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D-16, Fall 2000
Homework 2, due Thursday, November 2.

1. This question will give you more practice with the Ramsey Allocation Problem. In addition, it will expose you to the Uniform Taxation Result in public finance, a result like lies at the heart of our analysis of the Friedman rule. For now, why and how this is true should be a mystery and won't be clarified until later lectures.

   The following model is slightly more complicated than the one discussed in class, 10/26/00. The representative household solves the following problem:

   $$\max_{c_1,c_2,l} u(c_1, c_2, l),$$

   subject to

   $$c_1(1+\tau_1) + c_2(1+\tau_2) \leq zl,$$

   where l is labor, z is the marginal product of labor, τ_i are consumption tax rates and c_i are two different consumption goods, $i = 1, 2$. Also, u is differentiable, concave and increasing in its first two arguments, and decreasing in its third argument.

   Let government policy be denoted by $\pi = (\tau_1, \tau_2)$, and define the private sector allocation rule by:

   $$l(\pi), c_1(\pi), c_2(\pi).$$

   The government budget constraint is:

   $$g \leq c_1(\pi)\tau_1 + c_2(\pi)\tau_2$$

   (i) Write out the necessary and sufficient conditions which characterize the private sector allocation rules (hint: two Euler equations and a budget constraint will pin down the three unknowns, for each π). The domain for these rules is limited to those π's that are consistent with the government's budget constraint.

   (ii) Define the analog of the set, B, defined in class, 10/26/00.

(iii) Define the Ramsey Problem and the Ramsey Equilibrium for this economy.

Consider the following set:

$$\begin{aligned} D &= \{(c_1, c_2, l) : u_1 c_1 + u_2 c_2 + u_3 l = 0, \\ c_1 + c_2 + g &\leq zl\}. \end{aligned}$$

(iv) Prove $B = D$.

Define the Ramsey Allocation Problem:

$$\max_{c_1, c_2, l \in D} u(c_1, c_2, l)$$

(v) For a given triple, c_1, c_2, l, that solves this problem, show how one finds π that supports it as an equilibrium.

(vi) Suppose that u has the following structure:

$$u(c_1, c_2, l) = h(c_1, c_2) v(l),$$

where h is homogeneous of degree k and v is a monotone decreasing, concave function. Prove that the solution to the Ramsey problem is $\tau_1 = \tau_2$ (we'll see that in a class of monetary economies, this corresponds to $R = 0$, where R is the nominal rate of interest). (Hint: write out the Ramsey allocation problem in Lagrangian form, with a non-negative multiplier. Struggle with the first order conditions and notice that they imply $u_1/u_2 = 1$.)

2. Consider a version of the model of homework #1, in which $\alpha = 1/3$ and $\gamma = 1 - \alpha$.

(i) Show that aggregate output, Y, is related to aggregate capital and labor, K, N as follows:

$$Y = KN^{\frac{1-\alpha}{\alpha}}.$$

(ii) Show that the equilibrium level of employment, N_t, satisfies a first order difference equation, $v(N_t, N_{t+1}) = 0$, $t = 0, 1, 2, ...$. (Hint: combine the household intertemporal and intratemporal Euler equations, and substitute out for the wage and rental rates using the firm first order conditions.)

(iii) Show that the model has two steady states, in which the one associated with the higher level of employment is indeterminate and the lower one is determinate.

(iv) Show that $v(N, N') = 0$ has two 'branches'. That is, for each N, there are two N' that solve this difference equation. Draw the two branches in a diagram that has N on the horizontal axis and N' on the vertical axis.

(v) Construct a sunspot equilibrium around the indeterminate steady state. Do so by replacing the difference equation by $v(N_t, N_{t+1}) = \omega_{t+1}$, where $E_t\omega_{t+1} = 0$. Let $\omega_t \in (\underline{\omega}, \overline{\omega})$, with probability 1/2 each. Simulate a realization from this economy like this. Fix N_0 at an arbitrary value in a small neighborhood around the indeterminate steady state. Draw ω_1 and then solve for N_1 using the lower branch of v. Then, with this value of N_1 draw ω_2 and solve $v(N_1, N_2) = \omega_2$ for N_2, again, using the lower branch of v. Proceeding in this way, solve for $N_{t,}$ $t = 0, 1, 2, 3, 4, 5, ..., 20$. Solve for c_t, k_{t+1}, r_t, w_t $t = 0, 1, 2, ...$, where $k_0 = 1$. Report these results in a table. Explain why this is a realization from a stochastic equilibrium. (Hint: note that households and firms take r_t and w_t as stochastic processes beyond their control, drawn from the distribution induced by the above simulation procedure. Note that the household and firm responses are optimal, given the exogenous price processes you constructed.) The equilibrium is indexed by the choice of N_0 and the particular stochastic process, ω_t, used. Note that there are many other equilibria, corresponding to every other possible choice for N_0 and the stochastic process on ω_t, in the neighborhood of the indeterminate steady state.

(vi) Are there other types of sunspot equilibria, say which make use of the upper branch of v, and which are not near the indeterminate steady state?