1. Answer to second question. Establishing quasi-monotonicity is easy.

2. Consider an economy with a large number of identical households, each having preferences, \( \frac{1}{t} \sum_{t=0}^{\infty} u(c_t) \): Suppose the resource constraint is \( c_t + i_t \cdot f(k_t) \); where \( k_{t+1} = i_t + (1 - \varepsilon) k_t \); \( f \) is strictly increasing and concave, \( f'(k) \) ! 1 as \( k \rightarrow 0 \), \( f'(k) \) ! 0 as \( k \rightarrow 1, 0 < \varepsilon < 1 \):

Assume investment, \( i_t \); is irreversible, i.e., \( i_t \geq 0 \): In addition, suppose \( c_t; k_t \geq 0 \) and that \( k_0 > 0 \) is given. Consider the functional equation associated with this problem:

\[
v(k) = \max_{k_0 \in (k)} u(f(k) + (1 - \varepsilon) k \cdot k^0) + v(k^0)
\]

(a) State a set of assumptions on \( \bar{u} \) and \( u \) that guarantee there is a unique, differentiable, concave \( v \) that solves the above functional equation. For each property of \( v \); explain which assumptions are used to get it.

(b) Show that monotonicity of \( i(k) \), Assumption 4.6 in S-L., fails so that one of the conditions of Theorem 4.7 which guarantee strictly increasing \( v \); is not satisfied.

(c) Show that the feasible set for this economy satisfies the following `quasi-monotonicity property': if \( k \leq k' \); then \( \mu(i(k)) + (1 - \varepsilon)(k') \mu(i(k)) \mu(i(k')) \): Here, the sum of a set, say \( X \); and a number, say \( a \); is a new set, \( X + a \); where \( X + a \) \( \cup \) \( fX + a : x \in X \): g;

(d) Show: \( v \) is an increasing function in \( k \): (Hint: (i) following the basic strategy of the proof of Theorem 4.7, it’s enough to establish that the assumptions of Theorem 4.7 with the monotonicity assumption on \( i \); replaced by quasi-monotonicity guarantee \( Tw \) is increasing if \( w \) is; (ii) make use of the fact that if \( k^0 \geq i(k) \); then \( k^0 \geq k^0 + (1 - \varepsilon)(k') \geq k^0 \); \( R^0 > k^0 \); and \( f(k^0) + (1 - \varepsilon)k^0 \); \( R^0 > f(k) + (1 - \varepsilon)k \); \( k^0 \): Can you provide intuition for the fact that \( v \) is increasing even though \( i \); fails to satisfy monotonicity?
3. Consider the economy in question (1); except that $\varepsilon = 1$: Define a recursive competitive equilibrium for this economy, and display explicitly the household's value function ($V(K;k)$ in S-L, p. 30), policy function, $H(K;k)$; the aggregate law of motion for capital, $k^0 = h(k)$; and the aggregate pricing functions, $R(k)$ and $! (k)$; for this economy.