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Class Notes on Shleifer's 'Implementation Cycles', 1986 JPE.

1 Introduction

These notes explain one model, based on strategic complementarities, which can give rise to equilibria in which there are fluctuations in aggregate output. In contrast to the fluctuations in a real business cycle model, which are efficient, the fluctuations in the fluctuation equilibrium in the Shleifer model may be suboptimal in a welfare sense. There is another sense in which the two models are different: in the real business cycle model, booms are triggered by exogenous increases in technology, whereas in the Shleifer model booms can be triggered by expectations. Firms' expectation that output will be high can actually 'cause' output to be high. In this sense, the Shleifer model bears a strong similarity to the view about business cycles underlying the traditional 'IS-LM' vision of business cycles, which assigned a central role to 'animal spirits'.

The basic idea is simple. In 'normal' times firms earn zero profits and sell output at marginal cost. Periodically, some firms receive an idea about how to reduce marginal costs. When such firms first implement the idea, they enjoy monopoly profits. This is because, with no one else knowing the idea, it is not profitable for anyone else to jump in and undercut the price of the implementer. However, once a new idea is implemented, it cannot be kept secret for long. When the idea becomes generally known (after 'one period' in the Shleifer model), then monopoly profits are no longer possible, and price drops to the new lower level of marginal costs. These considerations create an incentive for a firm receiving a new idea to time the implementation of the idea carefully. Since monopoly profits can only be enjoyed for a short period, the firm has an incentive to implement the idea at a time when monopoly profits would be greatest. In the Shleifer model, such a time is in a boom, when aggregate employment and output are high. Thus, for example, a firm that thinks that monopoly profits may be larger in the near future may choose to delay the implementation of a new idea.

There is a strategic complementarity because the more other firms implement their ideas, the greater is the incentive for any one firm to do so. The reason is that the higher aggregate output that results when many other firms are actively implementing new ideas translates into higher demand for the products of every firm. For example, if everyone conjectures that everyone else will implement new ideas in the next period, so that the economy will be more active then, then it is in the interests of an individual firm with an idea to delay implementing it until then too. This mechanism can give rise to economic booms.

Note the importance of expectations in triggering booms in the Shleifer model. In this respect, the model differs sharply from the standard real business cycle model, in which booms are triggered by exogenous technology shocks.

An interesting feature of the model is that it suggests a mechanism by which idiosyncratic shocks at the individual firm level might result in aggregate fluctuations. It is a response to the Lucas intuition, discussed in class, and which is widely shared: that idiosyncratic events at the firm level would just wash out when viewed from the point of view of the aggregate economy. Shleifer's response is based on drawing attention to the distinction between the time that a new technological idea arrives in the firm, and the time the firm implements it. The exact timing of arrival of ideas may well be idiosyncratic at the firm level, in which case the economy-wide average rate of arrival of new ideas would be constant: firms discovering ideas for new products or labor-saving ways to produce output would be balanced by firms experiencing no progress or even regress. What the model emphasizes, however, is that it is not the arrival of new ideas *per se* that shifts up production functions. Rather, it is the *implementation* of the new ideas that does this. The model highlights the possibility that there may be plausible mechanisms in the economy which lead firms to implement new, technology shifting, ideas *at the same time*, even if the rate of arrival of new ideas is constant.

2 Households

Suppose that at date t, the representative household's preferences over consumption henceforth have the following form:

$$\sum_{j=t}^{\infty} \beta^{t-j} u(c_j),\ 0 < \beta < 1,$$

where u is strictly increasing and concave, and $c_t \geq 0$ denotes consumption. The budget constraint is:

$$c_j + \frac{B_{j+1}}{R_j} \leq B_j + w_j n_j + \pi_j, j = t, t+1, ...,$$

where n_j denotes labor, which must satisfy $0 \leq n_j \leq 1$. Also, B_{j+1}/R_j represents household lending, and we require $B_j \geq 0$. That is, households are not allowed to borrow. In addition, B_0 is given.

Suppose that $R_j \geq 1$ and $w_j \geq 0$ are such that at most finite utility is possible. Also, suppose that a particular sequence of consumption and debt holdings satisfy the initial condition and the non-negativity conditions on consumption, asset holdings, leisure and labor, and:

$$\begin{aligned} u_{cj} &= \beta R_j u_{c,j+1},\ j = t, t+1, t+2, ..., \quad (1) \\ \lim_{T\to\infty} \beta^T u_{c,T} B_T &= 0. \end{aligned}$$

Here, u_{ct} denotes the marginal utility of consumption. We know (see Theorem 4.19 in Stokey and Lucas) that these sequences solve the household's maximization problem. We also know that the first of the two conditions (the Euler equation) is *necessary* for an interior optimum.

3 Firms

The firm sector is composed of final good firms and intermediate good firms.

3.1 Final Good Firms

The representative, perfectly competitive, final good firm produces output, y_t, using inputs, $x_t(i) \geq 0$, $i \in (0, 1)$, using the following production function:

$$y_t = \exp\left[\int_0^1 \log(x_t(i)) di\right].$$

The price of the i^{th} intermediate input is $p_t(i)$. It is easily shown that the demand curve for the i^{th} intermediate input is:

$$p_t(i) = \frac{y_t}{x_t(i)}. \tag{2}$$

3.2 Intermediate Good Firms

There are two types of intermediate good producers, those with $i \in (0, 1/2)$, which we will label the 'South' and those with $i \in (1/2, 1)$, which we will label the 'North'. In each industry, $i \in (0, 1)$, there is a large number of potential producers, although only one of these actually produces $x_t(i)$. Time proceeds with $t = 1, 2, 3, ...$. In odd periods, one randomly selected firm (the 'innovator') in each industry in the South receives a new idea. In even periods, one randomly selected innovator in each industry in the North is randomly selected to receive a new idea.

Consider the situation in the South in period $t = 1$. I now describe the cost function for producing $x(i)$, for $i \in (0, 1/2)$. For *each* $i \in (0, 1/2)$, the innovator in that industry receives an idea which allows him/her to produce $x_1(i)$ units of the i^{th} intermediate input with $(c/\lambda)\, x_1(i)$ units of labor, where $c > 0$, $\lambda > 1$. Thus, the cost function available to the innovator is:

$$\text{cost}(x_1(i)) = \frac{w_1 c}{\lambda} x_1(i). \tag{3}$$

The other firms in industry i, the ones that do not receive an idea, have access to the following cost function:

$$\text{cost}(x_1(i)) = w_1 c x_1(i). \tag{4}$$

Thus, the idea received by the innovator in industry i is one that allows him/her to reduce the cost of production, by working with (3) rather than (4). The innovator can choose to implement the idea, i.e., work with (3), or it may instead choose to *not* implement the idea right away. In this case, it would use (4) in period 1. Whether the innovator implements in period 1 or not, it cannot charge more than $p_1(i) = w_1 c$. If it tried to, the other firms (the 'competitive fringe') would find it profitable to jump in and undercut the innovator's price. If it were not for the existence of the competitive fringe, the innovator would be a monopolist and try to charge an infinite price, given the nature of the demand curve.

I now describe how a period 1 innovator chooses between implementing in period 1 or in period 2. If the innovator implements the new idea in period 1, then its net revenues are $p_1(i)x_1(i) - w_1 c x_1(i)/\lambda = y_1 - w_1 c y_1/(p_1(i)\lambda)$, after substituting out for $x_1(i)$ using the demand function, (2). Then, substituting out for $p_1(i)$ using the pricing policy, $p_1(i) = w_1 c$,

$$\pi_1(i) = y_1 \left[1 - \frac{1}{\lambda}\right] > 0, \; x_1(i) = \frac{y_1}{w_1 c}.$$

If the innovator does not implement the new idea in period 1, then:

$$\pi_1(i) = 0, \; x_1(i) = \frac{y_1}{w_1 c}.$$

Why would an innovator forgo positive profits by not innovating immediately? The answer lies in the fact that an innovator can keep the new idea secret only during the first period of implementation. In the period after the new idea is first implemented, it becomes known to the competitive fringe and positive profits are no longer possible. So, an innovator may wish to delay implementation by one period if he/she thinks that more profits could be earned by delaying one period and innovating then. I do not consider the possibility that an innovator might want to delay implementation for more than one period.[1]

An innovator in period 1 would choose to implement in period 1 rather than in period 2 if:

$$y_1 \left[1 - \frac{1}{\lambda}\right] > \frac{y_2 \left[1 - \frac{1}{\lambda}\right]}{R_1},$$

or, after cancelling $1 - 1/\lambda$ from both sides,

$$y_1 > \frac{y_2}{R_1}. \tag{5}$$

Similarly, an innovator would prefer to delay implementation for one period if

$$y_1 < \frac{y_2}{R_1}. \tag{6}$$

[1]The student should think about how to prove that, in the two equilibria analyzed below, no one would ever want to delay implementing for more than one period.

Thus, if demand, y_2, in the next period is expected to be sufficiently high, then it makes sense to delay implementation until then, when the profits associated with implementation would be higher.

Now consider period 1 in the North. In this period, all firms in all industries, $i \in (1/2, 1)$, have access only to the cost function, $\text{cost}(x_1(i)) = cw_1x_1(i)$. Production occurs by one firm, although the others could jump in if they thought they could make profits. The existence of this competitive fringe guarantees that the producing firm sets price equal to marginal cost, $p_1(i) = cw_1$, and profits are zero.

In period 2 in the North, one firm in each industry, $i \in (1/2, 1)$, is randomly selected to receive a new idea. As in period 1 in the South, this firm, the 'innovator', is given access to a production technology, $\text{cost}((x_2(i)) = cw_2x_2(i)/\lambda$. Again, like in the South, the innovator can choose to implement the technology immediately, or to implement the cost function that was available in period 1. It implements right away if

$$y_2 > \frac{y_3}{R_2}. \tag{7}$$

Similarly, an innovator would prefer to delay implementation for one period if

$$y_2 < \frac{y_3}{R_2}. \tag{8}$$

The period 3 idea that arrives in the South gives the innovator (again, randomly selected) in industry $i \in (0, 1/2)$ access to the cost function, $cw_3x_3(i)/\lambda^2$. Since the previous idea was implemented (in period 1 or 2), the competitive fringe in the South in period 3 has access to the cost function, $cw_3x_3(i)/\lambda$. Mimicking the logic applied above, the period 3 innovator would make profits $y_3 [1 - 1/\lambda] > 0$ in period 3 and zero in period 4 if he/she chooses to implement in period 3. Profits would be zero in period 3 and $y_4 [1 - 1/\lambda]$ in period 4 if he/she implements in period 4 instead. The period 3 innovator will choose to implement in period 3 if

$$y_3 > \frac{y_4}{R_3}, \tag{9}$$

and will delay implementation until period 4 if

$$y_3 < \frac{y_4}{R_3}. \tag{10}$$

The following table indicates the cost functions used by firms manufacturing in the North and in the South, under the assumption that innovators implement new ideas immediately. The table also reports the quantity of intermediate goods produced in the North and the South.

Table 1
Wage Rate, Marginal Costs and Intermediate Good Output
in Immediate Implementation Equilibrium

	South			North			Wage Rat
t	Innovator	Fringe	$x_t(i)$, $i \in (0, 1/2)$	Innovator	Fringe	$x_t(i)$, $i \in (1/2, 1)$	w_t
1	$\frac{cw_1}{\lambda}$	cw_1	$\frac{y_1}{cw_1}$	cw_1	cw_1	$\frac{y_1}{cw_1}$	$\frac{1}{c}$
2	$\frac{cw_2}{\lambda}$	$\frac{cw_2}{\lambda}$	$\frac{y_2\lambda}{cw_2}$	$\frac{cw_2}{\lambda}$	cw_2	$\frac{y_2}{cw_2}$	$\frac{\lambda^{1/2}}{c}$
3	$\frac{cw_3}{\lambda^2}$	$\frac{cw_3}{\lambda}$	$\frac{y_3\lambda}{cw_3}$	$\frac{cw_3}{\lambda}$	$\frac{cw_3}{\lambda}$	$\frac{y_3\lambda}{cw_3}$	$\frac{\lambda}{c}$
4	$\frac{cw_4}{\lambda^2}$	$\frac{cw_4}{\lambda^2}$	$\frac{y_4\lambda^2}{cw_4}$	$\frac{cw_4}{\lambda^2}$	$\frac{cw_4}{\lambda}$	$\frac{y_4\lambda}{cw_4}$	$\frac{\lambda^{3/2}}{c}$
5	$\frac{cw_5}{\lambda^3}$	$\frac{cw_5}{\lambda^2}$	$\frac{y_5\lambda^2}{cw_5}$	$\frac{cw_5}{\lambda^2}$	$\frac{cw_5}{\lambda^2}$	$\frac{y_5\lambda^2}{cw_5}$	$\frac{\lambda^{4/2}}{c}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

It is possible to compute the wage rate in each period, by combining the information in Table 1 with the production function. Thus, substituting the period 1 level of intermediate good production from Table 1 into the production function, we obtain:

$$\log y_1 = \int_0^1 \log\left(\frac{y_1}{cw_1}\right) di = \log(y_1) - \log(cw_1).$$

Cancelling $\log y_1$ on both sides, we obtain $\log(cw_1) = 1$, or, $w_1 = 1/c$. Now consider w_2. Introducing the level of intermediate good production for period $t = 2$ in Table 2 into the production function:

$$\log y_2 = \int_0^{1/2} \log\left(\frac{y_2\lambda}{cw_2}\right) di + \int_{1/2}^1 \log\left(\frac{y_2}{cw_2}\right) di.$$

For the above equation to hold, it must be that $w_2 = \lambda^{1/2}/c$. The remaining entries for the wage rate in Table 1 are easily confirmed.

Now consider the case in which innovators in the South implement with a one-period delay, while innovators in the North implement immediately. This is covered in Table 2. The entries in this table are easy to confirm, and so I won't discuss them explicitly.

Table 2:
Wage Rate, Marginal Costs and Intermediate Good Output
When Southern Innovators Implement With Delay

	South			North			Wage Ra
t	Innovator	Fringe	$x_t(i),\ i \in (0, 1/2)$	Innovator	Fringe	$x_t(i),\ i \in (1/2, 1)$	w_t
1	cw_1	cw_1	$y_1/(cw_1)$	cw_1	cw_1	$y_1/(cw_1)$	$1/c$
2	cw_2/λ	cw_2	$y_2/(cw_2)$	cw_2/λ	cw_2	$y_2/(cw_2)$	$1/c$
3	cw_3/λ	cw_3/λ	$y_3\lambda/(cw_3)$	cw_3/λ	cw_3/λ	$y_3\lambda/(cw_3)$	λ/c
4	cw_4/λ^2	cw_4/λ	$y_4\lambda/(cw_4)$	cw_4/λ^2	cw_4/λ	$y_4\lambda/(cw_4)$	λ/c
5	cw_5/λ^2	cw_5/λ^2	$y_5\lambda^2/(cw_5)$	cw_5/λ^2	cw_5/λ^2	$y_5\lambda^2/(cw_5)$	λ^2/c
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

4 Equilibrium

The resource constraint for this economy is

$$c_t = y_t,\ t = 1, 2, 3, ... \ . \tag{11}$$

Clearing in the loan markets requires $B_t = 0$ for all t. A sequence of markets equilibrium is defined in the usual way. It is a sequence of prices and profits, $\{R_t, p_t(i), w_t, \pi_t; t = 1, 2, ...\}$, and quantities, $\{c_t, y_t, x_t(i), n_t, B_{t+1}; t = 1, 2, ...\}$, such that, given the prices, the quantities solve the household and firm problems at each date, and the resource constraint and loan market clearing condition are satisfied. Since hours worked do not enter the utility function, in equilibrium, $n_t = 1$ for all t. We only consider symmetric equilibria, in which all producers in the South behave alike and all producers in the North behave alike. I assume

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},\ \gamma > 0.$$

We will establish that there are parameter values for which there are at least two equilibria: the one exhibited in Table 1 and the one exhibited in Table 2. We refer to the equilibrium in which implementation is immediate as the 'Immediate Implementation Equilibrium'. We refer to the equilibrium in which Southern innovators delay for one period and Northern innovators implement immediately as the 'Simultaneous Implementation Equilibrium'.

In the first, growth is smooth, which in the latter fluctuations ('business cycles') are superimposed on top of growth.

For the immediate implementation equilibrium, we require, using (5) and (7),

$$R_t > \frac{y_{t+1}}{y_t},\ t = 1, 2, \ldots .$$

Combining this with the household's intertemporal Euler equation, (1), the parametric form for the utility function, and the resource constraint, we obtain:

$$\frac{1}{\beta} > \left(\frac{y_{t+1}}{y_t}\right)^{1-\gamma},\ t = 1, 2, \ldots .$$

For an equilibrium in which implementation in the South is delayed by one period, whereas the North always implements immediately, we require:

$$\begin{aligned} R_t &< \frac{y_{t+1}}{y_t},\ t \text{ odd} \\ R_t &> \frac{y_{t+1}}{y_t},\ t \text{ even,} \end{aligned}$$

or, after making use of the intertemporal Euler equation:

$$\begin{aligned} \frac{1}{\beta} &< \left(\frac{y_{t+1}}{y_t}\right)^{1-\gamma},\ t \text{ odd} \\ \frac{1}{\beta} &> \left(\frac{y_{t+1}}{y_t}\right)^{1-\gamma},\ t \text{ even.} \end{aligned}$$

Note that in the case of log utility, $\gamma = 1$, then *only* the immediate implementation equilibrium is possible. We now consider $\gamma \neq 1$. We consider each type of equilibrium separately:

4.1 Immediate Implementation Equilibrium

The previous subsection established that one or the other type of equilibrium exists if output exhibits an appropriate growth pattern. In this section we investigate whether this growth pattern constitutes an equilibrium.

In an immediate implementation equilibrium, the budget constraint in each period is:

$$y_t = \frac{1}{2}\pi_t + w_t,\ t = 1, 2, 3, \ldots . \tag{12}$$

where π_t is the profits earned by implementing innovators. Substituting out for profits using the fact, $\pi_t = y_t\left[1 - 1/\lambda\right]$, we obtain:

$$y_t = \frac{w_t}{1 - \frac{1}{2}(1 - \frac{1}{\lambda})},\ t = 1, 2, 3, ... \ . \tag{13}$$

This implies, taking the wage rate from Table 1:

$$y_1 = \frac{1}{c\left[1 - \frac{1}{2}(1 - \frac{1}{\lambda})\right]},$$

giving us output in the first period.

To obtain y_2, substitute the wage rate from Table 1 into (13):

$$y_2 = \frac{\lambda^{1/2}}{c\left[1 - \frac{1}{2}(1 - \frac{1}{\lambda})\right]}.$$

Proceeding in this way for all dates, we conclude:

$$y_t = \frac{\lambda^{(t-1)/2}}{c\left[1 - \frac{1}{2}(1 - \frac{1}{\lambda})\right]},\ t = 1, 2, 3, ... \ ,$$

so that:

$$\frac{y_{t+1}}{y_t} = \lambda^{\frac{1}{2}},\ \text{for } t = 1, 2, 3, ... \ .$$

This is an equilibrium if

$$\frac{1}{\beta} > \lambda^{\frac{1-\gamma}{2}}.$$

If $\gamma \geq 1$, this condition is definitely satisfied, since $0 < \beta < 1$ and $\lambda > 1$. We conclude that there is always an immediate implementation equilibrium if $\gamma \geq 1$.

4.2 Simultaneous Implementation Equilibrium

To determine if there is a simultaneous implementation equilibrium, we need to compute y_1, y_2, y_3, y_4, and verify that:

$$\frac{1}{\beta} < \left(\frac{y_2}{y_1}\right)^{1-\gamma},\ \frac{1}{\beta} > \left(\frac{y_3}{y_2}\right)^{1-\gamma},\ \frac{1}{\beta} < \left(\frac{y_4}{y_3}\right)^{1-\gamma},\ \text{etc.}$$

Note from Table 2 that there is no innovation in period 1, so that $y_1 = w_1$. But, according to Table 2, the wage rate in period 1 is $1/c$, so that:

$$y_1 = \frac{1}{c}.$$

In period 2 everyone implements, so that $y_2 = \pi_2 + w_2$, or $y_2 = y_2 \left[1 - 1/\lambda\right] + 1/c$, according to Table 2. Solving this for y_2:

$$y_2 = \frac{\lambda}{c}.$$

In period 3 no one implements, so that $y_3 = w_3 = \lambda/c$, according to Table 2, so that

$$y_3 = \frac{\lambda}{c}.$$

In period 4 everyone implements, so $y_4 = \pi_4 + w_4$, or

$$y_4 = \frac{\lambda^2}{c}.$$

Below, we show that boundedness for household utility in this equilibrium requires $\beta\lambda^{(1-\gamma)/2} < 1$. We conclude that simultaneous implementation is an equilibrium if:

$$\lambda^{\frac{(1-\gamma)}{2}} < \frac{1}{\beta} < \lambda^{1-\gamma},\ \frac{1}{\beta} > 1.$$

The second condition is of course always satisfied. The first one is tougher to meet. It requires $\gamma < 1$, since $\lambda > 1$.

There is a simple intuition underlying the fact that a high value of γ makes this equilibrium less likely. Recall that to induce innovators to delay implementing they must anticipate a high level of aggregate output for next period and the rate of interest from today to tomorrow must be low (see (6).) The problem is that there is a tension between these two requirements, and this becomes more severe the greater is γ. When output is high in the next period, relative to today, then households look forward to a high level of consumption growth from today to tomorrow. When the curvature in their utility function is high, then to get them to accept this intertemporal pattern in consumption and to discourage them from trying to smooth consumption by borrowing, implies a high equilibrium rate of interest. The higher the curvature in utility, the higher the interest rate, holding the rate of consumption growth fixed. This is why it is that when γ is large, it is harder for there to be an equilibrium in which innovators find it in their interest to delay implementing.

4.3 Multiple Equilibria

For the no delay and simultaneous implementation outcomes to *both* be equilibria requires:

$$\lambda^{\frac{1-\gamma}{2}} < \frac{1}{\beta} < \lambda^{1-\gamma}. \tag{14}$$

This condition can only be satisfied if $\gamma < 1$, since $\lambda > 1$. It is satisfied if $\beta = 1/1.002$, $\gamma = 0.9$ and $\lambda = 1.03$. In this case, $\lambda^{(1-\gamma)/2} = 1.0015$ and $\lambda^{1-\gamma} = 1.0030$.

The two equilibria are not likely to be equivalent in a welfare sense. They imply the following patterns of consumption and output. In the immediate implementation equilibrium, we have:

$$y_1 = \frac{1}{c}\frac{2\lambda}{1+\lambda}, \quad \frac{y_{t+1}}{y_t} = \lambda^{\frac{1}{2}}, \; t = 1, 2, 3, ...$$

The level of utility is just

$$v^1 = \sum_{t=1}^{\infty} \beta^{t-1} \frac{c_t^{1-\gamma}}{1-\gamma} = \frac{(y_1)^{1-\gamma}}{1-\beta\lambda^{(1-\gamma)/2}}\frac{1}{1-\gamma} = \left(\frac{1}{c}\frac{2\lambda}{1+\lambda}\right)^{1-\gamma} \frac{1}{1-\beta\lambda^{(1-\gamma)/2}}\frac{1}{1-\gamma}.$$

Note that this sum is well defined because $\beta\lambda^{(1-\gamma)/2} < 1$.

In the simultaneous implementation equilibrium, output and consumption in $t = 1$ is $1/c$. Thereafter,

$$\frac{y_{t+1}}{y_t} = \begin{cases} \lambda, & \text{for } t \text{ odd} \\ 1, & \text{for } t \text{ even.} \end{cases}$$

Thus,

$$\begin{aligned} v^2 &= \sum_{t=1}^{\infty} \beta^{t-1} \frac{c_t^{1-\gamma}}{1-\gamma} \\ &= \left[c_1^{1-\gamma} + \beta c_2^{1-\gamma} + \beta^2 c_3^{1-\gamma} + ...\right] \frac{1}{1-\gamma} \end{aligned}$$

$$
\begin{aligned}
&= c_1^{1-\gamma}\left[1+\beta\left(\frac{c_2}{c_1}\right)^{1-\gamma}+\beta^2\left(\frac{c_3}{c_2}\frac{c_2}{c_1}\right)^{1-\gamma}+\beta^3\left(\frac{c_4}{c_3}\frac{c_3}{c_2}\frac{c_2}{c_1}\right)^{1-\gamma}+...\right]\frac{1}{1-\gamma}\\
&= c_1^{1-\gamma}\left[1+\beta\lambda^{1-\gamma}+\beta^2\lambda^{1-\gamma}+\beta^3\lambda^{2(1-\gamma)}+\beta^4\left(\lambda^2\right)^{1-\gamma}+\beta^5\left(\lambda^3\right)^{1-\gamma}+\beta^6\left(\lambda^3\right)^{1-\gamma}+...\right]\frac{1}{1-}\\
&= c_1^{1-\gamma}\left[1+(1+\beta)\beta\lambda^{1-\gamma}\left\{1+\beta^2\lambda^{1-\gamma}+\beta^4\lambda^{2(1-\gamma)}+\beta^6\lambda^{3(1-\gamma)}...\right\}\right]\frac{1}{1-\gamma}\\
&= c_1^{1-\gamma}\left[1+(1+\beta)\beta\lambda^{1-\gamma}\left\{1+\psi+\psi^2+\psi^3...\right\}\right]\frac{1}{1-\gamma},
\end{aligned}
$$

where $\psi=\beta^2\lambda^{1-\gamma}$. Recall our assumption that at most finite utility is possible for the household. Here, this requires $\beta^2\lambda^{1-\gamma}<1$, or, $\beta\lambda^{(1-\gamma)/2}<1$. In this case, the geometric sum in the above formula converges, so that v^2 can be written (taking $c_1=1/c$ into account):

$$
\begin{aligned}
v^2 &= \left(\frac{1}{c}\right)^{1-\gamma}\left\{1+\frac{\beta(1+\beta)\lambda^{1-\gamma}}{1-\beta^2\lambda^{1-\gamma}}\right\}\frac{1}{1-\gamma}\\
&= \left(\frac{1}{c}\right)^{1-\gamma}\left\{\frac{1+\beta\lambda^{1-\gamma}}{1-\beta^2\lambda^{1-\gamma}}\right\}\frac{1}{1-\gamma},
\end{aligned}
$$

so that[2]

$$
\frac{v^1}{v^2}=\left(\frac{2\lambda}{1+\lambda}\right)^{1-\gamma}\frac{1+\beta\lambda^{(1-\gamma)/2}}{1+\beta\lambda^{1-\gamma}}.
$$

This is not easy to evaluate, since (14) requires that the second ratio be less than unity, while the first ratio is definitely greater than unity, since $\lambda>1$ and $\gamma<1$. Still, for the parameter values listed above, the ratio is $1.0007>1$, so that the immediate implementation equilibrium dominates, in a welfare sense, the delayed implementation equilibrium.

To see in detail why this is so, let $\lambda=1.03$, $c=1$. Then, in the immediate implementation equilibrium we obtain the following sequence of outputs:

$$y_1=1.0148,\ y_2=1.0299,\ y_3=1.0452,\ y_4=1.0608,\ y_5=1.0766,\ y_6=1.0926,\ ...$$

In the delayed implementation equilibrium, we have:

$$y_1=1.0000,\ y_2=1.0300,\ y_3=1.0300,\ y_4=1.0609,\ y_5=1.0609,\ y_6=1.0927,\ ...$$

[2]Warning: I have not carefully checked the utility formulas!

So, the delayed implementation equilibrium starts out in a relatively low level of output, as innovators decide to delay implementation until the second period. In the second period there is a surge in output that puts output a tiny bit higher than it is in the immediate implementation equilibrium. But then, there is no further output growth in period 3 because no one implements then, so again the delayed implementation equilibrium falls behind the immediate implementation equilibrium. Based on these numbers, it is not surprising that the immediate implementation equilibrium generates higher welfare than the delayed implementation equilibrium.