Homework #7
Economics 411
Due Wednesday, November 21.
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Consider the endogenous growth model with human capital discussed in class. One sector produces a homogeneous output good, which is transformed one-for-one into consumption and investment. The homogeneous output good is itself produced using a Cobb-Douglas production function:

$$c_t + k_{t+1} - (1-\delta) k_t = k_t^{\alpha} n_t^{1-\alpha}.$$

Another sector produces human capital according to the following accumulation equation:

$$h_{t+1} = h_t + \lambda (h_t - n_t),$$

where $\lambda > 0$, $c_t \geq 0$, $k_{t+1} \geq (1-\delta) k_t$, $0 \leq n_t \leq h_t$, and h_0, k_0 are given. Preferences are:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma},$$

$\gamma > 0$. To ensure boundedness, we require $\beta (1+\lambda)^{1-\gamma} < 1$. In class, the problem was reformulated in recursive form. It was shown that there are policy rules of the form, $x_{t+1} = f(x_t)$, $y_t = g(x_t)$, where $x_t = k_t/h_t$ and $y_t = h_{t+1}/h_t$.

1. Set $\alpha = 1/3$, $\delta = 0.10$, $\beta = 0.97$, $\lambda = 0.04$, $\gamma = 1.1$. Compute steady state values of x, y. How do these values change with α and with λ? Provide intuition.

2. Develop a formula for the date t price (in consumption units) of a unit of human capital, h_{t+1}. Develop a formula for the period $t+1$ payoff associated with an extra unit of h_{t+1} (hint: the payoff is the maximal increase in consumption that is possible in period $t+1$, while leaving the consumption opportunities unchanged in periods $t+2$ and later). The one-period rate of return on human capital acquired in period t is the payoff in period $t+1$ divided by the price in period t. Do the same for physical capital. Develop expressions for the rate of return on human and physical capital in terms of y_t and x_t.

3. Set up the problem of choosing the efficient allocations in Lagrangian form. Must the rate of return on physical and human capital be the same at all dates? Prove your answer.

4. Write down a set of functional equations that the equilibrium policy functions, $x_{t+1} = f(x_t)$ and $y_t = g(x_t)$, must satisfy. Use the perturbation method to develop first-order Taylor series approximations around steady state for f and g.Calculate and report the the first order Taylor series approximation.

5. For initial conditions that are close to steady state, does the physical to human capital ratio converge monotically to steady state? Explain. What is the rate of return on human and physical capital for x above steady state and for x below steady state? If you conclude that the system always converges to steady state, then report how long does it take to close 90% of the gap to steady state. Explain the formula you use for this calculation.

6. Describe a competitive decentralization for this economy and show that the equilibrium allocations coincide with the efficient allocations.