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ANSWERS TO FINAL EXAM

1. Answers to question 1

   (a) Let $\pi = (\delta, \tau)$ denote a government policy, and let $F(\pi) = (c_1(\pi), c_2(\pi), l(\pi), k(\pi))$, denote the equilibrium competitive allocations given policy π. A Ramsey equilibrium is a π^* and $F(\pi)$ where: (i) for any π, $F(\pi)$ maximizes the household's utility subject to its budget constraints, and (ii) π^* solves the problem, maximize, over π, $u(c_1(\pi) + c_2(\pi), l(\pi))$, subject to the government's budget constraint. The capital tax rate satisfies $(1 - \delta)R = 1$. Any tax rate higher than this would result in $k = 0$, and so no revenues from the capital tax. Any tax rate lower than this would result in $k = \omega$, with revenues to the government equal to $\delta R\omega$. The marginal effect of raising δ from such a low level operates like a lump sum tax, and so the government would never settle for such a low tax rate. Also, $k = \omega$ and $c_1 = 0$. At the Ramsey tax rate, $(1 - \delta)R = 1$, or $\delta = (R - 1)/R$, so that government revenues from taxing capital total $\delta R k = (R - 1)\omega < G$ by assumption. Since the Ramsey tax on capital is not enough to fund all of government spending, the Ramsey labor tax rate must be positive.

   (b) At the end of period 1, after $k = \omega$, the government has an incentive to increase δ above its Ramsey value, so that $\delta R\omega = G$. Note that this implies a deviation up in the capital tax rate, since $(R - 1)\omega < G \Rightarrow R - 1 < G/\omega \Rightarrow (R - 1)/R < G/(R\omega)$. Since by assumption, $R\omega > G$, $\delta \leq 1$ will work. Also, with this deviation from the Ramsey capital tax rate, it would be possible to set the labor tax rate to zero.

   To see that the government would raise utility by deviating, note that the household's first order condition for choosing labor is: $l = 1 - \tau$. Under the Ramsey policy and under a deviation, $c_1 = 0$, $k = \omega$, $c_2 = (1 - \delta)R\omega + (1 - \tau)l = (1 - \delta)R\omega + (1 - \tau)^2$. Thus, the policy of deviating solves

$$\max_{\delta,\tau} u(c_1 + c_2, l) = (1 - \delta)R\omega + \frac{1}{2}(1 - \tau)^2, \text{ s.t. } G = \delta R\omega + \tau(1 - \tau).$$

The government budget constraint can be rewritten, (1 ¡ ±)R! = R! ¡ G + ¿(1 ¡ ¿): Then, substituting out for ± into the government's objective:

$$\max_{¿} R! \ ¡ \ G + ¿(1 \ ¡ \ ¿) + \frac{1}{2}(1 \ ¡ \ ¿)^2;$$

or

$$\max_{¿} R! \ ¡ \ G + \frac{1}{2}(1 \ ¡ \ ¿^2):$$

This objective function is strictly decreasing in ¿ for 0 · ¿ · 1: Since ¿ > 0 under the Ramsey plan, it follows that utility is increased by reducing the labor tax rate from its Ramsey value.

(c) A sustainable equilibrium is a collection of numbers and two functions, ¿¤; ±¤; $\bar{c}_1$; c_2(±; ¿); l(±; ¿); $\bar{k}$; satisfying the following three properties: (i) the household problem is solved. That is, at date 1, $\bar{c}_1$; c_2(±¤; ¿¤); l(±¤; ¿¤); $\bar{k}$ solve the problem: max u(c_1 + c_2; l) over c_1; c_2; l; and k, subject to the period 1 and period 2 budget constraints and that the capital and labor tax rates are given by ±¤; ¿¤: The functions, c_2(±; ¿); l(±; ¿); solve for any ±; ¿; the household's period 2 maximization problem: max over c_2; l; the problem u($\bar{c}_1$ + c_2; l) subject to c_2 · (1 ¡ ±)R$\bar{k}$ + (1 ¡ ¿)l: (ii) ±¤; ¿¤ solve the government problem: maximize u($\bar{c}_1$ + c_2(±; ¿); l(±; ¿)) over ±; ¿; subject to the government budget constraint. In a sustainable equilibrium, ± = 1: If $\bar{k}$ > 0; then ± = 1 clearly is optimal since this maximizes revenues from what is a lump-sum tax. If $\bar{k}$ = 0; then all values of ± produce the same return for the government, as so ± = 1 is optimizing in this case too.