Homework #9
Economics 411, Winter 2005
Due Friday, March 11.
Christiano

1. Consider an economy in which households consume two types of goods, c_1 and c_2, and supply labor, l. The purchase of each good is taxed at a potentially different rate. The household problem is:

$$\max_{c_1,c_2,l} u(c_1, c_2, l)$$
$$\text{s.t. } zl \geq c_1(1+\tau_1) + c_2(1+\tau_2).$$

Here, z denotes the wage rate and the utility function satisfies the usual properties so that the first order conditions evaluated at equality characterize the houshold optimum. The solution to the household problem defines the private sector allocation rule:

$$c_1 = c_1(\tau_1, \tau_2), c_2 = c_2(\tau_1, \tau_2), l = l(\tau_1, \tau_2).$$

The Ramsey Problem solved by the government is:

$$\max_{\tau_1,\tau_2} u(c_1(\tau_1, \tau_2), c_2(\tau_1, \tau_2), l(\tau_1, \tau_2))$$
$$\text{s.t. } g \geq c_1(\tau_1, \tau_2)\tau_1 + c_2(\tau_1, \tau_2)\tau_2$$

(a) Define a Ramsey equilibrium for this economy.

(b) Let B denote the set of equilibrium allocations associated with some policy, τ_1, τ_2, that satisfies the government budget constraint. Let D denote the set of allocations that satisfy the government budget constraint and a particular implementability constraint. Identify the implementability constraint that makes $D = B$. (Hint: here you will have to show (i) if $(c_1, c_2, l) \in B$ then $(c_1, c_2, l) \in D$; (ii) if $(c_1, c_2, l) \in D$ then $(c_1, c_2, l) \in B$.)

(c) Explain why the allocations in D that produce the highest utility are the allocations in a Ramsey equilibrium (the allocations in D which maximize utility solve the 'Ramsey Allocation Problem').

(d) The *Uniform Taxation Result* is the following:

$$\begin{aligned} \text{if } u &= V(h(c_1, c_2), l), \ h \sim \text{homothetic} \\ \text{then } \tau_1 &= \tau_2. \end{aligned}$$

Prove this result (hint: study the Ramsey allocation problem and recall the calculations you had to do in (ii) of part b.)

2. Consider the two-period version of the Atkeson model, which was discussed in class and appears in a handout on the website. Atkeson shows that it is possible to have an equilibrium in which the current account is positive when output is low, and negative when it is high. Adopt functional forms for the utility (constant elasticity seems like a good bet) and probability functions (the handout supplies a candidate). Also, you need to adopt numerical values for Y^H and Y^L. Can you find a parameterization having the property that the current account is negative for $Y = Y^H$ and negative for $Y = Y^L$?

3. Consider an economy in which final output is produced by a perfectly competitive firm, which uses intermediate inputs, Y_{it}, $i \in (0,1)$:

$$Y_t = \left[\int_0^1 Y_{it}^{\rho} di \right]^{\frac{1}{\rho}}, \ 0 < \rho \leq 1.$$

The price of the i^{th} input is p_{it}, and the output price is p_t. The firm's problem is to maximize profits:

$$p_t Y_t - \int_0^1 p_{it} Y_{it} di,$$

taking all prices parametrically. This leads to the following first order condition:

$$Y_{it} = Y_t \left(\frac{p_t}{p_{it}} \right)^{\frac{1}{1-\rho}}, \ i \in (0,1).$$

Substituting this back into the final goods production function:

$$p_t = \left[\int_0^1 p_{it}^{\frac{\rho}{\rho-1}} di \right]^{\frac{\rho-1}{\rho}}$$

Each intermediate good is produced by a single producer, who sets price equal to marginal cost because of the existence of a competitive fringe. Any intermediate good firm that attempted to set a higher price would be bumped out of the market. Each intermediate good firm has a linear production function in labor, with marginal productivity equal to unity. What differentiates the intermediate good firms is that those with $i \in (0, \alpha)$ must borrow the wage bill in advance at gross rate of interest, R_t, while the rest can finance the wage bill out of receipts. Those firms have no financing requirements. As a result, the marginal cost of a unit of labor for firms, $i \in (0, \alpha)$ is $w_t R_t$ and the marginal cost of a unit of labor is w_t for the rest.

You should take the aggregate supply of labor by households, L, as a given number.

(a) Derive an expression for the output of final goods that has the following form:

$$Y = \phi(R)L,$$

provide a simple, closed form expression for $\phi(R)$. Show that $\phi(1) = 1$, $\phi'(1) = 0$. Evidently, the heterogeneous borrowing requirements of different agents has the potential to supply a theory of TFP.

(b) Consider a jump in the interest rate from $R = 1.05$ to 1.10. Is there a value of α or ρ that will associate this jump in R with something like a 10 percent drop in efficiency?