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D-16, Fall 2000

Homework 2, due Thursday, October 26.

Consider the nonstochastic version of the model in the Clarida-Gali-Gertler model on the web site (see also the closely related Woodford paper). The pricing equation, the household's Euler equation and the central bank's rule for setting the rate of interest are:

$$\begin{aligned}\delta \pi_{t+1} + \lambda y_t - \pi_t &= 0 \\ y_{t+1} - y_t - \frac{1}{\sigma}(r_t - \pi_{t+1}) &= 0 \\ \rho r_{t-1} + (1 - \rho)\beta \pi_{t+1} + (1 - \rho)\gamma y_t - r_t &= 0,\end{aligned}$$

where π_t is the rate of inflation, y_t is output (related to marginal cost), and r_t is the nominal rate of interest. For a detailed rationale of these equations see the CGG paper and (especially for the price equation), Woodford (these papers are on the class web site). Write this system in the form:

$$AY_{t+1} + BY_t = 0,$$

for $t = 1, \dots$ (note: in class the initial period is $t = 0$, but here I have changed it to $t = 1$). Here, $Y_t = (\pi_t, y_t, r_{t-1})$. Use the following parameter values, $\delta = 0.99$, $\sigma = 1$, $\lambda = 0.3$, $\gamma = 1$, $\rho = 0.5$, $\beta = 0.966$. Let the initial interest rate, r_0 , have its steady state value of 0.

1. Compute P and Λ in the eigenvector, eigenvalue decomposition of $-A^{-1}B$.
2. Compute the set of minimal state variable solutions (i.e., the set of 2×3 matrices with the property $DY_1 = 0 \implies DY_t = 0$ for all t).
3. Suppose the first period is $t = 1$, and time evolves for $t = 1, 2, 3, \dots, 20$. Construct the following 3 graphs. In each graph, display two lines of length 20 each, with $t = 1, \dots, 20$ on the horizontal axis. One line, a line of zeros, corresponds to the nonstochastic, steady state equilibrium of the model. The other is a convergent solution to the Euler equations in which π_1 is a little greater than zero. Note how the high inflation equilibrium, while eventually returning to steady state, displays a period of high inflation and high output.

4. Now, construct a sunspot equilibrium for the model. To do this, note that in the stochastic version of the model, the equations are satisfied after application of the E_t operator. Write the state-space representation of the model,

$$AY_{t+1} + BY_t = \omega_{t+1},$$

where ω_{t+1} is an arbitrary random variable with $E_t\omega_{t+1} = 0$ for all t . Identify a convergent solution to this model in which ω_t is a non-trivial random variable. By convergent, I mean a solution in which $E_t Y_{t+j} \rightarrow 0$ as $j \rightarrow \infty$ and $Var_t(Y_{t+j})$ is bounded as $j \rightarrow \infty$. A convergent solution is one in which all explosive eigenvalues have been suppressed. Draw ω_t from a random number generator in MATLAB and simulate the response of the three variables in Y_t to the sunspot. Graph this like in 1 above. Also, display a simulation of the response of the system in a sunspot solution that does not display convergence.