

Homework #6
 Economics D11-2
 Due Thursday, February 27, 1997.
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1. Suppose a typical household solves

$$\max_{c,n} u(c; n); \text{ subject to } c + (1 - \tau)wn; c, n \geq 0;$$

where w is the wage rate, τ is the labor tax rate, n is hours worked, and c is consumption. Suppose the production function is $f(n) = n$; so that the equilibrium wage rate is $w = 1$: Also, suppose the utility function has the following form:

$$u(c; n) = c + \frac{n^2}{2}$$

Suppose there is a government which has to finance a fixed level of expenditures, $g = \frac{1}{4}$; which are tossed into the ocean. The government's task is to choose $0 \leq \tau \leq 1$ to maximize the utility of the representative agent, subject to its budget constraint:

$$g = \tau wn:$$

- (a) Suppose the government commits itself to a value for τ before the households make their decisions (i.e., it solves the 'Ramsey problem'). What is the set of τ consistent with the government satisfying its budget constraint? What range of household utilities is associated with the elements in this set? What is the tax rate and utility level that solves the Ramsey problem?
- (b) Suppose the government selects a value for τ after households have committed themselves to a level of employment. How many sustainable equilibria are there? What are the associated utility levels?
- (c) Replace the government budget constraint by $g = T$; where T represents a lump-sum tax. Also, replace the household's budget constraint by

$$c + wn = T:$$

What is the welfare gain from going to the lump sum tax economy from the Ramsey problem?

2. Consider the following two-sector model. Consumption goods, c_t , are produced using the following production function:

$$c_t = k_{c,t}^\alpha (z_t l_{c,t})^{1-\alpha};$$

where $k_{c,t}$ and $l_{c,t}$ denote capital and labor used in the consumption goods sector, and z_t is a technology shock. The production function in the investment good sector is:

$$I_t = V_t k_{i,t}^\alpha (z_t l_{i,t})^{1-\alpha};$$

where $k_{i,t}$ and $l_{i,t}$ are defined as in the consumption good sector and

$$k_{t+1} = (1 - \delta)k_t + I_t;$$

Here, V_t is a technology shock that is specific to the investment good sector. Clearing in the labor and capital markets requires:

$$l_t = l_{i,t} + l_{c,t}; \quad k_t = k_{i,t} + k_{c,t};$$

The household and firm problems are defined as in question 2 of homework 4.

- (a) Show: it must necessarily be the case that $l_{c,t}$ and $l_{i,t}$ move in opposite directions in response to shocks (hint: write out the Household intratemporal Euler equation, replace the wage by the marginal product of labor in the consumption good sector, and stare). How could you change the model to break this (wildly counterfactual) implication?
- (b) Show that $P_t = 1/V_t$: Thus, if there is an upward trend in V_t ; then the price of investment goods falls.
- (c) Show that, in equilibrium:

$$\frac{l_{c,t}}{k_{c,t}} = \frac{l_{i,t}}{k_{i,t}};$$

(d) Show that this economy is equivalent to a one-sector economy in which the resource constraint is:

$$c_t + \frac{I_t}{V_t} = k_t^\alpha (z_t h_t)^{1-\alpha} :$$

3. Exercise 6.7a-d, SL, p. 157.
4. Consider a model in which utility is a function not just of market consumption, c ; and market labor effort, l ; but also of consumption of home produced goods or services, c_n ; and home labor effort, l_n : Specifically,

$$\log(c + c_n) = \alpha \log \left(\frac{l^{1+\bar{\alpha}}}{1+\bar{\alpha}} + l_n \right) ;$$

where $\alpha > 0$; $\bar{\alpha} > 0$: The home labor effort yields services via the home production function, $c_n = \bar{\alpha} l_n$: Show that this formulation implies a utility function in terms of market goods and labor having the following form:

$$\text{constant} + \alpha \log \left(c + \bar{\alpha} \frac{l^{1+\bar{\alpha}}}{1+\bar{\alpha}} \right) ,$$

where 'constant' and α are parameters. (Hint: recall how we got $F(k; k^0)$ for the version of the growth model in which utility is a function of labor effort, in addition to consumption.)