Homework #7
Economics 411
Due Wednesday, December 3
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1. Consider the endogenous growth model with human capital discussed in class. One sector produces a homogeneous output good, which is transformed one-for-one into consumption and investment. The homogeneous output good is itself produced using a Cobb-Douglas production function:

$$c_t + k_{t+1} - (1-\delta)\, k_t = k_t^{\alpha} n_t^{1-\alpha}.$$

Another sector produces human capital according to the following accumulation equation:

$$h_{t+1} = h_t + \lambda\,(h_t - n_t)\,,$$

where $\lambda > 0$, $c_t \geq 0$, $k_{t+1} \geq (1-\delta)\, k_t$, $0 \leq n_t \leq h_t$, and h_0, k_0 are given. Preferences are:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma},$$

$\gamma > 0$. To ensure boundedness, we require $\beta\,(1+\lambda)^{1-\gamma} < 1$. In class, the problem was reformulated in recursive form. It was shown that there are policy rules of the form, $x_{t+1} = f\,(x_t)\,,\ y_t = g\,(x_t)\,,$ where $x_t = k_t/h_t$ and $y_t = h_{t+1}/h_t$.

(a) Set $\alpha = 1/3$, $\delta = 0.10$, $\beta = 0.97$, $\lambda = 0.04$, $\gamma = 1.1$. Compute steady state values of x, y. How do these values change with α and with λ? Provide intuition.

(b) Develop a formula for the date t price (in consumption units) of a unit of human capital, h_{t+1}. Develop a formula for the period $t+1$ payoff associated with an extra unit of h_{t+1} (hint: the payoff is the maximal increase in consumption that is possible in period $t+1$, while leaving the consumption opportunities unchanged in periods $t+2$ and later). The one-period rate of return on human capital acquired in period t is the payoff in period $t+1$ divided by the price in period t. Do the same for physical capital. Develop expressions for the rate of return on human and physical capital in terms of y_t and x_t.

(c) Set up the problem of choosing the efficient allocations in Lagrangian form. Must the rate of return on physical and human capital be the same at all dates? Prove your answer.

(d) Write down a set of functional equations that the equilibrium policy functions, $x_{t+1} = f(x_t)$ and $y_t = g(x_t)$, must satisfy. Use the perturbation method to develop first-order Taylor series approximations around steady state for f and g. Calculate and report the the first order Taylor series approximation.

(e) For initial conditions that are close to steady state, does the physical to human capital ratio converge monotically to steady state? Explain. What is the rate of return on human and physical capital for x above steady state and for x below steady state? If you conclude that the system always converges to steady state, then report how long does it take to close 90% of the gap to steady state. Explain the formula you use for this calculation.

(f) Describe a competitive decentralization for this economy and show that the equilibrium allocations coincide with the efficient allocations.

2. Regarding the model of ideas and cycles, develop an expression for total profits of all firms at time t. Show that wages plus capital rental plus total profits equals Y_t, the total output of the homogeneous good.

3. Regarding the model of agency costs discussed in the last lecture.

   (a) Consider the version of the model in which it is the household that builds new capital and supplies it to a capital rental market (i.e., this is the standard neoclassical model except for the nonlinearity in the production function for producing new capital, a nonlinearity that is usually referred to as 'adjustment costs'.) Define a market in which households can buy and sell capital among each other, to make precise the idea of the 'price of capital'. Will there be trade in this market in equilibrium? Does the presence or absence of this market make any difference to the equilibrium quantity of consumption, output, employment, etc.? Derive an expression relating the market price of capital to the future discounted value of the proceeds from capital. Derive an expression relating the price of capital to the flow of investment and the stock of capital (this is the 'Tobin's q relation' in the model).

(b) Modify the model so that at each date t there are markets for output in each possible period $t+1$ state of nature. Allow households to have access to these markets, and also banks. Show how the presence of these markets leads to a different representation for the banks' zero profit condition, one in which it can make negative profits in some states and positive profits in others. Explain why it is that there will be trade in the state-contingent markets in equilibrium.

(c) It is possible to describe a stochastic neoclassical model whose allocations coincide exactly with those of the economy with the financial frictions. The neoclassical model must have adjustments in the resource constraint and the household must have a tax rate applied to the return on capital. Explain this isomorphic economy.