppose household preferences are as follows:

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\mu\left(s^t\right)u(c\left(s^t\right),n\left(s^t\right)),$$

where u has all the usual properties. The budget constraint is:

$$c\left(s^t\right)+k\left(s^t\right)-(1-\delta)\,k\left(s^{t-1}\right)\leq w\left(s^t\right)n\left(s^t\right)+r\left(s^t\right)k\left(s^{t-1}\right),$$

where $w\left(s^t\right)>0$ is the wage rate and $r\left(s^t\right)>0$ is the rental rate on capital, $k\left(s^{t-1}\right)$. In addition, the household must satisfy $c\left(s^t\right),\ k\left(s^t\right)\geq 0$, $0\leq n\left(s^t\right)\leq 1$. Prove that the inter- and intratemporal Euler equations, and the transversality condition displayed in class are sufficient for household optimization. (Hint: use the strategy of Theorem 4.15, though note that the setting here is somewhat different because of the presence of a budget constraint with arbitrary prices rather than a resource constraint.)

2. One of the pluses of the Real Business cycle model is that it predicts that the Solow residual, $y/(k^{s_k}l^{s_l})$, is procyclical, where s_k and s_l are, respectively, the share of income earned by capital and labor. In addition, k and l are the aggregate stocks of capital and labor and y is aggregate output. Evaluate the ability of the Christiano-Harrison model to account for the procylicality of the Solow residual.

3. Suppose households are as in the previous question. Output in the firm sector has the Dixit-Stiglitz- like structure. Final good firms produce output according to:

$$y=\exp\left[\int_0^1\log\left(x(i)\right)di\right]$$

The final good firm maximizes profits:

$$y-\int_0^1 p_jy_jdj.$$

The i^{th} intermediate good is produced by a monopolist using the following production function:

$$x(i) = \begin{cases} k(i)^a n(i)^{1-\alpha} - \phi, \text{ if } k(i)^a n(i)^{1-\alpha} \geq \phi \\ 0, \text{ otherwise} \end{cases},$$

where $\phi > 0$ is a fixed cost of production. Thus, if the monopolist is to sell $x(i) > 0$ units of goods, they must produce the fixed quantity, ϕ, first. The monopolist is competitive in the market for labor and capital and takes the rental rate on capital, r, and the wage rate, w, as given. The monopolist producing good $x(i)$ must make zero profits because there are potential entrants into the production of $x(i)$. They would enter and undercut any monopolist who attempted to make positive profits (in this respect, this is like the Romer model). The monopolist is competitive in factor markets and takes the rental rate of capital, r, and the wage rate, w, as given.

(a) Derive the demand curve for the j^{th} intermediate good. Consider the profit maximization problem of the j^{th} intermediate good producing monopolist. Suppose that there are *no* potential entrants into the production of the j^{th} intermediate good. Show that the profit maximization problem has no solution. For any finite price-quantity pair on the demand curve, profits are always increased by increasing the price level.

(b) Show that cost minimization by the j^{th} intermediate good producer, linear homogeneity of f, and the zero profit condition imply that output can be written

$$y_j = \frac{1}{\lambda_j} f(k_j, l_j),$$

where λ_j is the firm markup of price over marginal cost.

(c) Show that the zero profit condition implies the markup must fall when the firm produces more output. Provide the intuition for this result.

(d) Explain why it is that in equilibrium, final output has the following representation:

$$y = \frac{1}{\lambda} f(k, l),$$

where l is household labor supply, k is the supply of capital by households, and μ is the markup. Does this model have the potential to account for the procyclical Solow residual?

(e) Let r_t and w_t denote the equilibrium rental rate of capital and wage rate, respectively.

i. Show that:

$$r_t = \alpha \frac{k_t^a n_t^{1-\alpha} - \phi}{k_t}, \; w_t = (1-\alpha) \frac{k_t^a n_t^{1-\alpha} - \phi}{n_t},$$

where k and n denote the economy-wide average stock of capital and labor, respectively.

ii. Recall the definition of strategic complementarity given in class, 'if everyone else is more active, then the return to any given agent of being more active is increased.' Explain why it is that in this example, when $\phi = 0$ there is no strategic complementarity, but when $\phi > 0$ there may be. (Hint: make use of the household's optimality conditions which require (i) the date t household investment decision equates the household's intertemporal marginal rate of substitution in consumption from t to $t+1$ with r_{t+1}[1] and (ii) the date t employment decision equates the household's marginal rate of subsitution between consumption and leisure to w_t.)

iii. Explain why the same strategic complementarity may emerge in the Christiano-Harrison model, when $\gamma > 0$, but cannot occur when $\gamma = 0$.

iv. Recall the John Bryant example, in which the presence of strategic complementarity implies a continuum of equilibria. Recall that preferences matter in that example too. Not *all* beliefs about the level of activity of everyone else can be an equilibrium. Based on the above example, explain how increased curvature in household utility reduces the chances of equilibrium multiplicity.

[1] Actually,

$$E_t m_{t,t+1} r_{t+1} = 1,$$

where $m_{t,t+1}$ is the marginal rate of subsitution in consumption from t to $t+1$.

4. (You will need MATLAB for this question.) Consider the deterministic version of the model in class with an externality. Preferences have the following form:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t), \beta = 1.03^{-.25}, \text{ and } u(c_t, n_t) = \log(c_t) + \psi \log(1 - n_t).$$

The household budget constraint is:

$$c_t + k_{t+1} - (1 - \delta)k_t = r_t k_t + w_t n_t, \ \delta = .02,$$

where r_t, w_t denote the rental rate on capital and the wage rate, respectively. Firms operate the following technology:

$$y_t = A_t k_t^{\alpha} n_t^{1-\alpha}, \ \alpha = 0.33.$$

Here,

$$A_t = Y_t^{\gamma},$$

where Y_t denotes the economy-wide level of output, in per capita terms. Firms are competitive, and maximize profits, which are zero in equilibrium.

(a) Derive the household static and dynamic Euler equations. Derive the firm Euler equation.

(b) In the intertemporal and intratemporal household Euler equations, substitute out the rental rate on capital and the wage rate for labor, using the firm first order conditions

$$\begin{aligned} \text{`intra'} &: \quad v_h(n_t, k_t, k_{t+1}) = 0 \\ \text{`inter'} &: \quad v_k(k_t, k_{t+1}, k_{t+2}, n_t, n_{t+1}) = 0. \end{aligned}$$

Linearize these equations about the steady state values of employment and the firm's stock of capital. In computing the steady states, fix steady state employment at 1/3 and choose the value of ψ that rationalizes that. Initially, fix $\gamma = 0$.

(c) Substitute out for labor in the dynamic Euler equation, using the static Euler equation. This will give you one (linear) dynamic Euler equation in k_t, k_{t+1}, k_{t+2},

$$\begin{aligned} 0 &= v(k_t, k_{t+1}, k_{t+2}) \\ &= v_0 \tilde{k}_t + v_1 \tilde{k}_{t+1} + v_2 \tilde{k}_{t+2}, \end{aligned}$$

where $\tilde{k}_t = k_t - k_t^*$. Find two values of λ, say λ_1 and λ_2, such that $\tilde{k}_{t+1} = \lambda_i \tilde{k}_t$ solve this equation. (These two values of λ correspond to two values of g', the derivative of the equilibrium policy rule, $k_{t+1} = g(k_t)$, obtained by the first order perturbation method discussed in class.) For $\gamma = 0$, this is just a standard real business cycle model, so one of the two values of λ is inside the unit circle and the other is outside.

(d) Show that as γ is increased, both roots move inside the unit circle. Explain why *both* $\tilde{k}_{t+1} = \lambda_1 \tilde{k}_t$ and $\tilde{k}_{t+1} = \lambda_2 \tilde{k}_t$ correspond to equilibria of the original unlinearized economy, for $\tilde{k}_0$ close to zero.

(e) Note that there are more than two solutions to $v(k_t, k_{t+1}, k_{t+2}) = 0$. There is a *continuum* of solutions, each having the property that $\tilde{k}_t \to 0$:

$$\tilde{k}_t = \left(\tilde{k}_0 - a\right) \lambda_1^t + a \lambda_2^t,$$

where $\tilde{k}_0$ is the initial deviation from k^* and a is arbitrary. Explain why we have now identified a continuum of equilibria for the original economy, for $\tilde{k}_0$ close enough to 0. Note that these equilibria exist because γ was made large enough, i.e., strategic complementarities are large enough.