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D11-2, Winter 1996

FINAL EXAM

The exam is in three equally weighted parts. If a question seems ambiguous, state why, sharpen it up and answer the revised question. You have 2 hours. Good luck!

1. Consider the following two-period economy. The typical household's preference are given by $u(c_1 + c_2; l)$; where c_i denotes consumption in period i; i = 1; 2; and l denotes period 2 labor e®ort. The household's ¯rst and second period budget constraints are given by:

$$c_1 + k \;\; \cdot \;\; !$$
$$c_2 \;\; \cdot \;\; (1 \text{ ¡ } \pm)Rk + (1 \text{ ¡ } ¿)l;$$

where ! is the household's endowment, R is the rental rate on capital, ± is the tax rate on capital, ¿ is the tax rate on labor, and k is capital. Assume that if the household is indi®erent between c_1 and c_2 , then it sets $c_1 = 0$: The household chooses c_1 and k at the beginning of period 1, and c_2 and l at the beginning of period 2:

The government selects values for ± and ¿ to maximize the utility of the typical household, subject to the following constraints:

$$G \cdot \pm Rk + ¿l; \;\; 0 \cdot \pm \cdot 1; \;\; 0 \cdot ¿ \cdot 1;$$

where G denotes per-capita government spending. Assume:

$$(R \text{ ¡ } 1)! < G; \;\; R! > G:$$

(a) Suppose the government has the ability to commit to a ±; ¿ policy before period 1. De¯ne a Ramsey equilibrium, the equilibrium concept relevant for this scenario. What value does the capital tax rate take on in a Ramsey equilibrium? What values do k and c_1 take on? Is the labor tax rate positive? Explain carefully.

(b) Suppose that at the end of period 1, the government has an opportunity to deviate from the Ramsey policy. Would it choose to lower or raise the capital rate, or not change it at all? Would it choose a positive labor tax rate? Show that the government could raise the representative household's utility by deviating from the Ramsey plan. (For this, you may find it convenient to adopt the following parametric form for household utility: $u(c_1+c_2; l) = c_1+c_2 - 0.5l^2$.)

(c) Suppose it is known before period 1 that the government has no ability to commit to a δ, τ policy. Define a sustainable equilibrium, which is a useful equilibrium concept for this scenario. What is the value of δ in this equilibrium?

2. Consider an economy with no uncertainty (either fundamental or non-fundamental) in which, at each date t, the typical household solves the following problem:

$$\max_{\{c_j, k_{j+1}, l_j\}_{j=t}^{\infty}} \sum_{j=t}^{\infty} \beta^{j-t}\left[\log(c_j) + \sigma \log(1 - l_j)\right]$$

subject to

$$c_j + k_{j+1} - (1-\delta)k_j \le r_j k_j + w_j l_j + \int_0^1 \pi_{ij}\, di; \quad j \ge t;$$

and $c_j \ge 0$; $0 \le l_j \le 1$; where $0 \le \delta, \beta \le 1$; $\sigma > 0$.

Final output is produced by perfectly competitive firms using the following production technology:

$$y_t = \left(\int_0^1 y_{it}^{\lambda}\, di\right)^{\frac{1}{\lambda}}; \quad 0 < \lambda < 1;$$

where y_{it}, $i \in (0, 1)$ denotes intermediate goods. These firms sell y_t at a price normalized at unity, and buy y_{it} in competitive markets at prices p_{it}. They maximize profits in the usual way.

Intermediate goods firms produce y_{it} using the following increasing returns to scale production function:

$$y_{it} = a k_{it} l_{it}^{\nu}; \quad a, \nu > 0.$$

They rent k_{it} and l_{it} in competitive factor markets at rental rates r_t and w_t, respectively. At date t they solve:

$$\pi_{it} = \max_{k_{it}, l_{it}} p_{it} y_{it} - r_t k_{it} - w_t l_{it}.$$

The resource constraints for this economy are:

$$\begin{aligned} c_t + k_{t+1} - (1-\delta)k_t &\leq y_t, \\ \int_0^1 k_{it}\,di &= k_t, \quad \int_0^1 l_{it}\,di = l_t. \end{aligned}$$

You may restrict your attention just to equilibria in which firms behave symmetrically.

(a) What restrictions on λ are needed to ensure that the monopolist's problem is well defined?

(b) Define a sequence of markets equilibrium for this economy.

(c) Establish a set of conditions which can be used to determine whether a given sequence, $\{l_t; t = 0, 1, 2, \ldots\}$, forms a part of an equilibrium.

(d) If $\nu = 1$, is it possible to have 'regime switching' equilibria, in which allocations and prices suddenly appear to obey a different set of laws? What if $\nu = 2$? Explain.

3. The typical household can engage in two types of activities: producing current output and studying at home. Although time spent studying at home sacrifices current production, it augments future output by increasing the household's future stock of human capital, k_{t+1}. The household has one unit of time available to split between home study and current production. Any given amount of human capital accumulation, k_{t+1}/k_t, leaves an amount of time, h_t, left over for producing current output, where $h_t = \phi(k_{t+1}/k_t)$. Here, ϕ is strictly decreasing, strictly concave, and continuously differentiable, with

$$\begin{aligned} \phi(1-\delta) &= 1 \text{ for some } \delta \in (0,1), \\ \phi(1+\lambda) &= 0 \text{ for some } \lambda > 0. \end{aligned}$$

The variable, h_t, must satisfy $0 \leq h_t \leq 1$. This implies that the household cannot set k_{t+1}/k_t greater than $1+\lambda$ or less than $1-\delta$.

A household's effective labor input into production is the product of its time and human capital: $h_t k_t$. Total output is related to effective labor input by

$$f(h_t k_t) = (h_t k_t)^{\alpha}, \quad \alpha \in (0,1).$$

The resource constraint for this economy is

$$c_t \leq f(h_t k_t),$$

and the initial level of human capital, k_0, is given. The utility value of a given sequence of consumption, c_t, is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \text{ where } u(c_t) = c_t^{\sigma}/\sigma, \ \sigma < 0.$$

(a) Express the planning problem for this economy as a sequence problem (SP). Write out the associated functional equation (FE).

(b) Show that $v(k) = Ak^{\sigma\alpha}$ is a solution to FE, and that the maximum is attained by the policy function, $g(k) = \theta k$ for some $(1-\delta) < \theta < (1+\lambda)$. (You may assume, without proof, that the maximum of the FE problem occurs in the interior of the relevant constraint set. Explain why this assumption is useful.)

(c) Suppose there are two separate economies, which differ only in how patient households are. In the more patient economy the discount rate is $\bar{\beta}$ and in the less patient economy, the discount rate is $\beta < \bar{\beta}$. Show that in the economy with more patient households, the growth rate of human capital is greater.