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D11-2, Winter 1997
Homework 3. Due: Wednesday, January 29.
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1. Answer to second question. Establishing quasi monotonicity is easy.

2. Consider an economy with a large number of identical households, each having preferences, $\sum_{t=0}^{1} \beta^t u(c_t)$: Suppose the resource constraint is $c_t + i_t \cdot f(k_t)$; where $k_{t+1} = i_t + (1 - \delta)k_t$; f is strictly increasing and concave, $f'(k) \to 1$ as $k \to 0$; $f'(k) \to 0$ as $k \to 1$, $0 < \delta < 1$: Assume investment, i_t; is irreversible, i.e., it must be that $i_t \geq 0$: In addition, suppose $c_t; k_t \geq 0$ and that $k_0 > 0$ is given. Consider the functional equation associated with this problem:

$$v(k) = \max_{k' \in \Gamma(k)} u(f(k) + (1 - \delta)k - k') + \beta v(k')$$
$$\Gamma(k) = \{k' : (1 - \delta)k \cdot k' \cdot f(k)\}:$$

   (a) State a set of assumptions on β and u that guarantee there is a unique, differentiable, concave v that solves the above functional equation. For each property of v; explain which assumptions are used to get it.

   (b) Show that monotonicity of $\Gamma(k)$, Assumption 4.6 in S-L, fails so that one of the conditions of Theorem 4.7 which guarantee strictly increasing v; is not satisfied.

   (c) Show that the feasible set for this economy satisfies the following `quasi-monotonicity property': if $\tilde{k} \geq k$; then $\Gamma(k) + (1 - \delta)(\tilde{k} - k) \subseteq \Gamma(\tilde{k})$: Here, the sum of a set, say X; and a number, say a; is a new set, $X + a$; where $X + a \equiv \{x + a : x \in X\}$:

   (d) Show: v is an increasing function in k: (Hint: (i) following the basic strategy of the proof of Theorem 4.7, it's enough to establish that the assumptions of Theorem 4.7 with the monotonicity assumption on Γ replaced by quasi-monotonicity guarantee Tw is increasing if w is; (ii) make use of the fact that if $k' \in \Gamma(k)$; then $\tilde{k}' = k' + (1 - \delta)(\tilde{k} - k) \in \Gamma(\tilde{k})$, $\tilde{k}' > k'$; and $f(\tilde{k}) + (1 - \delta)\tilde{k} - \tilde{k}' > f(k) + (1 - \delta)k - k'$.) Can you provide intuition for the fact that v is increasing even though Γ fails to satisfy monotonicity?

3. Consider the economy in question (1); except that $\pm = 1$: De¯ne a recursive competitive equilibrium for this economy, and display explicitly the household's value function ($V(K;k)$ in S-L, p. 30), policy function, $H(K;k)$; the aggregate law of motion for capital, $k' = h(k)$; and the aggregate pricing functions, $R(k)$ and $!(k)$; for this economy.