

Homework #6
 Economics 411, fall 2006
 Due Thursday, November 9
 Christiano

1. Consider the model economy associated with Romer's model of growth through specialization. That is, preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}, \gamma > 0.$$

The technology for producing final goods is:

$$y_t = \int_0^{M_t} x_t(i)^\alpha di, \quad M_t > 0, \quad 0 < \alpha < 1. \quad (1)$$

To produce $x_t(i)$ units of the i^{th} intermediate good requires

$$\frac{1}{2}(1 + x_t(i)^2) \quad (2)$$

units of capital if $x_t(i) > 0$ and zero units of capital if $x_t(i) = 0$. The following constraint must be satisfied:

$$\int_0^{M_t} \frac{1}{2}(1 + x_t(i)^2) di = k_t, \quad (3)$$

where k_t is the beginning-of-period t aggregate stock of capital. The initial capital stock, $k_0 > 0$, is given. The resource constraint is:

$$c_t + I_t \leq y_t, \quad (4)$$

and the aggregate capital accumulation technology is given by:

$$k_{t+1} = (1 - \delta)k_t + I_t.$$

The efficient allocations for this economy solve the planning problem, maximize utility with respect to $\{M_t, k_{t+1}, y_t, c_t, x_t(i), i \in (0, M_t)\}_{t=0}^{\infty}$, subject to the various constraints.

- (a) Explain why economic efficiency dictates $x_t(i) = \bar{x}_t$ for $i \in (0, M_t)$. From here on, you may simply assume $x_t(i) = \bar{x}_t$ for all $i \in (0, M_t)$.
- (b) Show that the planning problem for the Romer economy coincides with the planning problem for the Ak model. In particular, show that the problem can be written,

$$\max_{\{k_{t+1} \in \Gamma(k_t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t F(k_t, k_{t+1}),$$

where

$$F(k, k') = \frac{[(A + 1 - \delta)k - k']^{1-\gamma}}{1 - \gamma} = \max_{\bar{x}_t, M_t} \frac{c_t^{1-\gamma}}{1 - \gamma}.$$

The last maximization is subject to (1)-(4), and the given values of k_t, k_{t+1} . Display an expression for the value of A in terms of model parameters. In addition to verifying the form of F , show what the constraint set, Γ , is.

- (c) Identify a set of parameter values under which positive growth is efficient, although the growth rate in the market decentralization analyzed in class is zero.
- (d) The problem with monopoly power is that it results in an inefficiently low level of activity (in the Romer model, the root of this inefficiency is the monopoly power that leads monopolists to pay a rental rate on capital that is less than its marginal product). In the Romer model we have just seen that this manifests itself in the form of inefficiently low growth. The pace at which new varieties of specialized inputs (e.g., specialized manufactured goods, specialized labor) are introduced is too slow in the market economy. Some sort of intervention in the market economy is desirable. One possibility is to subsidize the activities of monopolists. Accordingly, let $p(i)x(i)$ be the revenues of the i^{th} monopolist in the absence of taxes or subsidies. A subsidy rate, τ_t , raises the revenues of the i^{th} monopolists to $p(i)x(i)(1 + \tau_t)$. The total cost, G_t , to the government of this subsidy scheme is

$$G_t = \int_0^{M_t} p(i)x(i)\tau_t di.$$

Suppose G_t is financed by a lump sum tax applied to households. That is, the household budget constraint is modified as follows:

$$c_t + k_{t+1} - (1 - \delta)k_t = r_t k_t + w_t n_t - T_t,$$

where T_t represents taxes paid by the representative household to the government. Suppose the government balances its budget period by period:

$$T_t = G_t.$$

Find the subsidy rate, τ_t , that causes the allocations in the market economy to coincide with the efficient allocations.

These results have to be interpreted with caution. You have identified an ideal form of government intervention, which makes the private market economy efficient. However, the intervention we investigated abstracts from any social inefficiencies induced by having to raise the revenues needed to finance the subsidy to monopolists. We abstracted from this by assuming that the tax on households is administered in lump-sum form. In practice, such taxes are not available.

The only taxes we have are attached to specific economic activities (like the income tax) and so they distort those specific activities. So, the problem of ‘fixing’ the inefficiency in the Romer model is actually more complicated than this question makes it out to be.

2. Consider the model of Matsuyama, in the handout available on the website. Matsuyama denotes a situation in which $k < \theta F / (1 - \alpha)$ as a ‘Solow regime’ (I referred to it as a ‘neoclassical regime’) and a situation in which $k > \theta F / (1 - \alpha)$ as a ‘Romer regime’. Let

$$G = \beta \left[\alpha \left(\frac{\theta F}{1 - \alpha} \right)^{\alpha - 1} + 1 - \delta \right].$$

- (a) Suppose $G < 1$. Show that there is a steady state value of k in the Solow regime, call it k^* . That is, for any $M_{-1} > 0$ if the initial stock of capital is $K_0 = k^* M_{-1}$, then there is a no growth equilibrium with

$$K_{t+1} = K_0, \text{ for } t = 0, 1, 2, \dots$$

Note that in this steady state equilibrium, there is never any innovation. This regime is more likely the larger is F , which makes sense because this represents the fixed cost of innovation.

Let $M_{-1} = 1$, $\beta = 1/1.03$, $\alpha = 0.36$, $F = 100$, so that $G = 0.8833$, after rounding. Compute k^* . If K_0 is in a sufficiently small neighborhood of $k^* M_{-1}$, show that there exists an equilibrium in which $\lim_{t \rightarrow \infty} K_t = k^* M_{-1}$. (Hint: consider the household’s intertemporal Euler equation after substituting out for the rental rate of capital using equation (12) in the handout. Use the following facts: (i) for K_t sufficiently close to $k^* M_{-1}$, the Taylor series expansion of this equation about $K_t = K_{t+1} = K_{t+2} = k^* M_{-1}$ is an arbitrarily good approximation to this equation, and write this as

$$V_0 \tilde{K}_t + V_1 \tilde{K}_{t+1} + V_2 \tilde{K}_{t+2} = 0, \quad t = 0, 1, 2, \dots$$

where $\tilde{K}_t \equiv K_t - k^* M_{-1}$; (ii) the set of solutions to a linear difference equation like this is given by $\tilde{K}_t = (\tilde{K}_0 - a)\lambda_1^t + a\lambda_2^t$, for $t = 0, 1, 2, \dots$, for arbitrary a , where the λ_i ’s solve:

$$V_0 + V_1 \lambda_i + V_2 \lambda_i^2 = 0, \quad i = 1, 2;$$

- (b) Suppose $G > 1$. Show that there is a steady state value of k in the Romer regime, call it k^{**} . That is, given $M_{-1} > 0$ and $K_0 > 0$, there is an equilibrium in which

$$\frac{K_t}{M_{t-1}} = k^{**}, \quad \frac{c_{t+1}}{c_t} = \frac{K_{t+1}}{K_t} = \frac{M_t}{M_{t-1}} = G, \quad t = 0, 1, 2, \dots$$

Provide a formula for computing k^{**} and verify $k^{**} > \theta F / (1 - \alpha)$.

- (c) Think about the possibility of equilibria that fluctuate between the Romer and Solow regimes: in a Solow regime the relatively low amount of physical capital results in a high rental rate on capital. This discourages innovation but encourages capital accumulation (just like in the neoclassical growth model when you are below steady state). When capital becomes relatively abundant (so that $k > \theta F/(1 - \alpha)$) then innovators have an incentive to enter: the Romer regime begins and M starts to grow. If M grows fast enough relative to K (this will depend upon parameter values) then k is driven back down towards the Solow regime, and the process starts all over again. Along such a growth path there will be alternating periods of fast growth during which there is no innovation and slow growth, during which there is a lot of innovation. Interestingly, the same conditions that encourage high growth in capital and output, i.e., a high rental rate of capital, discourage innovation. This model generates all sorts of empirical hypotheses that would be interesting to test (patent applications come in bursts, and at times of low growth?).