Economics D11-1
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Model Introduced in Lecture #1 and in Recitation

1. The Growth Model

   The simple growth model we developed in class last time:

$$\max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t),$$

   $0 < \beta < 1$,subject to

$$c_t + k_{t+1} - (1-\delta)k_t \leq f(k_t),\ 0 < \delta < 1,$$

   and

$$c_t \geq 0,\ k_{t+1} \geq 0, \text{ for all } t = 0, 1, .$$

   and

$$k_0 > 0,\ \text{fixed}.$$

   Assumptions: u, f increasing, differentiable, concave.

2. Canonical Model

$$\max_{\{x_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t F(x_t, x_{t+1})$$

   subject to:

$$\begin{aligned} x_t &\in X \subseteq R^l \\ x_{t+1} &\subset \Gamma(x_t),\ t = 0, 1, ... \\ \Gamma &: X \to X \\ F &: A \to R \\ A &= \{(x, y) : x \in X,\ y \in \Gamma(x)\} \\ x_0 &\in X,\ \text{given}. \end{aligned}$$

3. Assumptions:

$A4.3$	:	X convex subset of R^l, Γ non-empty, compact, continuous.
$A4.4$	:	$F : A \to R$ bounded, continuous, $0 < \beta < 1$.
$A4.5$	:	for each y, $F(\cdot, y)$ is strictly increasing for each fixed y.
$A4.7$	:	F strictly concave.
$A4.9$	:	F continuously differentiable on interior of A.

Result. Suppose $x_{t+1}^* \in int\left(\Gamma(x_t^*)\right)$, $x_0^* = x_0$ and:

$$\begin{aligned} E &: \quad F_2(x_{t-1}^*, x_t^*) + \beta F_1(x_t^*, x_{t+1}^*) = 0, \; t = 0, 1, 2, ... \\ TV &: \quad \lim_{t \to \infty} \beta^t F_1(x_t^*, x_{t+1}^*) x_t^* = 0. \end{aligned}$$

Then,

$$\sum_{t=0}^{\infty} \beta^t F(x_t^*, x_{t+1}^*) \geq \sum_{t=0}^{\infty} \beta^t F(x_t, x_{t+1}),$$

for all other feasible x_t, $t = 0, 1, 2, ...$, where an infinite sum is interpreted as the limit of the following sequence:

$$\lim_{T \to \infty} \sum_{t=0}^{T} \beta^t F(x_t, x_{t+1}).$$