Econ 411
Lawrence Christiano

Agency Costs in the Ownership and Management of Capital

Many economic activities require financing. That is, someone must free up resources by suppressing consumption and then waiting for a period time until output occurs. In some cases, this waiting time is quite short. A hot dog vendor at a baseball game acquires the raw materials for hot dogs in the morning, and the rewards come in during the afternoon as the game is played. In other cases, the waiting time can be very long. For example, the rewards of constructing an apartment building do not occur fully until many years have passed. In the neoclassical model, finance is required in the construction of capital because resources are allocated in one period and the return does not occur until the next period. Finance in the neoclassical model does not entail any particular complications because the person suppressing consumption and the person building capital are the same.

In practice, borrowers and lenders are different people and there is reason to believe that there is a conflict of interest between them. The reason is that when borrowers put the funds they receive to work 'stuff happens', in which case the output that was expected does not occur. For example, in the process of building an apartment building it may be discovered that the ground for the building site is softer than expected, raising the cost of construction. Alternatively, there could be a labor strike that results in lower output than expected. It is likely that the 'stuff' that happens is better understood by the borrower than the lender. The lender may have to allocate a substantial amount of resources to sort out what happened in case the borrower declares that 'stuff happened' and that he or she cannot pay back a loan.

The previous considerations suggest that when borrowers and lenders are different people, a simple sharing rule contract will not be the best arrangement between them. In that arrangement, the lender transfers resources to the borrower and the borrower returns a prespecified share of the proceeds to the lender. This is not a good arrangement because the borrower has an incentive to hide profits and declare that output was low. With this type of arrangement, it is possible that the lender would have to expend considerable resources monitoring borrowers.

In a seminal paper Robert Townsend (1979, 1988) suggested a better arrangement.[1] He posited that lenders would offer borrowers a 'standard debt contract'. The name of the contract reflects that it resembles real-world loan contracts. The standard debt contract specifies a loan amount and an interest rate. If the borrower pays the interest rate, then there is no need for bank monitoring. If the borrower declares he or she cannot pay the interest, then he or she is monitored and the bank takes whatever the borrower has. Free entry in banking drives bank profits to zero. Competition in the lending market ensures

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[1] Other work on includes Bernanke and Gertler (1989), Covas and den Haan (2007), Fuerst (1995), Carlstrom and Fuerst (1997), Gale and Hellwig (1985), Kiyotaki and Moore (1997), Kwark (2002), Cooley, Marimon, and Quadrini (2004), Levin, Natalucci and Zakrajsek (2004) and Hopenhayn and Werning (2008). Extensions to open-economy settings include De Graeve (2007) and Gertler, Gilchrist, and Natalucci (2003). An analysis of the US Great Depression using financial frictions like those in this handout appears in Christiano, Motto and Rostagno (2003).

that the loan amount and interest rate in the standard debt contract optimize the welfare of borrowers, subject to specified constraints including the zero profit condition on banks.

We introduce the standard debt contract into the neoclassical model, following the setup in Bernanke, Gerlter and Gilchrist (1999). The economy is composed of households, goods-producing firms, capital producers, entrepreneurs and banks. Good-producing firms produce goods using labor and capital. Labor is hired in competitive markets from households and capital is rented in competitive markets from entprepreneurs. Entrepreneurs own the capital stock. They acquire ownership in part using their own resources, or 'net worth'. The balance is borrowed from a bank in a competitive lending market with free entry into banking. There exist frictions between the bank and the entrepreneur because the entrepreneur who purchases capital suffers an idiosyncratic productivity shock that is observed by the entrepreneur, but is only observable to the bank after it pays a monitoring cost. This resembles the environment considered by Townsend, and so I assume that the bank and entrepreneur use a standard debt contract. In principle, the contract varies across entrepreneurs with different levels of net worth. For this reason, I adopt the assumption that entrepreneurs with each different level of net worth go to a different bank for their loan. We show that because of special assumptions made in this environment, all entrepreneurs pay the same interest rate, regardless of their net worth. In addition, the debt to net worth ratio of each entrepreneur is the same across all entrepreneurs, regardless of the level of their net worth.

The bank obtains the funds it loans to the entrepreneur by issuing bonds to households. In the process of intermediation between households and entprepreneurs, some resources are lost as banks monitor the fraction of the entrepreneurs who declare that it is infeasible to repay their bank loans. We now describe the economy in detail.

Let $s_t \in S$ denote the realization of aggregate uncertainty in period t and let $s^t = (s_0, ..., s_t)$ denote the history of aggregate uncertainty up to and including period t. Also, $\mu(s^t)$ denotes the proability of history s^t.

1. Households, Goods-Producing Firms and Capital Producers

There is a continuum of identical households. The budget constraint of the representative household is:

$$c\left(s^t\right) + b\left(s^t\right) \leq \left(1 + R\left(s^t\right)\right) b\left(s^{t-1}\right) + w\left(s^t\right) l\left(s^t\right) + T^h\left(s^t\right),$$

where $R(s^t)$ denotes the interest earned on deposits with the bank, $b(s^{t-1})$ denotes the stock of those deposits acquired in s^{t-1}, and $w(s^t)$, $l(s^t)$ and $T^h(s^t)$ denote the wage rate, employment and lump sum taxes in s^t. Subject to this budget constraint and a no-Ponzi condition, households seek to maximize utility:

$$\sum_{t=0}^{\infty} \sum_{s^t}^{\infty} \beta^t \mu\left(s^t\right) u\left(c\left(s^t\right), l\left(s^t\right)\right),$$

where $\mu(s^t)$ denotes the probability of history s^t. Legal regulations stipulate that the interest rate paid by banks to households must be state non-contingent. That is,

$$R\left(s^t, s_{t+1}\right) = R\left(s^t, s'_{t+1}\right) \text{ for all } s_{t+1}, s'_{t+1} \in S. \tag{1.1}$$

Households' first order conditions, in addition to the transversality condition, are:

$$u_c\left(s^t\right) \;=\; \beta \sum_{s^{t+1}|s^t} u_c\left(s^{t+1}\right)\left(1 + R\left(s^{t+1}\right)\right) \tag{1.2}$$

$$\frac{-u_l\left(s^t\right)}{u_c\left(s^t\right)} \;=\; w\left(s^t\right), \tag{1.3}$$

where $u_c\left(s^t\right)$ and $-u_l\left(s^t\right)$ denote the marginal utilities of consumption and leisure, respectively, in s^t.

Goods-producing firms have the following linear homogeneous technology:

$$y\left(s^t\right) = k\left(s^{t-1}\right)^\alpha \left(z\left(s^t\right) l\left(s^t\right)\right)^{1-\alpha}. \tag{1.4}$$

They rent capital and hire labor in perfectly competitive markets at rental rate, $r\left(s^t\right)$, and wage rate, $w\left(s^t\right)$, respectively. Optimization implies:

$$y_k\left(s^t\right) = r\left(s^t\right), \;\; y_l\left(s^t\right) = w\left(s^t\right), \tag{1.5}$$

where $y_k\left(s^t\right)$ and $y_l\left(s^t\right)$ denote the marginal productivities of capital and labor, respectively, in s^t. The object, $z\left(s^t\right)$, is the sole source of aggregate uncertainty in this economy.

In s^t, the representative, competitive capital producer purchases investment goods, $x\left(s^t\right)$, and old capital, $k\left(s^{t-1}\right)$, to produce new capital, $k\left(s^t\right)$, using the following linear homogeneous technology:

$$k\left(s^t\right) = (1-\delta)\,k\left(s^{t-1}\right) + x\left(s^t\right) - \Phi\left(\frac{x\left(s^t\right)}{k\left(s^{t-1}\right)}\right) k\left(s^{t-1}\right). \tag{1.6}$$

The function, Φ, is an 'adjustment cost technology', which introduces curvature in the trade-off between consumption an additional capital. One implication is that the price of capital deviates from unity. The competitive market prices of $k\left(s^{t-1}\right)$ and $k\left(s^t\right)$ are $P_k\left(s^t\right)$ and $P_{k'}\left(s^t\right)$, respectively, in s^t. Capital producer optimization leads to the following conditions:

$$P_k\left(s^t\right) \;=\; \frac{1}{1 - \Phi'\left(\frac{x(s^t)}{k(s^{t-1})}\right)}\left[1 - \delta - \Phi\left(\frac{x\left(s^t\right)}{k\left(s^{t-1}\right)}\right) + \Phi'\left(\frac{x\left(s^t\right)}{k\left(s^{t-1}\right)}\right)\frac{x\left(s^t\right)}{k\left(s^{t-1}\right)}\right] \tag{1.7}$$

$$P_{k'}\left(s^t\right) \;=\; \frac{1}{1 - \Phi'\left(\frac{x(s^t)}{k(s^{t-1})}\right)}. \tag{1.8}$$

2. Entrepreneurs and Banks

We now turn to the discussion of the entrepreneurs. Before going into the details, I present an overview of the situation of the entrepreneurs. Entrepreneurs specialize in owning and renting out the stock of capital. In period s^t they purchase the new capital that will produce in (s^t, s_{t+1}) for each $s_{t+1} \in S$ (see Figure 1). In s^{t+1} entrepreneurs earn rent on their capital in competitive rental markets with goods-producers. After goods production has occurred in s^{t+1}, entrepreneurs sell the capital they purchased in s^t to capital producers who combine

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this capital with investment goods to produce new capital. At this point, entrepreneurial loans from banks are paid off or, if necessary, the entrepreneur declares bankruptcy. A fraction, $1 - \gamma$, of entrepreneurs are randomly selected to exit the economy. The fraction, γ, who survive receive a small transfer payment and a new group of entrepreneurs is born, each of whom also receives the small transfer payment. The entrepreneurs who exit consume a fraction of whatever net worth they have, and the rest is transferred to households.

Figure 1: A Period in the Life of an Entrepreneur

Period s^t Period s^{t+1}

Entrepreneurial relationship with bank ends either with full repayment of debt, or with bankruptcy.

Entrepreneur supplies capital services to capital services rental market

* End of period t: Using net worth and loans, entrepreneur purchases new, end-of-period s^t stock of capital from capital goods producers. Entrepreneur experiences idiosyncratic disturbance to its newly purchased capital.

Entrepreneur sells used capital to capital producers

If entrepreneur survives another period, goes to *.

2.1. $N-$type Entrepreneurs

We now discuss the entrepreneurial bank loans. At the end of s^t there is a continuum of entrepreneurs, each with a different level of net worth. The only thing that differentiates entrepreneurs at this point is their net worth. Nothing else about an entrepreneur is relevant. We suppose that for each possible level of net worth there are many entrepreneurs. Call these $N-$type entrepreneurs. These entrepreneurs participate in a competitive loan market with banks which specialize in $N-$type entrepreneurial loans. Legal regulations constrain the type of loan contracts that may be traded. Loan market contracts extended to $N-$type entrepreneurs are required by law to specify an interest rate, $Z^N \left(s^{t+1} \right)$, and a loan amount, $B^N \left(s^t \right)$. (Note, the interest rate that the entrepreneur pays may be contingent upon the realization of aggregate uncertainty in period $t+1$.) The loan amount extended to an $N-$type entrepreneur satisfies:

$$B^N \left(s^t \right) = P_{k'} \left(s^t \right) k^N \left(s^t \right) - N \left(s^t \right). \tag{2.1}$$

After the $N-$type entrepreneur purchases $k^N \left(s^t \right)$ units of capital, he draws a non-negative random variable, ω, and $k^N \left(s^t \right)$ becomes $k^N \left(s^t \right) \omega$. The random variable, ω, is drawn in-

dependently from the same mean-unity distribution by all $N-$type entrepreneurs. The entrepreneur's draw of ω is private information. An outsider (e.g., the bank) can determine the value of the entrepreneur's draw of ω only by paying a monitoring cost. The rate of return on capital received by the $N-$type entrepreneur who draws ω is:

$$\frac{\omega r\left(s^{t+1}\right)+\omega P_{k}\left(s^{t+1}\right)}{P_{k'}\left(s^{t}\right)}=(1+R^{k}\left(s^{t+1}\right))\omega, \tag{2.2}$$

say. The denominator in this rate of return is the price, $P_{k'}\left(s^{t}\right)$, paid by the entrepreneur for one unit of new capital in s^{t}. The numerator represents the payoff the entrepreneur receives from holding that capital into s^{t+1}. Because the lower bound on ω is zero, there will always be some $N-$type entrepreneurs for whom it is infeasible to pay back the bank loan. These are entrepreneurs who experience an ω below $\bar{\omega}^{N}\left(s^{t+1}\right)$, where

$$\bar{\omega}^{N}\left(s^{t+1}\right)\left(1+R^{k}\left(s^{t+1}\right)\right)P_{k'}\left(s^{t}\right)k^{N}\left(s^{t}\right)=Z^{N}\left(s^{t+1}\right)B^{N}\left(s^{t}\right). \tag{2.3}$$

$N-$type entrepreneurs who draw $\omega<\bar{\omega}^{N}\left(s^{t+1}\right)$ must pay all their revenues,

$$\omega\left(1+R^{k}\left(s^{t+1}\right)\right)P_{k'}\left(s^{t}\right)k^{N}\left(s^{t}\right),$$

to the bank. In this case, the bank must monitor the entrepreneur, at cost

$$\mu\omega\left(1+R^{k}\left(s^{t+1}\right)\right)P_{k'}\left(s^{t}\right)k^{N}\left(s^{t}\right),$$

where μ is a monitoring parameter. Random monitoring is illegal.

At the point in s^{t} when $N-$type banks extend loan contracts to $N-$type entrepreneurs, I suppose they have access to funds in a market with households at interest rate, $R\left(s^{t+1}\right)$. Because of competition, banks take the rate of return, $R\left(s^{t+1}\right)$, as given. As noted above, there is a legal restriction which makes $R\left(s^{t+1}\right)$ contingent on s^{t} but not s_{t+1}. The cash flow, $\pi^{N}\left(s^{t+1}\right)$, of an $N-$type bank in s^{t+1} is

$$\begin{aligned}
\pi^{N}\left(s^{t+1}\right) &= \left[1-F\left(\bar{\omega}^{N}\left(s^{t+1}\right)\right)\right]Z^{N}\left(s^{t+1}\right)B^{N}\left(s^{t}\right) \tag{2.4}\\
&+ (1-\mu)\int_{0}^{\bar{\omega}^{N}\left(s^{t+1}\right)}\omega dF\left(\omega\right)\left(1+R^{k}\left(s^{t+1}\right)\right)P_{k'}\left(s^{t}\right)k^{N}\left(s^{t}\right)\\
&- \left(1+R\left(s^{t+1}\right)\right)B^{N}\left(s^{t}\right).
\end{aligned}$$

Here, the first two terms after the equality represent receipts from non-bankrupt and bankrupt (net of monitoring costs) entrepreneurs, respectively. The last term represents payments to households for loans. We assume that there do not exist markets in period s^{t} in which banks can purchase funds for delivery in (s^{t},s_{t+1}), for $s_{t+1}\in S$. Since banks cannot pay out funds they do not have, this implies $\pi^{N}\left(s^{t},s_{t+1}\right)\geq0$ for each $s_{t+1}\in S$. Suppose $p^{N}\left(s^{t+1}\right)>0$ is the weight assigned to $\pi^{N}\left(s^{t+1}\right)$ when the bank evaluates profits in s^{t}. With free entry into banking, profits in period s^{t} must be zero:

$$\sum_{s_{t+1}|s^{t}}p^{N}\left(s^{t+1}\right)\pi^{N}\left(s^{t+1}\right)=0.$$

But, $p^N(s^{t+1}) > 0$ and $\pi^N(s^{t+1}) \geq 0$ and zero profits implies

$$\pi^N\left(s^t, s_{t+1}\right) = 0 \text{ for each } s_{t+1} \in S. \tag{2.5}$$

If households participated in state contingent markets for goods, it would be possible for bank profits to be non-zero in different states of nature, with profits being negative sometimes and positive in other times. I have explored this specification of the zero profit condition and found that its quantitative implications are roughly the same as the quantitative implications of the kind of zero profit condition used here.

Substituting out for $Z^N(s^{t+1}) B^N(s^t)$ in (2.4) from (2.3), imposing (2.5), using (2.1), and rearranging I obtain:

$$1 + R\left(s^{t+1}\right) = \left(1 - \tau^k\left(s^{t+1}\right)\right)\left(1 + R^k\left(s^{t+1}\right)\right). \tag{2.6}$$

Here,

$$1 - \tau^k\left(s^{t+1}\right) \equiv \frac{\bar{k}^N\left(s^t\right)}{\bar{k}^N\left(s^t\right) - 1}\left[\Gamma(\bar\omega^N\left(s^{t+1}\right)) - \mu G(\bar\omega^N\left(s^{t+1}\right))\right], \tag{2.7}$$

where

$$G(\bar\omega^N\left(s^{t+1}\right)) \equiv \int_0^{\bar\omega^N\left(s^{t+1}\right)} \omega dF(\omega).$$

$$\Gamma(\bar\omega^N\left(s^{t+1}\right)) \equiv \bar\omega^N\left(s^{t+1}\right)\left[1 - F(\bar\omega^N\left(s^{t+1}\right))\right] + G(\bar\omega^N\left(s^{t+1}\right))$$

$$\bar{k}^N\left(s^t\right) \equiv \frac{P_{k'}\left(s^t\right) k^N\left(s^t\right)}{N\left(s^t\right)}.$$

Here, $\bar{k}^N(s^t)$ represents the leverage of the $N-$type entrepreneur and $\Gamma(\bar\omega^N(s^{t+1}))$ is the expected share of profits, net of monitoring costs, accruing to the bank. It is useful to work out the derivative of Γ:

$$\Gamma'(\bar\omega^N\left(s^{t+1}\right)) = 1 - F(\bar\omega^N\left(s^{t+1}\right)) - \bar\omega^N\left(s^{t+1}\right) F'(\bar\omega^N\left(s^{t+1}\right)) + G'(\bar\omega^N\left(s^{t+1}\right)) \tag{2.8}$$
$$= 1 - F(\bar\omega^N\left(s^{t+1}\right)) > 0.$$

We assume that the period s^t loan contract optimizes the $N-$type entrepreneur's expected state at the termination of the loan contract, when the loan is either repaid in full, or the entrepreneur experiences bankruptcy. The entrepreneur's state at this point in s^{t+1} is the total return from the purchase of capital, net of bank loans. We denote this expectation, conditional on s^{t+1}, by $V^N(s^{t+1})$, so that

$$V^N\left(s^{t+1}\right) = \int_{\bar\omega^N(s^{t+1})}^{\infty}\left[\left(1 + R^k\left(s^{t+1}\right)\right)\omega P_{k'}\left(s^t\right) k^N\left(s^t\right) - Z^N\left(s^t, s_{t+1}\right) B^N\left(s^t\right)\right] dF(\omega) \tag{2.9}$$
$$= \left\{\int_{\bar\omega^N(s^{t+1})}^{\infty}\left[\omega - \bar\omega^N\left(s^{t+1}\right)\right] dF(\omega)\left(1 + R^k\left(s^{t+1}\right)\right)\right\} P_{k'}\left(s^t\right) k^N\left(s^t\right)$$
$$= \left[1 - \Gamma(\bar\omega^N\left(s^{t+1}\right))\right]\left(1 + R^k\left(s^{t+1}\right)\right)\bar{k}^N\left(s^t\right) N\left(s^t\right).$$

The first equality uses (2.3) and the last equality makes use of:

$$1 = \int_0^{\infty} \omega dF(\omega) = \int_{\bar\omega^N(s^{t+1})}^{\infty} \omega dF(\omega) + G(\bar\omega^N\left(s^{t+1}\right)).$$

Thus, expected entrepreneurial net worth at the end of the loan contract, evaluated at s^t is:

$$\sum_{s_{t+1}|s^t} \mu\left(s^{t+1}\right) V^N\left(s^{t+1}\right)$$

$$= \sum_{s_{t+1}|s^t} \mu\left(s^{t+1}\right)\left[1 - \Gamma(\bar{\omega}^N\left(s^{t+1}\right))\right]\left(1 + R^k\left(s^{t+1}\right)\right)\bar{k}^N\left(s^t\right) N\left(s^t\right).$$

Note that expected entrepreneurial utility is strictly increasing in leverage, $\bar{k}^N\left(s^t\right)$, and strictly decreasing in $\bar{\omega}^N\left(s^t, s_{t+1}\right)$ (see (2.8)) for each $s_{t+1} \in S$. The contract between $N-$type entrepreneurs and banks optimizes entrepreneurial net worth at the end of the loan contract with respect to $\bar{k}^N\left(s^t\right)$ and $\bar{\omega}^N\left(s^t, s_{t+1}\right)$ for each $s_{t+1} \in S$, subject to (2.5) and taking $N\left(s^t\right)$, $P_{k'}\left(s^t\right)$, $R\left(s^{t+1}\right)$, $R^k\left(s^{t+1}\right)$ as given. Note that given $\bar{k}^N\left(s^t\right)$ and $\bar{\omega}^N\left(s^t, s_{t+1}\right)$, the loan amount can be recovered from the definition of leverage and (2.1), and the entrepreneur's interest rate can be recovered from (2.3). Writing this optimal contracting problem in Lagrangian form, I obtain:

$$\max_{\bar{\omega}^N(s^{t+1}),\bar{k}^N(s^t)} \sum_{s_{t+1}|s^t} \mu\left(s^{t+1}\right)\left\{\left[1 - \Gamma(\bar{\omega}^N\left(s^{t+1}\right))\right]\left(1 + R^k\left(s^{t+1}\right)\right)\bar{k}^N\left(s^t\right)\right.$$

$$\left. + \lambda^N\left(s^{t+1}\right)\left[\bar{k}^N\left(s^t\right)\left[\Gamma(\bar{\omega}^N\left(s^{t+1}\right)) - \mu G(\bar{\omega}^N\left(s^{t+1}\right))\right]\left(1 + R^k\left(s^{t+1}\right)\right) - \left(1 + R\left(s^{t+1}\right)\right)\left(\bar{k}^N\left(s^t\right) - 1\right)\right]\right\},$$

where $\lambda^N\left(s^{t+1}\right)$ is the multiplier on the zero cash flow condition, (2.5). Note that $k\left(s^t\right)$ has been replaced with $\bar{k}^N\left(s^t\right)$ in this problem. There is no loss in this because $N\left(s^t\right)$ and $P_{k'}\left(s^t\right)$ are taken as given. Also, both expected entrepreneurial utility and $\pi\left(s^{t+1}\right)$ have been scaled by $N\left(s^t\right)$ in the optimal contract problem. The first order condition with respect to $\bar{k}^N\left(s^t\right)$ is:

$$\sum_{s_{t+1}|s^t} \mu\left(s^{t+1}\right)\left\{\left[1 - \Gamma(\bar{\omega}^N\left(s^{t+1}\right))\right]\left(1 + R^k\left(s^{t+1}\right)\right)\right.$$

$$\left. + \lambda^N\left(s^{t+1}\right)\left[\left[\Gamma(\bar{\omega}^N\left(s^{t+1}\right)) - \mu G(\bar{\omega}^N\left(s^{t+1}\right))\right]\left(1 + R^k\left(s^{t+1}\right)\right) - \left(1 + R\left(s^{t+1}\right)\right)\right]\right\} = 0.$$

The first order condition with respect to $\bar{\omega}^N\left(s^t, s_{t+1}\right)$ for a specific $s_{t+1} \in S$ is (after dividing by $\left(1 + R^k\left(s^{t+1}\right)\right)\bar{k}^N\left(s^t\right)$):

$$\Gamma'(\bar{\omega}^N\left(s^t, s_{t+1}\right)) = \lambda^N\left(s^{t+1}\right)\left[\Gamma'(\bar{\omega}^N\left(s^t, s_{t+1}\right)) - \mu G'(\bar{\omega}^N\left(s^t, s_{t+1}\right))\right].$$

Solving this expression for $\lambda^N\left(s^{t+1}\right)$ and then substituting out for $\lambda^N\left(s^{t+1}\right)$ in the first order condition for $\bar{k}^N\left(s^t\right)$, the latter reduces to:

$$\sum_{s_{t+1}|s^t} \mu\left(s^{t+1}\right)\left\{\left[1 - \Gamma(\bar{\omega}^N\left(s^{t+1}\right))\right]\left(1 + R^k\left(s^{t+1}\right)\right)\right. \tag{2.10}$$

$$+ \frac{\Gamma'(\bar{\omega}^N\left(s^t, s_{t+1}\right))}{\Gamma'(\bar{\omega}^N\left(s^t, s_{t+1}\right)) - \mu G'(\bar{\omega}^N\left(s^t, s_{t+1}\right))}$$

$$\left. \times \left[\left[\Gamma(\bar{\omega}^N\left(s^{t+1}\right)) - \mu G(\bar{\omega}^N\left(s^{t+1}\right))\right]\left(1 + R^k\left(s^{t+1}\right)\right) - \left(1 + R\left(s^{t+1}\right)\right)\right]\right\}.$$

This single expression, together with the zero cash condition in each $s_{t+1} \in S$:

$$\bar{k}^N\left(s^t\right)\left[\Gamma(\bar{\omega}^N\left(s^{t+1}\right)) - \mu G(\bar{\omega}^N\left(s^{t+1}\right))\right]\left(1 + R^k\left(s^{t+1}\right)\right) = \left(1 + R\left(s^{t+1}\right)\right)\left(\bar{k}^N\left(s^t\right) - 1\right),$$
(2.11)

are the equations which determine $\bar{k}^N\left(s^t\right)$ and $\bar{\omega}^N\left(s^t, s_{t+1}\right)$ for each $s_{t+1} \in S$. A key feature of these equations is that they are the same for all N. Thus, the optimal loan contract between entrepreneurs and banks for each $N-$type have the property:

$$\bar{\omega}^N\left(s^t, s_{t+1}\right) = \bar{\omega}\left(s^t, s_{t+1}\right), \ \bar{k}^N\left(s^t\right) = \bar{k}\left(s^t\right), \ \text{for each } N.$$
(2.12)

Using (2.3) it is then easy to verify that $Z^N\left(s^{t+1}\right) = Z\left(s^{t+1}\right)$ for all N too. In particular, every entrepreneur, regardless of his net worth, receives a loan contract with the same rate of interest and with a loan amount that is the same fraction of his net worth.

2.2. Aggregating Over $N-$type Entrepreneurs

We now discuss the evolution of the aggregate net worth of all entrepreneurs. Let $f(N; s^t)$ denote the density of $N-$type entrepreneurs, so that aggregate net worth at the time in s^t when loan contracts are made is:

$$\bar{N}\left(s^t\right) = \int_0^\infty N f(N; s^t) dN.$$

In period s^{t+1}, the average net worth of $N-$type entrepreneurs, right after their relationship with the bank comes to an end, but before it is determined whether they exit the economy or continue is, by (2.9):

$$V^N\left(s^{t+1}\right) = \left[1 - \Gamma(\bar{\omega}^N\left(s^{t+1}\right))\right]\left(1 + R^k\left(s^{t+1}\right)\right)\bar{k}^N\left(s^t\right)N\left(s^t\right).$$

Integrating this over all $N-$type entrepreneurs and taking into account (2.12):

$$\begin{aligned}
V\left(s^{t+1}\right) &\equiv \int_0^\infty V^N\left(s^{t+1}\right) f(N; s^t) dN \\
&= \left[1 - \Gamma(\bar{\omega}\left(s^{t+1}\right))\right]\left(1 + R^k\left(s^{t+1}\right)\right)\bar{k}\left(s^t\right)\bar{N}\left(s^t\right) \\
&= \left[1 - \Gamma(\bar{\omega}\left(s^{t+1}\right))\right]\left(1 + R^k\left(s^{t+1}\right)\right)P_{k'}\left(s^t\right)k\left(s^t\right),
\end{aligned}$$

where $k\left(s^t\right)$ is the aggregate stock of capital in period s^t :

$$k\left(s^t\right) = \int_0^\infty k^N\left(s^t\right) f(N; s^t) dN.$$

Writing out $V\left(s^{t+1}\right)$ in more detail,

$$
\begin{aligned}
V\left(s^{t+1}\right) &= \left(1+R^k\left(s^{t+1}\right)\right) P_{k'}\left(s^t\right) k\left(s^t\right) - \left(1+R^k\left(s^{t+1}\right)\right) P_{k'}\left(s^t\right) k\left(s^t\right) \Gamma(\bar{\omega}\left(s^{t+1}\right)) \\
&= \left(1+R^k\left(s^{t+1}\right)\right) P_{k'}\left(s^t\right) k\left(s^t\right) \\
&\quad - \left(1+R^k\left(s^{t+1}\right)\right) P_{k'}\left(s^t\right) k\left(s^t\right) \left\{ \bar{\omega}^N\left(s^{t+1}\right) \left[1 - F(\bar{\omega}^N\left(s^{t+1}\right))\right] + \int_0^{\bar{\omega}\left(s^{t+1}\right)} \omega dF(\omega) \right\} \\
&= \left(1+R^k\left(s^{t+1}\right)\right) P_{k'}\left(s^t\right) k\left(s^t\right) \\
&\quad - \left(1+R^k\left(s^{t+1}\right)\right) P_{k'}\left(s^t\right) k\left(s^t\right) \\
&\quad \times \left\{ \bar{\omega}^N\left(s^{t+1}\right) \left[1 - F(\bar{\omega}^N\left(s^{t+1}\right))\right] + (1-\mu) \int_0^{\bar{\omega}\left(s^{t+1}\right)} \omega dF(\omega) + \mu \int_0^{\bar{\omega}\left(s^{t+1}\right)} \omega dF(\omega) \right\} \\
&= \left(1+R^k\left(s^{t+1}\right)\right) P_{k'}\left(s^t\right) k\left(s^t\right) \\
&\quad - \left\{ \left(1+R\left(s^{t+1}\right)\right) B^N\left(s^t\right) + \left(1+R^k\left(s^{t+1}\right)\right) P_{k'}\left(s^t\right) k\left(s^t\right) \mu \int_0^{\bar{\omega}\left(s^{t+1}\right)} \omega dF(\omega) \right\},
\end{aligned}
$$

after imposing the zero cash flow condition and making use of (2.3) and (2.4). Finally, using (2.1)

$$
\begin{aligned}
V\left(s^{t+1}\right) &= \left(1+R^k\left(s^{t+1}\right)\right) P_{k'}\left(s^t\right) k\left(s^t\right) \\
&\quad - \left[1 + R\left(s^{t+1}\right) + \frac{\left(1+R^k\left(s^{t+1}\right)\right) P_{k'}\left(s^t\right) k\left(s^t\right) \mu \int_0^{\bar{\omega}\left(s^{t+1}\right)} \omega dF(\omega)}{P_{k'}\left(s^t\right) k\left(s^t\right) - \bar{N}\left(s^t\right)} \right] \left[P_{k'}\left(s^t\right) k\left(s^t\right) - \bar{N}\left(s^t\right) \right]
\end{aligned}
$$

At this point, γ entrepreneurs are randomly selected to survive and $(1-\gamma)$ are selected to exit. In addition, all entrepreneurs receive a transfer of $T\left(s^{t+1}\right)$:

$$
\begin{aligned}
\bar{N}\left(s^{t+1}\right) &= \gamma\{ \left(1+R^k\left(s^{t+1}\right)\right) P_{k'}\left(s^t\right) k\left(s^t\right) \\
&\quad - \left[1 + R\left(s^{t+1}\right) + \frac{\left(1+R^k\left(s^{t+1}\right)\right) P_{k'}\left(s^t\right) k\left(s^t\right) \mu \int_0^{\bar{\omega}\left(s^{t+1}\right)} \omega dF(\omega)}{P_{k'}\left(s^t\right) k\left(s^t\right) - \bar{N}\left(s^t\right)} \right] \left[P_{k'}\left(s^t\right) k\left(s^t\right) - \bar{N}\left(s^t\right) \right] \} \\
&\quad + T\left(s^{t+1}\right).
\end{aligned}
\tag{2.13}
$$

This is the law of motion for aggregate net worth.

The expressions determining $\bar{\omega}\left(s^{t+1}\right)$ and $\bar{k}\left(s^t\right) = P_{k'}\left(s^t\right) k\left(s^t\right) / \bar{N}\left(s^t\right)^2$ are (2.10) after deleting the superscript, N :

$$
\sum_{s_{t+1}|s^t} \mu\left(s^{t+1}\right) \{ \left[1 - \Gamma(\bar{\omega}\left(s^{t+1}\right))\right] \left(1+R^k\left(s^{t+1}\right)\right) \tag{2.14}
$$

$$
+ \frac{\Gamma'(\bar{\omega}\left(s^t, s_{t+1}\right))}{\Gamma'(\bar{\omega}\left(s^t, s_{t+1}\right)) - \mu G'(\bar{\omega}\left(s^t, s_{t+1}\right))} \left[\left[\Gamma(\bar{\omega}\left(s^{t+1}\right)) - \mu G(\bar{\omega}\left(s^{t+1}\right))\right] \left(1+R^k\left(s^{t+1}\right)\right) - \left(1+R\left(s^{t+1}\right)\right) \right] \},
$$

and (2.11) for each $s_{t+1} \in S$:

$$
\bar{k}\left(s^t\right) \left[\Gamma(\bar{\omega}\left(s^{t+1}\right)) - \mu G(\bar{\omega}\left(s^{t+1}\right))\right] \left(1+R^k\left(s^{t+1}\right)\right) = \left(1+R\left(s^{t+1}\right)\right) \left(\bar{k}\left(s^t\right) - 1\right). \tag{2.15}
$$

---

[2] Here, we have used the fact that $k^N\left(s^t\right) / N\left(s^t\right) = \bar{k}\left(s^t\right)$ or all N implies $k\left(s^t\right) / \bar{N}\left(s^t\right) = \bar{k}\left(s^t\right)$.

The three equilibrium conditions provided by the entrepreneurial relationship with banks are (2.13)-(2.15). The $1-\gamma$ entrepreneurs who are selected for exit in s^{t+1} consume a fraction, Θ, of their net worth:

$$C^e\left(s^{t+1}\right) = \Theta(1-\gamma)V\left(s^{t+1}\right).$$

The complementary fraction, $(1-\Theta)\left(1-\gamma\right)V\left(s^{t+1}\right),$ is given to households as a transfer payment. It is convenient to write this in terms of aggregate net worth:

$$C^e\left(s^{t+1}\right) = \frac{\Theta(1-\gamma)}{\gamma}\left[\bar{N}\left(s^{t+1}\right) - T\left(s^{t+1}\right)\right].$$

3. Aggregate Constraints

In period s^t, the uses of goods include household and entrepreneurial consumption, investment and resources used in monitoring. This must equal to the supply of goods, $y\left(s^t\right)$:

$$c\left(s^t\right) + \frac{\Theta(1-\gamma)}{\gamma}\left[\bar{N}\left(s^t\right) - T\left(s^t\right)\right] + x\left(s^t\right) + \mu \int_0^{\bar{\omega}\left(s^t\right)} \omega dF(\omega)\left(1+R^k\left(s^t\right)\right)P_{k'}\left(s^t\right)k\left(s^t\right) = y\left(s^t\right).$$
(3.1)

The constraint governing transfer payments is:

$$T^h\left(s^t\right) = (1-\Theta)\left(1-\gamma\right)V\left(s^t\right) - T\left(s^t\right).$$

The loan market clearing condition is that the bonds purchased by households, $b\left(s^t\right),$ equal the bonds supplied by banks, $B\left(s^t\right) = P_{k'}\left(s^t\right)k\left(s^t\right) - \bar{N}\left(s^t\right)$:

$$b\left(s^t\right) = P_{k'}\left(s^t\right)k\left(s^t\right) - \bar{N}\left(s^t\right).$$

4. General Equilibrium

An equilibrium is a set of allocations, $\left\{y\left(s^t\right), l\left(s^t\right), x\left(s^t\right), c\left(s^t\right), k\left(s^t\right), \bar{N}\left(s^t\right), \bar{\omega}\left(s^t\right)\right\},$ and prices and rates of return, $\left\{R^k\left(s^t\right), R\left(s^t\right), r\left(s^t\right), w\left(s^t\right), P_k\left(s^t\right), P_{k'}\left(s^t\right)\right\},$ which have the property that (i) optimality for households, goods producing firms, entrepreneurs and banks are satisfied; and (ii) goods and loan market clearing occurs.

The equations that characterize the 13 equilibrium objects are, in addition to (1.1), the following 12: the definition of the rate of return on capital, (2.2) the two household first order conditions, (1.2) and (1.3), the production function, (1.4), the two firm first order conditions, (1.5), the law of motion for capital, (1.6), the two capital producer optimization conditions, (1.7) and (1.8), the three entrepreneur/bank conditions, (2.13)-(2.15), and the resource constraint, (3.1).

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