Homework #8
Economics 411, Winter 2005
Due Friday, March 4.
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1. Consider the model in the last question of homework #7, parameterized so that the steady state is indeterminate. Suppose that the allocations and prices in the equilibrium of the model are random, even though there are no disturbances in the economy that affect preferences or technology. Then, the intertemporal Euler equation of the household is:

$$\sum_{s_{t+1}} \mu\left(s^{t+1}|s^t\right)\left\{u_c\left(s^t\right) - \beta u_c\left(s^{t+1}\right)\left[r\left(s^{t+1}\right) + (1-\delta)\right]\right\} = 0,$$

where s_t is the time t exogenous uncertainty, s^t is the realized history of that uncertainty, and $\mu\left(s^{t+1}|s^t\right)$ is the conditional probability of history s^{t+1} given s^t. Also, $r\left(s^{t+1}\right)$ is the rental rate of capital in history s^{t+1}. Note that this Euler equation can equivalently be written

$$u_c\left(s^t\right) - \beta u_c\left(s^{t+1}\right)\left[r\left(s^{t+1}\right) + (1-\delta)\right] = \omega\left(s^{t+1}\right),$$

where $\omega\left(s^{t+1}\right)$ is a random variable that satisfies:

$$\sum_{s_{t+1}} \mu\left(s^{t+1}|s^t\right)\omega\left(s^{t+1}\right) = 0$$

   (a) Do the relevant substitutions and show that the Euler equations that must be satisfied in equilibrium are:

$$\begin{array}{lll} \text{'intra'} & : & v_h(n_t, k_t, k_{t+1}) = 0 \\ \text{'inter'} & : & v_k(k_t, k_{t+1}, k_{t+2}, n_t, n_{t+1}) = u_{t+1}. \end{array}$$

   where u_{t+1} has the property that $E_t u_{t+1} = 0$.

   (b) Linearize v_h and v_k about steady state, substitute out for employment to obtain

$$v_0\tilde{k}_t + v_1\tilde{k}_{t+1} + v_2\tilde{k}_{t+2} = u_{t+1},$$

where $\tilde{k}_t = k_t - k^*$, and k^* is the steady state stock of capital. Suppose

$$u_{t+1} = \begin{cases} \sigma, \text{ w.p. } 1/2 \\ -\sigma, \text{ w.p. } 1/2 \end{cases}.$$

Note that this stochastic process satisfies $E_t u_{t+1} = E u_{t+1} = 0$.

i. Set $\sigma = 0$ and show that given $\tilde{k}_0$, $\tilde{k}_t \to 0$ as $t \to \infty$, for an arbitrary choice of $\tilde{k}_1$.

ii. Set $\sigma > 0$ and let $\tilde{k}_0$ be given. Show that as $t \to \infty$, $\tilde{k}_t \in \left[\tilde{k}^l, \tilde{k}^u\right]$ with probability 1 where

$$-\infty < \tilde{k}^l < \tilde{k}^u < \infty.$$

Derive explicit formulas for $\tilde{k}^l$ and $\tilde{k}^u$ in terms of σ and v_0, v_1, v_2, and explain why these are independent of the choice of $\tilde{k}_1$. Show that $\tilde{k}^l \to \tilde{k}^u$ as $\sigma \to 0$.

(c) Draw a random sequence, u_2, u_3, ..., u_{20}, from the above distribution (Hint: do this by drawing from the uniform [0,1] distribution, rand, in MATLAB, and setting $u_t = -\sigma$ if the draw is less than 1/2 and setting $u_t = \sigma$ if the draw is greater than 1/2). Let $\tilde{k}_0 = 0$, so that the system begins in the nonstochastic steady state. Set $\tilde{k}_1$ as you wish.

(d) Given the choice of $\tilde{k}_1$, compute the remaining quantities and prices in equilibrium, for periods 1, 2, 3, ..., 20. That is, compute r_t, w_t, n_t, k_{t+1}, c_t for $t = 1, 2, ..., 20$. This is done using the intratemporal Euler equation, the firm first order conditions and the resource constraint. Plot these numbers.

(e) Can you find values of σ and $\tilde{k}_1$ so that the numbers satisfy the various non-negativity constraints? Can you find values of σ and $\tilde{k}_1$, with $\sigma > 0$, so that the numbers satisfy the non-negativity constraints with probability 1 (i.e., regardless of what sequence of shocks you draw)?

(f) Explain intuitively, why it is that for the original economy (i.e., before linearization), the presence of an indeterminate steady state implies a non-trivial sunspot equilibrium exists in a sufficiently small neighborhood of that steady state (hint: remember the sufficiency of Euler and transversality conditions).

2. Consider the Shleifer model.

   (a) We established conditions in class which guarantee that a simultaneous implementation equilibrium exists. We did so under the assumption that innovators are not permitted to delay more than one period. Prove that they would never delay additional periods.

   (b) Construct a numerical example illustrating an equilibrium in which the South always implements immediately, while the North always implements with a one period delay. Write in the numbers for output, profits and wage rate for each period.