Deriving the Basic Shape of the Zero Profit Function Analytically

- First, some simple notation.

- Then, the results.

Some Notation and Results

- Let:

$$G(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega dF(\omega) = \left[\int_0^{\bar{\omega}} \omega \overbrace{\frac{dF(\omega)}{F(\bar{\omega})}}^{\text{density of } \omega \text{, conditional on } \omega \leq \bar{\omega}} \right] F(\bar{\omega}) = \overbrace{E[\omega | \omega < \bar{\omega}]}^{\text{expected value of } \omega \text{, conditional on } \omega < \bar{\omega}} F(\bar{\omega})$$

$$\Gamma(\bar{\omega}) \equiv \bar{\omega}[1 - F(\bar{\omega})] + \int_0^{\bar{\omega}} \omega dF(\omega) = \bar{\omega}[1 - F(\bar{\omega})] + E[\omega | \omega < \bar{\omega}] F(\bar{\omega})$$

- Result:

$$G'(\bar{\omega}) = \frac{d}{d\omega} \int_0^{\bar{\omega}} \omega dF(\omega) \overbrace{=}^{\text{Leibniz's rule}} \bar{\omega} F'(\bar{\omega})$$

$$\Gamma'(\bar{\omega}) = 1 - F(\bar{\omega}) - \bar{\omega} F'(\bar{\omega}) + G'(\bar{\omega}) = 1 - F(\bar{\omega}) \geq 0$$

$$\Gamma''(\bar{\omega}) = -F'(\bar{\omega}) < 0$$

$\rightarrow \Gamma(\bar{\omega})$ increasing and concave

- Result:

$$\int_0^{\bar{\omega}} \omega dF(\omega) + \int_{\bar{\omega}}^{\infty} \omega dF(\omega) = 1$$

$$E[\omega|\omega < \bar{\omega}]F(\bar{\omega}) + E[\omega|\omega > \bar{\omega}][1 - F(\bar{\omega})] = 1$$

$$\rightarrow E[\omega|\omega > \bar{\omega}][1 - F(\bar{\omega})] = 1 - E[\omega|\omega < \bar{\omega}]F(\bar{\omega})$$

- Then:

$$1 - \Gamma(\bar{\omega}) = 1 - \bar{\omega}[1 - F(\bar{\omega})] - E[\omega|\omega < \bar{\omega}]F(\bar{\omega})$$

$$= 1 - E[\omega|\omega < \bar{\omega}]F(\bar{\omega}) - \bar{\omega}[1 - F(\bar{\omega})]$$

$$= E[\omega|\omega > \bar{\omega}][1 - F(\bar{\omega})] - \bar{\omega}[1 - F(\bar{\omega})]$$

$$= (E[\omega|\omega > \bar{\omega}] - \bar{\omega})[1 - F(\bar{\omega})] \geq 0$$

- Conclude: $0 \leq \Gamma(\bar{\omega}) \leq 1$, for all $\bar{\omega} \geq 0$.

Limiting Properties

- According to our previous result:

$$0 \le \overbrace{\bar{\omega}[1 - F(\bar{\omega})] + \int_0^{\bar{\omega}} \omega dF(\omega)}^{\Gamma(\bar{\omega})} \le 1, \text{ for all } \bar{\omega} \ge 0$$

- So that,

$$\bar{\omega}[1 - F(\bar{\omega})] \le 1 - \int_0^{\bar{\omega}} \omega dF(\omega) \to 0, \text{ as } \bar{\omega} \to \infty$$

- But, $0 \le \bar{\omega}[1 - F(\bar{\omega})] \le 1 - \int_0^{\bar{\omega}} \omega dF(\omega)$, so

$$\lim_{\bar{\omega}\to\infty} \bar{\omega}[1 - F(\bar{\omega})] = 0.$$

- Conclude:

$$\lim_{\bar{\omega}\to\infty} \Gamma(\bar{\omega}) = \lim_{\bar{\omega}\to\infty} \bar{\omega}[1 - F(\bar{\omega})] + \lim_{\bar{\omega}\to\infty} G(\bar{\omega})$$

$$= 0 + 1 = 1.$$

More Limiting Properties

- Obvious results:

$$\lim_{\bar{\omega}\to\infty} G(\bar{\omega}) = 1, \ \lim_{\bar{\omega}\to 0} G(\bar{\omega}) = 0, \text{ where } G(\bar{\omega}) = \int_0^{\bar{\omega}} \omega dF(\omega)$$

$$\lim_{\bar{\omega}\to 0} \Gamma(\bar{\omega}) = \lim_{\bar{\omega}\to 0}\left(\bar{\omega}[1 - F(\bar{\omega})] + \int_0^{\bar{\omega}} \omega dF(\omega) \right) = 0$$

- Finally,

$$\lim_{\bar{\omega}\to 0} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] = 0$$

$$\lim_{\bar{\omega}\to\infty} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] = 1 - \mu$$

Expressing Zero Profit Condition In Terms of New Notation

$$\overbrace{(1-F(\bar{\omega}))\bar{\omega}+(1-\mu)\int_0^{\bar{\omega}}\omega dF(\omega)}^{\text{share of entrepreneurial profits (net of monitoring costs) given to bank}} = \frac{1+R}{1+R^k}\frac{L-1}{L}$$

$$\Gamma(\bar{\omega})-\mu G(\bar{\omega}) = \frac{1+R}{1+R^k}\frac{L-1}{L}$$

$$L = \frac{1}{1-\frac{1+R^k}{1+R}\left[\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right]}$$

- Formula for *L* indicates that we want to know about $q(\bar{\omega}) \equiv \Gamma(\bar{\omega}) - \mu G(\bar{\omega})$
- The hazard function is increasing for log normal *F* (see BGG):

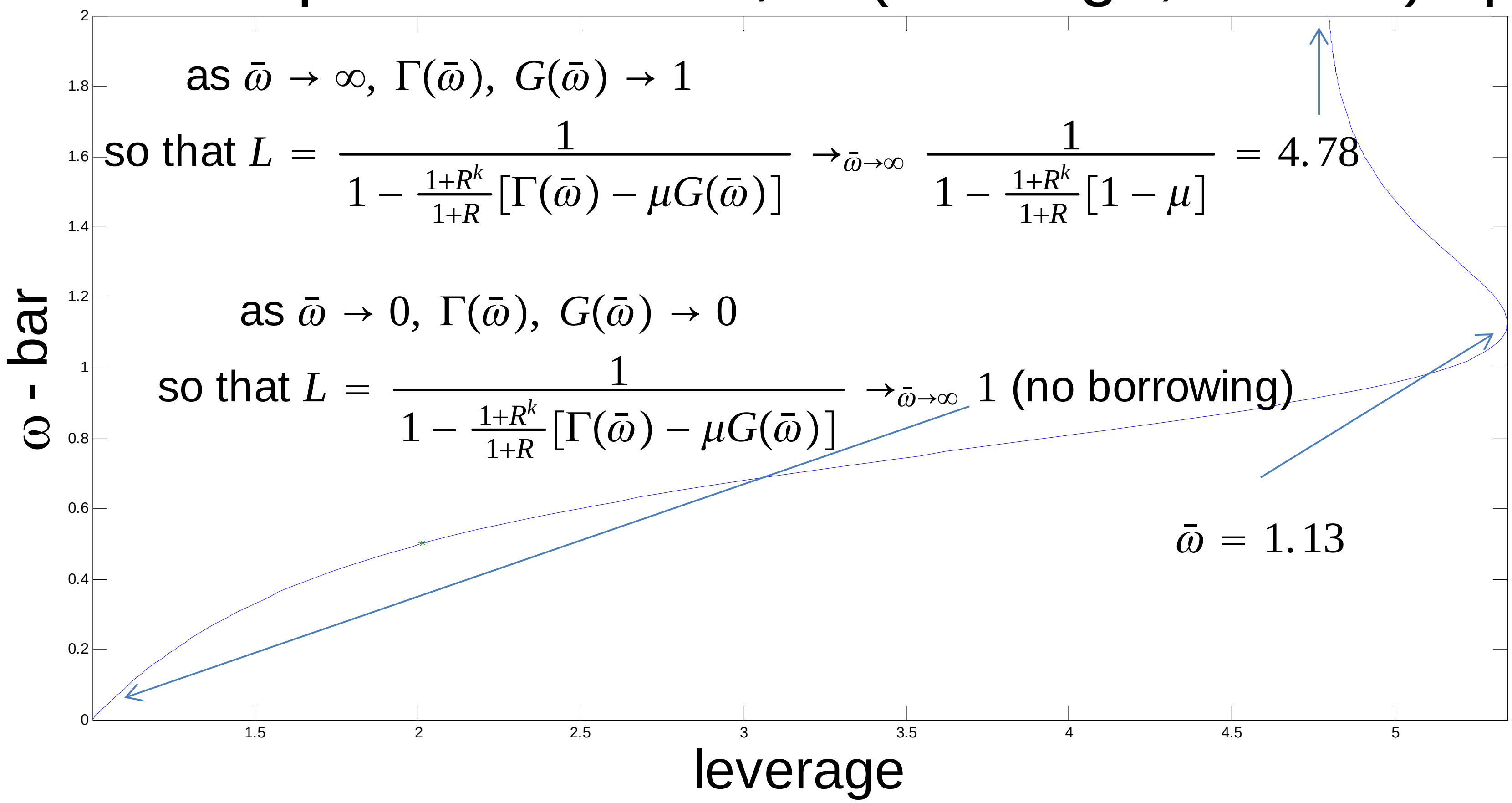
$$h(\bar{\omega}) = \frac{\bar{\omega}F'(\bar{\omega})}{1-F(\bar{\omega})}$$

- Differentiate $q(\bar{\omega})$:

$$\begin{aligned} q'(\bar{\omega}) &= 1 - F(\bar{\omega}) - \mu\bar{\omega}F'(\bar{\omega}) \\ &= 1 - F(\bar{\omega}) - \mu h(\bar{\omega})(1 - F(\bar{\omega})) \\ &= [1 - F(\bar{\omega})][1 - \mu h(\bar{\omega})] \end{aligned}$$

- So, $q(\bar{\omega})$ initially rises and then falls. *L* does too, explaining the basic shape of the zero profit function (see BGG(1999, p. 1382)).

Bank zero profit condition, in (leverage, ω - bar) space



conclude: possible equilibrium $\bar{\omega}$'s, $[0, 1.13]$