Homework #7
Due Friday, November 13.
D11-1, Fall 1998
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1. One reason why the rate of return on capital may not be falling, despite the fact that capital continues to grow, is that there are increasing returns to scale in production. Generally, increasing returns to scale leads gives rise to monopoly power. However, increasing returns does not *necessarily* lead to monopoly power. It is possible to build a theory of growth in which the rate of return on capital remains high along a growth path because of increasing returns, and yet there is perfect competition. This question explores such a model. Increasing returns and perfect competition are compatible if the increasing returns are *external* to the firm, while the firm production function exhibits constant returns to scale in the factors that the firm controls. For a paper which pushes for the notion that a version of such a model with exogenous shocks to technology represents a good business cycle model, see Murphy, Kevin M., Andrei Shleifer, and Robert W. Vishny, 1989, "Building blocks of market clearing business cycle models," NBER Macroeconomics Annual 1989, edited by Olivier J. Blanchard and Stanley Fischer, MIT Press.

   Following is a description of the economy. The typical household (there are lots of them, all identical) chooses consumption, $\widetilde{c}_t$, hours worked, $\widetilde{n}_t$, and gross investment, $\widetilde{k}_{t+1} - (1-\delta)\widetilde{k}_t$, to maximize $\sum_{j=0}^{\infty} \beta^t u(\widetilde{c}_t, \widetilde{n}_t)$ subject to its budget constraint,

$$\widetilde{c}_t + \widetilde{k}_{t+1} - (1-\delta)\widetilde{k}_t = r_t\widetilde{k}_t + w_t\widetilde{n}_t + \pi_t,$$

   for $t = 1, \ldots, \infty$. It takes the initial stock of capital, $\widetilde{k}_0$, as given, as well as market prices and profits, $r_t, w_t, \pi_t, t = 1, \ldots, \infty$. The utility function is:

$$u(c,n) = \log(c) + \sigma \log(1-n).$$

   There is a representative firm which hires labor and capital and uses these to produce output, $\widetilde{y}_t$, using the production technology,

$$\widetilde{y}_t = A_t F(\widetilde{k}_t, \widetilde{n}_t)$$

where

$$F(\widetilde{k}_t, \widetilde{n}_t) = \widetilde{k}_t^{\theta} \widetilde{n}_t^{(1-\theta)},$$

and

$$A_t = y_t^{\gamma},\ 1 > \gamma \geq 0,$$

y_t denotes the average, economy-wide, output of all firms. The firm takes y_t and, hence, A_t, as exogenous and beyond its control. The firm chooses $\widetilde{k}_t$ and $\widetilde{n}_t$ to maximize profits:

$$\pi_t = \widetilde{y}_t - r_t \widetilde{k}_t - w_t \widetilde{n}_t,$$

treating prices and A_t as given and beyond its control. The object, A_t, is an *externality.* It captures the notion that a high degree of activity in the economy (i.e., high y_t) shifts individual firms' production functions up. This may reflect that when there is a high amount of activity new ideas are being generated more rapidly, and these are freely transferable across firms. It may reflect that at a high level of activity, it is profitable for specialized suppliers to come into existence. Another possibility, emphasized by Diamond ('Aggregate Demand Management in Search Equilibrium', JPE, 1982) is that it reflects a search externality that exists because as the number of potential trading partners increases, it becomes easier for any one trader to find a partner. Whatever it is that accounts for the presence of y_t in the production function, if it enters with a sufficiently large value of γ, it should be clear that it may offset the depressing effect of investment on the marginal product of capital and thus permit sustained growth.

A sequence of markets competitive equilibrium is a set of prices and profits, $\{r_t, w_t, \pi_t\}$, and allocations, $\{\widetilde{c}_t, \widetilde{k}_t, \widetilde{n}_t\}$, such that the typical household and firm maximizes (with perfect foresight about later prices) and markets clear. For the goods market, market clearing means that the aggregate resource constraint must be satisfied:

$$c_t + k_{t+1} - (1-\delta)k_t \leq A_t F(k_t, n_t),$$

for all t, where a variable without a tilde, $\widetilde{\cdot}$, denotes its average, economy-wide value. Since everyone is identical, it seems natural to consider only equilibria in which everyone does the same thing, i.e., tilde'd variables equal their un-tilde'd counterparts.

(a) What is the value of profits, π_t, in equilibrium? Explain.

(b) Write out the first order conditions that the allocations in competitive equilibrium must satisfy. Write them in such a way that prices and profits have been substituted out. Write them in terms of aggregate, economy-wide variables.

(c) Write out the planning problem for this economy in sequence form. Write the first order conditions for this problem, which are analogous to the objects you derived for the competitive equilibrium above. Any interesting differences?

(d) Define a steady-state *balanced growth path* as a situation in which

$$c_{t+1} = \lambda c_t, k_{t+1} = \lambda k_t, n_t = n,$$

where λ is the gross growth rate of the economy.

(e) Suppose $\theta/(1-\gamma) = 1$. Show that $\lambda > 1$ is possible.

2. This question explores the restrictions on preferences and technology that the existence of balanced growth path requires. Consider the following growth model:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t)$$

subject to the resource constraint:

$$c_t + k_{t+1} - (1-\delta)k_t \leq f(k_t, \gamma^t n_t), \ 0 < \alpha < 1,$$
$$k_{t+1}, c_t \geq 0, 0 \leq n_t \leq 1, \gamma > 1.$$

Also,

$$u(c_t, 1-n_t) = \{(1-\sigma)c_t^{\frac{\rho-1}{\rho}} + \sigma(1-n_t)^{\frac{\rho-1}{\rho}}\}^{\frac{\rho\psi}{\rho-1}}/\psi, \text{ for } \rho > 0, \neq 1,$$

and

$$u(c_t, 1-n_t) = [c_t^{(1-\sigma)}(1-n_t)^{\sigma}]^{\psi}/\psi, \text{ for } \rho = 1,$$

where the (constant) elasticity of substitution between c_t and $(1-n_t)$ is ρ, and

$$f(k,n) = \begin{cases} \{(1-\alpha)k^{\frac{\nu-1}{\nu}} + \alpha n^{\frac{\nu-1}{\nu}}\}^{\frac{\nu}{\nu-1}}, \text{ for } \nu > 0, \neq 1 \\ k^{(1-\alpha)}n^{\alpha}, \text{ for } \nu = 1, \end{cases}$$

where ν is the elasticity of substitution between n and k in production.

(a) Define a balanced growth path as an equilibrium in which

$$n_t = n, \frac{k_{t+1}}{k_t} = \frac{c_{t+1}}{c_t} = \gamma.$$

Show that $\rho = 1$ is required for there to be a balanced growth path for this economy.

(b) Show that when $\rho = 1$, there is a scaling of the variables for this problem, in which the problem boils down to the standard growth model with no steady state growth.

3. Consider the following two-sector model. Consumption goods, c_t, are produced using the following production function:

$$c_t \leq k_{c,t}^{\alpha} \left(l_{c,t}\right)^{1-\alpha},$$

where $k_{c,t}$ and $l_{c,t}$ denote capital and labor used in the consumption goods sector. The production function in the investment good sector is:

$$I_t \leq V_t k_{i,t}^{\alpha} \left(l_{i,t}\right)^{1-\alpha},$$

where $k_{i,t}$ and $l_{i,t}$ are defined as in the consumption good sector and

$$k_{t+1} = (1-\delta)k_t + I_t.$$

Here, V_t is a technology shock that is specific to the investment good sector. Clearing in the labor and capital markets requires:

$$l_t = l_{i,t} + l_{c,t}, \;\; k_t = k_{i,t} + k_{c,t}.$$

The household and firm problems are defined as usual.

(a) Show that $P_t = 1/V_t$. Thus, if there is an upward trend in V_t, then the price of investment goods falls.

(b) Show that, in equilibrium:

$$\frac{l_{c,t}}{k_{c,t}} = \frac{l_{i,t}}{k_{i,t}}.$$

(c) Show that this economy is equivalent to a one-sector economy in which the resource constraint is:

$$c_t + \frac{I_t}{V_t} = k_t^{\alpha} \left(z_t h_t\right)^{1-\alpha}.$$