Homework #8
Economics 411, Fall 2006
Due Friday, November 24.
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1. Consider a model in which a final good is produced using intermediate goods. The final good, y, is produced by a competitive, representative firm using the following homogeneous technology:

$$y = \exp \int_0^1 [\log y_j] \, dj.$$

The firm maximizes profits:

$$y - \int_0^1 p_j y_j dj,$$

taking p_j as given. Here, the price of the final good has been normalized at unity. The j^{th} intermediate good is produced by a monopolist using the following technology:

$$\begin{aligned} y_j &= \begin{cases} f(k_j, l_j) - \phi & f(k_j, l_j) \geq \phi \\ 0 & \text{otherwise} \end{cases}, \qquad (1) \\ f(k_j, l_j) &= k_j^{\alpha} l_j^{1-\alpha}, \ 0 < \alpha < 1. \end{aligned}$$

Thus, if the monopolist is to sell y_j units of goods, they must produce the fixed quantity, ϕ, first. The monopolist is competitive in the market for labor and capital and takes the rental rate on capital, r, and the wage rate, w, as given.

(a) Derive the demand curve for the j^{th} intermediate good. Consider the profit maximization problem of the j^{th} intermediate good firm. Show that it has no solution. That is, for any finite price-quantity pair on the demand curve, profits are always increased by increasing the price level.

(b) Suppose that there are other potential entrants into the production of the j^{th} intermediate good, and that they have access to the same technology, (1). Explain why this implies that the profits of the intermediate good producer must be zero.

(c) Show that cost minimization by the j^{th} intermediate good producer, linear homogeneity of f, and the zero profit condition imply that output can be written

$$y_j = \frac{1}{\mu_j} f(k_j, l_j),$$

where μ_j is the firm markup, the ratio of price to marginal cost, λ_j:

$$\mu_j = \frac{p_j}{\lambda_j}.$$

(d) Show that the zero profit condition implies the markup must fall when the firm produces more output. Provide the intuition for this result.

(e) Explain why it is that in equilibrium, final output has the following representation:

$$y = \frac{1}{\mu} f(k, l),$$

where l is household labor supply, k is the supply of capital by households, and μ is the markup. Suppose that k, l and y fluctuate in response to shocks outside of the firm sector. Explain how. according to this theory, conventional empirical measures of disembodied technical change are severely misled. This type of theory is sometimes referred to as a 'theory of TFP' (Total Factor Productivity, another name for disembodied technology). It's a theory about how conventional measures of TFP may be recovering some kind of endogenous variable, rather than true, exogenous technology.

2. Consider an economy in which final output is produced by a perfectly competitive firm, which uses intermediate inputs, Y_{it}, $i \in (0, 1)$:

$$Y_t = \left[\int_0^1 Y_{it}^{\rho} di \right]^{\frac{1}{\rho}}, \ 0 < \rho \leq 1.$$

The price of the i^{th} input is p_{it}, and the output price is p_t. The firm's problem is to maximize profits:

$$p_t Y_t - \int_0^1 p_{it} Y_{it} di,$$

taking all prices parametrically. This leads to the following first order condition:

$$Y_{it} = Y_t \left(\frac{p_t}{p_{it}} \right)^{\frac{1}{1-\rho}}, \ i \in (0,1).$$

Substituting this back into the final goods production function:

$$p_t = \left[\int_0^1 p_{it}^{\frac{\rho}{\rho-1}} di \right]^{\frac{\rho-1}{\rho}}$$

Each intermediate good is produced by a single producer, who sets price equal to marginal cost because of the existence of a competitive fringe. Any intermediate good firm that attempted to set a higher price would be bumped out of the market. Each intermediate good firm has a linear production function in labor, with marginal productivity equal to unity. What differentiates the intermediate good firms is that those with $i \in (0, \alpha)$ must borrow the wage bill in advance at gross rate of interest, R_t, while the rest can finance the wage bill out of receipts. Those firms have no financing requirements. As a result, the marginal cost of a unit of labor for firms, $i \in (0, \alpha)$ is $w_t R_t$ and the marginal cost of a unit of labor is w_t for the rest.

You should take the aggregate supply of labor by households, L, as a given number.

(a) Derive an expression for the output of final goods that has the following form:

$$Y = \phi(R)L,$$

provide a simple, closed form expression for $\phi(R)$. Show that $\phi(1) = 1$, $\phi'(1) = 0$. Evidently, the heterogeneous borrowing requirements of different agents has the potential to supply a theory of TFP.

(b) Consider a jump in the interest rate from $R = 1.05$ to 1.10. Is there a value of α or ρ that will associate this jump in R with something like a 10 percent drop in efficiency?

3. Consider a model in which utility is a function not just of market consumption, c, and market labor effort, l, but also of consumption

of home produced goods or services, c_n, and home labor effort, l_n. Specifically,

$$\log\left(c + c_n\right) - \gamma \log\left(\frac{l^{1+\psi}}{1+\psi} + l_n\right),$$

where $\gamma, \psi > 0$. The home labor effort yields services via the home production function, $c_n = \psi_0 l_n$. Show that this formulation implies a utility function in terms of market goods and labor having the following form:

$$\text{constant} + a \log\left(c - \psi_0 \frac{l^{1+\psi}}{1+\psi}\right),$$

where 'constant' and a are parameters. (Hint: recall how we got $F(k, k')$ for the version of the growth model in which utility is a function of labor effort, in addition to consumption.)