Christiano
411, Fall, 2007

FINAL EXAM

Allocate your time to the following four questions in proportion to the number of points available. If a question seems ambiguous, state why, sharpen it up and answer the revised question. Good luck!

1. (35) A representative, competitive firm produces a homogeneous final good, Y, using the following production function:

$$Y = \left[\int_0^1 y_i^{\lambda} di\right]^{\frac{1}{\lambda}},\ 0 < \lambda < 1,$$

where y_i is the quantity of the i^{th} intermediate good. The final good producer takes the price of y_i, P_i, as given.

The i^{th} intermediate good is produced by a single monopolist which sets price, P_i, and quantity, y_i to maximize profits. The intermediate good firm has production technology:

$$y_i = A k_i^{\alpha} N_i^{1-\alpha},\ 0 < \alpha < 1,\ A > 0,$$

where k_i, N_i denote capital and employment, respectively, used by the i^{th} monopolist. Each monopolist is a price taker in factor markets and let r and w denote the rental rate on capital and wage rate on labor, respectively.

With the Cobb-Douglas production function, marginal cost for a producer that pays rental rate $\tilde{r}$ and wage rate $\tilde{w}$ is:

$$MC(\tilde{r}, \tilde{w}) = \xi \tilde{r}^{\alpha} \tilde{w}^{1-\alpha},$$

where ξ is a function of A and α.

Monopolists, $i \in (0, \gamma)$ are subject to a 'financial friction': they must borrow the funds to pay rk_i and wN_i at the beginning of the period. Here, $0 < \gamma < 1$. So, at the end of the period these firms owe the bank principle plus interest equal to $Rrk_i + RwN_i$, where R is a gross rate of interest (i.e., a number like 1.08 for '8 percent interest'). Monopolists with $i \in (\gamma, 1)$ face no financial friction and simply pay rk_i and wN_i at the end of the period for its factor payments.

Consider an econometrician who has a time series record on aggregate capital, k, and labor, N, where

$$k = \int_0^1 k_i di, \;\; N = \int_0^1 N_i di.$$

Also, the econometrician knows the true value of α. The econometrician estimates a time series record of total factor productivity using aggregate output, labor and capital using the following formula:

$$TFP = \frac{Y}{k^{\alpha} N^{1-\alpha}}.$$

Derive a formula relating TFP to A, R, γ, λ. What would the business cycle properties of R have to be for an RBC theorist to conclude that the data are government by an RBC model with procyclical, neutral technical change?

(Hint: use the efficiency conditions of the final and intermediate good producers, as well as the production functions, to develop a simple relationship between the outputs and inputs of financially constrained and unconstrained monopolists.)

2. (10) Suppose there is a given total amount of some finite resource, X. This resource can be allocated among a range of inputs, $x(i)$, $i \in (0, M)$ subject to the resource constraint:

$$\int_0^M x(i)\, di \leq X.$$

Suppose the inputs can be converted into output according to the following technology:

$$y = n^{1-\alpha} \int_0^M x(i)^{\alpha}\, di,\ 0 < \alpha < 1,$$

where $n > 0$ is labor. Explain the sense in which the present environment is one in which there are gains from specialization.

3. (20) Consider the following competitive, two-period lived overlapping generations economy. People work and save when young. Their time endowment when young is unity. When, old households do not work and they pay for consumption out of the rent from capital accumulated while young. The population is constant, and the number of young born in each period is equal to the number of old who die in the same period. Let c_t^t and c_{t+1}^t denote the period t and $t+1$ consumptions, respectively, of agents born in period t. Let w_t denote the period t wage rate. Preferences of households are given by $u\left(c_t^t, c_{t+1}^t\right)$ and these are increasing and concave. The young supply one unit of labor inelastically. The budget constraint of the young and old are, respectively,

$$\begin{aligned} c_t^t + k_{t+1} &\leq w_t \\ c_{t+1}^t &\leq r_{t+1}k_{t+1} + (1-\delta)\, k_{t+1} \end{aligned}$$

where r_{t+1} denotes the rental rate of capital in period $t+1$ and w_t denotes the wage rate in period t. All quantities chosen by the household are required to be non-negative. The initial generation of old people owns the initial stock of capital, k_0, and simply consume the income from this stock. A representative, competitive firm chooses capital and labor in each period to maximize profits, using the following technology:

$$f(k_t, n) = b\left[\alpha k_t^\rho + (1-\alpha)\, n_t^\rho\right]^{\frac{1}{\rho}},\ 0 < \rho < 1,\ b > 0,$$

where n_t denotes labor.

(a) Explain the presence of $(1-\delta)\, k_{t+1}$ on the right side of old people's budget constraint. Where does this income come from?

(b) Provide a formal, sequence of markets definition of equilibrium for this economy. Be sure to provide a resource constraint for the economy and explain how new capital is produced.

(c) Prove that growth is technologically feasible for some $b > 0$.

(d) Prove that growth (i.e., k_{t+1}/k_t for all $t > t^*$, some t^*) is not possible in equilibrium.

4. (35) Preferences of the typical household are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where

$$u(c_t) = \frac{c^{1-\nu} - 1}{1-\nu}, \ \nu > 0.$$

The household accumulates private capital using the following accumulation technology:

$$k_{p,t+1} = (1-\delta_p)k_{p,t} + i_{p,t},$$

where $k_{p,t}$ is the beginning-of-period t stock of capital, $i_{p,t}$ is period t gross investment, and $0 < \delta_p < 1$. The initial stock of capital, $k_{p,0}$, is given. The household is endowed with $n > 0$ units of labor time, and must satisfy $c_t, k_{p,t} \geq 0$.

The technology for producing output is operated by a perfectly competitive, representative firm:

$$y_t = k_{gt}^{\gamma} k_{pt}^{\alpha} n_t^{(1-\alpha)}, \ 0 < \alpha < 1, \gamma \geq 0,$$

where k_{gt} is the per capita stock of government-provided capital. The household and firm takes the sequence $\{k_{gt}\}$ as given and beyond their control.

The technology for accumulating government capital is:

$$k_{g,t+1} = (1-\delta_g)k_{gt} + i_{gt},$$

where i_{gt} denotes per capita gross investment by the government, and $0 < \delta_g < 1$. The government finances investment by levying taxes, T_t, in the amount:

$$i_{gt} = T_t.$$

Taxes are levied on households in *lump-sum* form: each household must pay T_t, regardless of what decisions it takes regarding consumption, investment, or labor effort.

Finally, the resource constraint for this economy is:

$$c_t + i_{pt} + i_{gt} \leq y_t.$$

(a) Suppose the government chooses its sequence of public investment so that

$$k_{gt} = sk_{pt}, \; t \geq 1, \; s > 0. \tag{1}$$

Sharpen up the statement of the household and firm problems, and define a sequence of markets equilibrium.

(b) Suppose $\gamma = 1 - \alpha$. Show that s can be chosen so that there is a steady-state balanced growth path for this economy, in which all quantity variables but employment display the same positive growth rate. Explain intuitively, why steady state growth is possible in this case, but is not when $\gamma = 0$.

(c) Display an optimization problem, the solution of which gives the efficient allocations for this economy. Write out the Euler equations for this problem. Are they sufficient for a maximum to the problem? Explain your answer.

(d) Suppose $\gamma = 1 - \alpha$. Suppose the economy is in a sequence-of-markets equilibrium, and that the government must optimally choose a sequence, i_{gt}, $t = 0, 1, 2, \ldots$. Would that sequence be consistent with the specification for public investment in (1)? Explain your answer.