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D11-2, Winter 1998

FINAL EXAM

The exam is in three equally weighted parts. If a question seems ambiguous, state why, sharpen it up and answer the revised question. You have 2 hours. Good luck!

1. Consider the following two-period economy. The typical household's preference are given by $u(c_1 + c_2, l)$, where c_i denotes consumption in period i, $i = 1, 2$, and l denotes period 2 labor effort. The household's first and second period budget constraints are given by:

$$\begin{aligned} c_1 + k &\leq \omega \\ c_2 &\leq (1-\delta)Rk + (1-\tau)l, \end{aligned}$$

where ω is the household's endowment, R is the rental rate on capital, δ is the tax rate on capital, τ is the tax rate on labor, and k is capital. Assume that if the household is indifferent between c_1 and c_2 , then it sets $c_1 = 0$. The household chooses c_1 and k at the beginning of period 1, and c_2 and l at the beginning of period 2.

The government selects values for δ and τ to maximize the utility of the typical household, subject to the following constraints:

$$G \leq \delta Rk + \tau l, \; 0 \leq \delta \leq 1, \; 0 \leq \tau \leq 1,$$

where G denotes per-capita government spending. Assume:

$$(R-1)\omega < G, \; R\omega > G.$$

(a) Suppose the government has the ability to commit to a δ, τ policy before period 1. Define a Ramsey equilibrium, the equilibrium concept relevant for this scenario. What value does the capital tax rate take on in a Ramsey equilibrium? What values do k and c_1 take on? Is the labor tax rate positive? Explain carefully.

(b) Suppose that at the end of period 1, the government has an opportunity to deviate from the Ramsey policy. Would it choose to lower or raise the capital rate, or not change it at all? Would it choose a positive labor tax rate? Show that the government could raise the representative household's utility by deviating from the Ramsey plan. (For this, you may find it convenient to adopt the following parametric form for household utility: $u(c_1+c_2, l) = c_1+c_2-0.5l^2$.)

(c) Suppose it is known before period 1 that the government has no ability to commit to a δ, τ policy. Define a sustainable equilibrium, which is a useful equilibrium concept for this scenario. What is the value of δ in this equilibrium?

2. Consider the following model. Preferences of the typical household are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where

$$u(c_t) = \frac{c^{1-\nu} - 1}{1-\nu}, \ \nu > 0.$$

The household accumulates private capital using the following accumulation technology:

$$k_{p,t+1} = (1-\delta_p)k_{p,t} + i_{p,t},$$

where $k_{p,t}$ is the beginning-of-period t stock of capital, $i_{p,t}$ is period t gross investment, and $0 < \delta_p < 1$. The initial stock of capital, $k_{p,0}$, is given. The household is endowed with $n > 0$ units of labor time, and must satisfy $c_t, k_{p,t} \geq 0$.

The technology for producing output is

$$y_t = k_{gt}^{\gamma} k_{pt}^{\alpha} n_t^{(1-\alpha)}, \ 0 < \alpha < 1, \gamma \geq 0,$$

where k_{gt} is the per capita stock of government-provided capital. The household and firm take the sequence $\{k_{gt}\}$ as given and beyond their control.

The technology for accumulating government capital is:

$$k_{g,t+1} = (1-\delta_g)k_{gt} + i_{gt},$$

where i_{gt} denotes per capita gross investment by the government, and $0 < \delta_g < 1$. The government finances investment by levying taxes, T_t, in the amount:

$$i_{gt} = T_t.$$

Taxes are levied on households in *lump-sum* form: each household must pay T_t, regardless of what decisions it takes regarding consumption, investment, or labor effort.

Finally, the resource constraint for this economy is:

$$c_t + i_{pt} + i_{gt} \leq y_t.$$

(a) Suppose the government chooses its sequence of public investment so that

$$k_{gt} = sk_{pt}, \ t \geq 1, \ s > 0.$$

Sharpen up the statement of the household and firm problems, and:

i. define a sequence of markets equilibrium.

ii. define a date 0 Arrow-Debreu equilibrium.

iii. define a recursive competitive equilibrium.

(b) Suppose $\gamma = 1 - \alpha$. Show that s can be chosen so that there is a steady-state balanced growth path for this economy, in which all quantity variables but employment display the same positive growth rate. Explain intuitively, why steady state growth is possible in this case, but is not when $\gamma = 0$.

(c) Display an optimization problem, the solution of which gives the efficient allocations for this economy. Write out the Euler equations for this problem. Are they sufficient for a maximum to the problem? Explain your answer.

(d) Suppose $\gamma = 1 - \alpha$. Suppose the economy is in a sequence-of-markets equilibrium, and that the government must optimally choose a sequence, i_{gt}, $t = 0, 1, 2, \ldots$. Would that sequence be consistent with the specification for public investment in (1)? Explain your answer.

3. Consider the following two-sector model. Consumption goods, c_t, are produced using the following production function:

$$c_t \leq k_{c,t}^{\alpha} \left(z_t l_{c,t}\right)^{1-\alpha},$$

where $k_{c,t}$ and $l_{c,t}$ denote capital and labor used in the consumption goods sector, and z_t is a technology shock. The production function in the investment good sector is:

$$I_t \leq V_t k_{i,t}^{\alpha} \left(z_t l_{i,t}\right)^{1-\alpha},$$

where $k_{i,t}$ and $l_{i,t}$ are defined as in the consumption good sector and

$$k_{t+1} = (1-\delta)k_t + I_t.$$

Here, V_t is a technology shock that is specific to the investment good sector. Clearing in the labor and capital markets requires:

$$l_t = l_{i,t} + l_{c,t}, \;\; k_t = k_{i,t} + k_{c,t}.$$

Let the household's utility function be strictly concave and exhibit unit elasticity of substitution between consumption, c_t, and leisure $1 - l_t$. Let the firm problem be defined in the usual way.

(a) Show: it must necessarily be the case that $l_{c,t}$ and $l_{i,t}$ move in opposite directions in response to shocks (hint: write out the Household intratemporal Euler equation, replace the wage by the marginal product of labor in the consumption good sector, and stare). How could you change the model to break this counterfactual implication? Explain.

(b) Show that $P_t = 1/V_t$. Thus, if there is an upward trend in V_t, then the price of investment goods falls.

(c) Show that, in equilibrium:

$$\frac{l_{c,t}}{k_{c,t}} = \frac{l_{i,t}}{k_{i,t}}.$$

(d) Show that this economy is equivalent to a one-sector economy in which the resource constraint is:

$$c_t + \frac{I_t}{V_t} = k_t^{\alpha} \left(z_t h_t\right)^{1-\alpha}.$$