

Economics 411
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**Notes on: Krusell and Rios-Rull's Vested Interests in a Positive
Theory of Stagnation and Growth
(RESTUD, vol. 63, no. 2, 1996, pp. 301-329)**

Any analysis of economic growth cannot avoid the fact that some countries don't grow at all, or even shrink. This handout explores one explanation that is based on the observation that growth involves the selection of new technologies, and each time a new technology is selected there are winners and losers. The losers are the vested interests, the people that have invested in the old technology. They have an incentive to block the implementation of a new technology. The purpose of these notes is to provide a sketch of the theory of vested interests developed by Krusell and Rios-Rull.

For the theory to be interesting, there must be several different types of people. In addition, the theory is articulated in an overlapping generations economy. This seems natural, since one associates new technologies with young people, and vested interests with older people.

1 The Model

The model is a three-period overlapping generations model. The population is constant. Each generation is associated with a number on the unit interval, $[0,1]$. So, we will say that there are (measure) one young, one middle-aged, and one old agent in any period. Thus, in any date there are three people: one young, one middle-aged and one old. The lifetime utility of an agent is:

$$c_1 + \beta c_2 + \beta^2 c_3,$$

where c_i is consumption at age i . Each person is endowed with one unit of labor, and this is supplied inelastically each period to the market. Agents cannot borrow or lend, so their consumption in any particular period equals whatever income they earn in that period.

There is potentially a range of Cobb-Douglas technologies, each one indexed by τ :

$$f_\tau(\varphi_{m,\tau}, \varphi_{n,\tau}) = A_\tau \varphi_{m,\tau}^\alpha \varphi_{n,\tau}^{1-\alpha}, \quad 0 < \alpha < 1, \quad (1)$$

where $\varphi_{m,\tau}$ is the number (measure) of managers people working on τ and $\varphi_{n,\tau}$ is the number of unskilled ('non-managers') people working on τ . We denote the entire distribution of workers across production functions by the vector, φ . In (1) the parameter, τ , indexes the vintage of the technology. The technology with $\tau = 1$ corresponds to the newest vintage during a particular period. A higher value of τ indicates an older vintage. Productivity improves with vintage:

$$\frac{A_\tau}{A_{\tau+1}} = \gamma, \quad \gamma > 1.$$

A technology which is vintage τ in the current period becomes vintage $\tau + 1$ in the next period if a new vintage technology becomes available in the next period (this requires a process of innovation which will be discussed momentarily). If in the next period there is no new vintage technology, then vintage τ technology in the current period remains vintage τ technology in the next period.

In order to bring a new technology into play, an agent must choose to innovate it. The innovator must spend two periods of life receiving zero income, and in the last period of life, the innovator manages the new technology. According to (1), a manager receives a higher return, the more unskilled workers come to work in the new technology.

There are two ways to become a manager on a technology that has been operating more than one period. One is to be apprenticed to that technology for two periods of life. During these two periods, the apprentice works as an unskilled worker in the technology where he is an apprentice. A second way to become a manager is to work as an unskilled person in the first period of life, and then learn a particular vintage in the second period. During this second period, the worker cannot be an unskilled worker and receives no income. By this option an agent can learn a vintage quickly, but at the cost of foregoing income in the current period. Because agents only have three periods of life, the soonest one can be a manager is in old age.

Finally, anyone can be an unskilled worker in any available vintage at any time. To summarize, at the beginning of life, young people can participate in one of the following four careers:

1. be unskilled in each of the three periods of life
2. be an apprentice in the first two periods in a given vintage technology, draw unskilled pay during these periods, and then be a manager for this technology in the third period
3. be unskilled in the first period, pick a particular vintage technology to learn quickly in the second period, and then be a manager in that technology in the third period
4. be an innovator in the first two periods, not drawing any pay then, and be skilled in the third period.

Now for some notation to help us be more precise about the environment. The type of an agent (or, his state at the start of a period) is denoted x . Here, $x = 1$ means young agents, $x = (2, \tau)$ and $x = (3, \tau)$ refers to middle aged and old agents with experience with vintage τ technologies. Also, $x = 3$ signifies old agents with skill in no technology. That is, in the past they worked as unskilled agents in technologies of different vintages (ruling out career 2) and did not do fast learning in middle age (i.e., are not in career 3). Also, $\mu_x \geq 0$ is the mass of agents of type x . So,

$$\mu_1 = 1, \quad \sum_{\tau=0}^{\infty} \mu_{2,\tau} = 1, \quad \mu_3 + \sum_{\tau=1}^{\infty} \mu_{3,\tau} = 1.$$

Here, we allow for $\tau = 0$ in $\mu_{2,\tau}$. By $\mu_{2,0}$, we indicate the measure of middle-aged agents who innovated in their youth. In the case, $\mu_{3,\tau}$, the range of summation begins with $\tau = 1$, reflecting that no one would start a new innovation project in middle age (this would have to be indexed by $\tau = 0$). Although the range of summation extends to $\tau = \infty$, in practice $\mu_{i,\tau} > 0$ for only a few values of τ , for $i = 2, 3$. Note also that there is no notation, μ_2 . That is, there are no middle-aged agents without skills in some vintage technology. They had to work *somewhere* in their youth. As noted above, it is possible for an agent to arrive in old age with no skill. This happens in the event that they worked in two different vintage technologies in the previous two periods, and they were not in a fast learning program in middle age. The whole set of μ 's is given by the vector, μ .

The choice taken by an agent in any particular period is denoted y , where

$$y = (y_a, y_\tau), \quad y_a \in \{m, n, d, i\}.$$

Here, y_a refers to the choice of working as a manager (e.g., this is an option in the third period of life for agents who chose careers 2-4 above), or as an unskilled worker (e.g., this is an option in each period for every agent...working as an unskilled worker is implied by career choices 1-3 above, though not by 4), or to learn without being currently productive, d (this is an option in the second period of life under career choice 3 above), or to innovate, as in career choice 4. Also, y_τ refers to the technology the agent chooses to be attached to. As just indicated, the feasible set of choices for an agent is constrained to some extent by past choices, i.e., by the agent's type, x . For example, an $x = 3$ agent cannot choose to be a manager in the current period.

Agent choices are also constrained by government policy:

$$\pi = \begin{cases} 1, & \text{adoption of a new technology is forbidden} \\ 0, & \text{new technologies may be adopted} \end{cases}.$$

Specifically, if in a given period $\pi = 1$, then agents may not begin the innovation process in the current period (innovations lawfully begun in previous periods can go forward). So, given policy and x , an agent has a set of possible choices,

$$y \in \Gamma(x, \pi).$$

Evidently, a person's current state, x , and choice, y , determine his next period's state:

$$x' = \xi(x, y, \mu; H_\varphi). \quad (2)$$

In principle the law of motion of x may seem as if it would only require x and y . However, momentarily we will explain why x' depends in part on whether other agents innovated in the previous two periods. Whether or not they did so can be recovered from the distribution of workers across skills and vintages. This in turn is a function of the aggregate state:

$$\varphi = H_\varphi(\mu).$$

The objects, φ , represent economy-wide data that are beyond the control of individual agents. Of course, they can be obtained by aggregating individual decisions, and in equilibrium there will have to be a consistency between H_φ and the aggregate of individual decisions.

An example will help to illustrate the notation. Suppose $x = 1$ and $y = (u, 1)$. That is, we consider a young agent who chooses in the current period to be unskilled in vintage $\tau = 1$. Then in the next period $x' = (2, 1)$ in case no new vintage technology becomes productive in the next period. If a new vintage technology does become productive in the next period, then $x' = (2, 2)$. This is because a technology which is vintage $\tau = 1$ in the current period drops in the ranking in the next period if there is a new technology and does not otherwise.

The state of the economy in any particular period is μ and the state for an individual agent is μ, x .

2 Equilibrium Conditional On a Given Policy

Here, we assume that policy is simply a given function of the aggregate state of the economy:

$$\pi = \Psi(\mu).$$

Managers hire unskilled workers to work on particular vintage technologies. The managers are residual claimants, after paying unskilled workers their marginal productivities, in terms of wages. Because of the assumed functional form of production, workers will receive a share, $1 - \alpha$, of total product while managers receive a share, α . Everyone must earn the same lifetime income in equilibrium.

We adopt a recursive formulation of agents' problem. The young or middle-aged agent's problem is:

$$v(x, \mu) = \max_{y \in \Gamma(x, \pi)} \{w(y, \varphi) + \beta v(x', \mu')\}, \quad (3)$$

where

$$\begin{aligned} x' &= \xi(x, y, \mu; H_\varphi) \\ \varphi &= H_\varphi(\mu) \\ \mu' &= H_\mu(\mu) \\ \pi &= \Psi(\mu). \end{aligned}$$

The first of the above four expressions repeats from (2) the law of motion of the individual agent's state. The second is the mapping from the aggregate state to the distribution of workers across vintage technologies that was discussed above. The third object is the law of motion of the aggregate state, μ . The individual agent must know $H \equiv (H_\mu, H_\varphi)$ in order for his problem to be well defined. Let

$$h(x, \mu) = \arg \max_y \{w(y, \varphi) + \beta v(x', \mu')\}$$

Note that $h(x, \mu)$ provides the decision of each particular agent, x , for every possible μ . With this in hand, one can construct φ - and, hence, $H_\varphi(\mu)$ - the measure of agents in each vintage technology. Similarly, with $h(x, \mu)$ one can construct $\mu' = H_\mu(\mu)$. In a rational expectations equilibrium, the h chosen by agents must be consistent with the H that they took as given in solving their problem. This will be reflected in our definition of equilibrium below.

We now define the function, $w(y, \varphi)$, in (3). As noted before, this corresponds to the agent's current consumption. For unskilled agents, this is simply the wage rate (we scale this by A_1 ¹). In particular, consider $y = (u, \tau)$. This is a choice to be unskilled on a vintage τ technology. For such a choice:

$$w((u, \tau), \varphi) = (1 - \alpha) \gamma^{1-\tau} \varphi_{m,\tau}^\alpha \varphi_{n,\tau}^{-\alpha}.$$

Note that this is the marginal product of labor, scaled by A_1 . The agent on fast-track learning receives nothing. That is, $y = (d, \tau)$ and

$$w((d, \tau), \varphi) = 0.$$

Now consider the old agents. If $x = 3$, so that the old agent is unskilled, then

$$v(3, \mu) = \max_{\tau} w((u, \tau), \varphi).$$

The old agent who is skilled in vintage τ has the option of managing a vintage τ production function, or working as an unskilled worker. As a manager, the agent receives profits, which is revenues after paying unskilled workers. The manager operating a technology with vintage τ chooses labor, l , to solve

$$R(\tau, u, \varphi) = \max_l \gamma^{1-\tau} l^{1-\alpha} - w((u, \tau), \varphi) l.$$

Let

$$l(\tau, u, \varphi) = \max_l \gamma^{1-\tau} l^{1-\alpha} - w((u, \tau), \varphi) l.$$

We can now define equilibrium:

Definition 1 *A private sector equilibrium, given Ψ , is a set of functions, v , h , H , w such that (i) (v, h) solve the agent's problems, given H and w , (ii) household choices, h , are consistent with $H_\varphi(\mu)$ and H_μ , (iii) the wage rate, corresponds to the marginal product of labor, (iv) the market for unskilled labor clears, $l(\tau, u, \varphi) = \varphi_{n,\tau}$, where $\varphi_{n,\tau}$ corresponds to the appropriate element in H_φ .*

To gain insight into this equilibrium and into the conflict of interests among agents in the model, consider two special cases of Ψ . One is the stagnation equilibrium, and corresponds to $1 = \Psi(\mu)$ for all μ and the other corresponds to the innovation equilibrium, $0 = \Psi(\mu)$ for all μ . Krusell and Rios-Rull refer to the stagnation equilibrium as the medieval equilibrium, since it involves no innovation and only involves apprentices and managers (or, in their language, masters).

¹The notes follow Krusell and Rios-Rull in this scaling. However, this appears flawed and deserves more thought.

2.0.1 Stagnation

Conjecture that in the stagnation equilibrium there is only one technology in operation. All agents choose to be unskilled in the first two periods of life. When $\alpha \geq 1/3$ then all agents follow the same career pattern: they are apprentices in the first two periods and become managers in old age. When $\alpha < 1/3$ then only a fraction, φ_m , of agents become managers in old age. The rest are unskilled in old age. In the first case, all agents have identical lives and so trivially they all have the same utility. In the second case, there are two different career paths: 1 and 2 above. These must generate the same lifetime utility and so it must be that the payment received by a manager in this case is the same as that received by the unskilled agent.

In this case, $\tau = 1$ always, since no new technology is ever developed. In each period, there are two unskilled, so that $\varphi_n = 2$, and there is one manager, so that $\varphi_m = 1$. Then, the wage rate earned by the unskilled is:

$$(1 - \alpha) \left(\frac{1}{2} \right)^\alpha .$$

Each manager employs two unskilled workers, so that a manager's profits are the output of the production technology given the one manager and the two apprentices, minus the payment to the two apprentices:

$$2^{1-\alpha} - (1 - \alpha) \left(\frac{1}{2} \right)^\alpha 2 = \alpha 2^{1-\alpha} .$$

A person who has served two periods as an apprentice could choose to be unskilled in the third period. A manager is willing to manage instead being an unskilled worker as long as profits are no less than the wage:

$$\alpha 2^{1-\alpha} \geq (1 - \alpha) \left(\frac{1}{2} \right)^\alpha .$$

Rearranging, this reduces to,

$$\alpha \geq \frac{1}{3} .$$

So, indeed, in the case $\alpha \geq 1/3$, all old agents will be managers.

Consider now the case where $\alpha < 1/3$. In this case, $\varphi_m < 1$ old agents become managers and $1 - \varphi_m$ become unskilled workers. Thus, the total number of unskilled workers is $2 + 1 - \varphi_m$. So, the wage rate is

$$(1 - \alpha) \left(\frac{\varphi_m}{2 + 1 - \varphi_m} \right)^\alpha .$$

Each manager employes $(2 + 1 - \varphi_m) / \varphi_m$ unskilled workers, so the profits of an individual manager are:

$$\left(\frac{2 + 1 - \varphi_m}{\varphi_m} \right)^{1-\alpha} - (1 - \alpha) \left(\frac{\varphi_m}{2 + 1 - \varphi_m} \right)^\alpha \left(\frac{2 + 1 - \varphi_m}{\varphi_m} \right) = \alpha \left(\frac{2 + 1 - \varphi_m}{\varphi_m} \right)^{1-\alpha} .$$

In this case, the utility of an individual manager and an individual unskilled worker are the same:

$$\alpha \left(\frac{2 + 1 - \varphi_m}{\varphi_m} \right)^{1-\alpha} = (1 - \alpha) \left(\frac{\varphi_m}{2 + 1 - \varphi_m} \right)^\alpha.$$

After rearranging, this reduces to:

$$\varphi_m = 3\alpha.$$

We conclude that when $\alpha > 1/3$, then all old agents become manager while when $\alpha \leq 1/3$, a fraction, 3α become managers. In the latter case, managers and unskilled workers receive the same current period payment.

It is interesting to inquire how people would feel about eliminating the injunction against innovations.

2.1 Growth

Now consider the steady state equilibrium with $\pi = 0$, i.e., innovation is always permitted. By steady state, we mean that labor effort and the proportions of the population that is skilled, innovating, being apprenticed, etc. are constant. Consider a growth equilibrium in which there are always two technologies being developed and only one is actually being used. Thus, the unskilled switch vintages twice in their lifetimes. Suppose that a fraction, $\bar{\varphi}$, of the young choose to innovate and develop new technologies and the remainder, $1 - \bar{\varphi}$, choose to remain unskilled. Then, in any period $\bar{\varphi}$ of the population are old innovator-managers, $1 - \bar{\varphi}$ are old unskilled, $1 - \bar{\varphi}$ are middle-aged unskilled and $1 - \bar{\varphi}$ are young unskilled. Thus is, $\bar{\varphi}$ are managers and $3(1 - \bar{\varphi})$ are unskilled in each period. With inputs constant over time and a new technology in each period, each period's technology produces γ more than the previous one. For example, if total output in a given period is 1, it is γ in the next period and γ^2 in the following period.

The wage rate in three consecutive periods is the marginal product of labor, which is a function of the ratio of managers to unskilled workers and of γ :

$$(1 - \alpha) \left(\frac{\bar{\varphi}}{3(1 - \bar{\varphi})} \right)^\alpha, \quad (1 - \alpha)\gamma \left(\frac{\bar{\varphi}}{3(1 - \bar{\varphi})} \right)^\alpha, \quad (1 - \alpha)\gamma^2 \left(\frac{\bar{\varphi}}{3(1 - \bar{\varphi})} \right)^\alpha.$$

Thus, the lifetime utility of an agent born in the first of these periods is:

$$(1 - \alpha) \left(\frac{\bar{\varphi}}{3(1 - \bar{\varphi})} \right)^\alpha + (1 - \alpha)\beta\gamma \left(\frac{\bar{\varphi}}{3(1 - \bar{\varphi})} \right)^\alpha + (1 - \alpha)\beta^2\gamma^2 \left(\frac{\bar{\varphi}}{3(1 - \bar{\varphi})} \right)^\alpha.$$

Each manager employs

$$3 \frac{1 - \bar{\varphi}}{\bar{\varphi}}$$

unskilled workers. So, the profits earned by one manager in the third period are:

$$R = \gamma^2 \left(3 \frac{1 - \bar{\varphi}}{\bar{\varphi}} \right)^{1-\alpha} - (1 - \alpha)\gamma^2 \left(\frac{1 - \bar{\varphi}}{3} \frac{\bar{\varphi}}{1 - \bar{\varphi}} \right)^\alpha 3 \frac{1 - \bar{\varphi}}{\bar{\varphi}} = \alpha\gamma^2 \left(3 \frac{1 - \bar{\varphi}}{\bar{\varphi}} \right)^{1-\alpha}.$$

Since careers are chosen freely, we require that the lifetime utility of a worker and an innovator be the same. The lifetime utility of the innovator, from the standpoint of youth, is $\beta^2 R$. So, equality of lifetime utility requires:

$$\begin{aligned}
(1 - \alpha) \left(\frac{1}{3} \frac{\bar{\varphi}}{1 - \bar{\varphi}} \right)^\alpha [1 + \beta\gamma + \beta^2\gamma^2] &= \beta^2 R \\
&= \beta^2\gamma^2 \left(3 \frac{1 - \bar{\varphi}}{\bar{\varphi}} \right)^{1 - \alpha} - (1 - \alpha) \beta^2\gamma^2 \left(\frac{1}{3} \frac{\bar{\varphi}}{1 - \bar{\varphi}} \right)^\alpha 3 \frac{1 - \bar{\varphi}}{\bar{\varphi}} \\
&= \beta^2\gamma^2 \left(\frac{1}{3} \frac{\bar{\varphi}}{1 - \bar{\varphi}} \right)^{\alpha - 1} - (1 - \alpha) \beta^2\gamma^2 \left(\frac{1}{3} \frac{\bar{\varphi}}{1 - \bar{\varphi}} \right)^{\alpha - 1} \\
&= \alpha \beta^2\gamma^2 \left(\frac{1}{3} \frac{\bar{\varphi}}{1 - \bar{\varphi}} \right)^{\alpha - 1}.
\end{aligned}$$

Then,

$$(1 - \alpha) \left(\frac{1}{3} \frac{\bar{\varphi}}{1 - \bar{\varphi}} \right) [1 + \beta\gamma + \beta^2\gamma^2] = \alpha \beta^2\gamma^2$$

or,

$$\frac{\bar{\varphi}}{1 - \bar{\varphi}} = \frac{3}{1 - \alpha} \frac{\alpha \beta^2\gamma^2}{[1 + \beta\gamma + \beta^2\gamma^2]}$$

so that

$$\begin{aligned}
\bar{\varphi} &= \frac{\frac{3}{1 - \alpha} \frac{\alpha \beta^2\gamma^2}{[1 + \beta\gamma + \beta^2\gamma^2]}}{1 + \frac{3}{1 - \alpha} \frac{\alpha \beta^2\gamma^2}{[1 + \beta\gamma + \beta^2\gamma^2]}} \\
&= \frac{1}{1 + \left(\frac{1 - \alpha}{3\alpha} \right) \frac{1 + \beta\gamma + \beta^2\gamma^2}{\beta^2\gamma^2}}
\end{aligned} \tag{4}$$

The following table may help clarify the nature of the conjectured equilibrium. It displays the income across different cohorts in periods t , $t + 1$ and $t + 2$.

Table: Earnings In Growth Equilibrium					
Generation	Cohorts	Number in Cohort	Earnings Per Person in Cohort, Over Three Periods		
			t	$t + 1$	$t + 2$
Old	Managers	$\bar{\varphi}$	$\alpha \left(3 \frac{1 - \bar{\varphi}}{\bar{\varphi}} \right)^{1 - \alpha}$	$\alpha \gamma \left(3 \frac{1 - \bar{\varphi}}{\bar{\varphi}} \right)^{1 - \alpha}$	$\alpha \gamma^2 \left(3 \frac{1 - \bar{\varphi}}{\bar{\varphi}} \right)^{1 - \alpha}$
	Unskilled	$1 - \bar{\varphi}$	$(1 - \alpha) \left(\frac{\bar{\varphi}}{3(1 - \bar{\varphi})} \right)^\alpha$	$(1 - \alpha) \gamma \left(\frac{\bar{\varphi}}{3(1 - \bar{\varphi})} \right)^\alpha$	$(1 - \alpha) \gamma^2 \left(\frac{\bar{\varphi}}{3(1 - \bar{\varphi})} \right)^\alpha$
Middle Aged	Innovator	$\bar{\varphi}$	0	0	0
	Unskilled	$1 - \bar{\varphi}$	$(1 - \alpha) \left(\frac{\bar{\varphi}}{3(1 - \bar{\varphi})} \right)^\alpha$	$(1 - \alpha) \gamma \left(\frac{\bar{\varphi}}{3(1 - \bar{\varphi})} \right)^\alpha$	$(1 - \alpha) \gamma^2 \left(\frac{\bar{\varphi}}{3(1 - \bar{\varphi})} \right)^\alpha$
Young	Innovator	$\bar{\varphi}$	0	0	0
	Unskilled	$1 - \bar{\varphi}$	$(1 - \alpha) \left(\frac{\bar{\varphi}}{3(1 - \bar{\varphi})} \right)^\alpha$	$(1 - \alpha) \gamma \left(\frac{\bar{\varphi}}{3(1 - \bar{\varphi})} \right)^\alpha$	$(1 - \alpha) \gamma^2 \left(\frac{\bar{\varphi}}{3(1 - \bar{\varphi})} \right)^\alpha$

With the value for $\bar{\varphi}$ in (4) do we have an equilibrium? For this to be an equilibrium, it must be that no one can increase their utility by deviating from the actions that have been prescribed for them. For example, old managers

clearly would not choose to be unskilled workers in old age, since their earnings as managers exceeds the unskilled wage when they are in old age. To completely verify that we have an equilibrium, there remain additional optimality conditions to check. It turns out that for the above scenario to be an equilibrium, the model parameters must be restricted. These restrictions are explored in the following homework questions at the end.

1. Verify that innovators in middle-age would not choose to spend their middle-age quickly learning the vintage technology then in place, and then manage it in their old age (i.e., verify that career option #3 is not better for innovators). In this case, two vintage technologies would coexist during their old age. Develop and explain a set of restrictions on model parameters that guarantee that this is the case.

2. Verify that the condition you developed in the first question guarantees that no one would choose career option #2. That is no one would work unskilled in youth and unskilled in the same technology in middle-age, and then manage that same technology in old age.

3. Are there additional restrictions needed for the conjectured equilibrium allocations to be an actual equilibrium?