Money: Facts and Frictions
(Christiano, Eichenbaum and Evans)

- Monetary Facts (Based on a Different VAR than Handbook Analysis)
- Monetary Frictions
- What Combination of Frictions is Necessary to Account for the Facts?
  - Construct a Macro Model With the Monetary Frictions
  - Will Require Discussion of Other Frictions, Beside Monetary Frictions.
- What’s At Stake?
  - Want a Good Model for Policy Questions.
  - The Nature of the Frictions Matter for Optimal Policy Design.
- Order of Presentation:
  (1) Overview of Analysis, General Ideas.
  (2) More Detailed Discussion of Analysis.
(1) Monetary Facts Suggested by VAR Analysis. After a Positive Monetary Shock:

(a) hump-shaped, response of output with peak effect after about 2 years.
(b) hump-shaped response inflation, with peak response after about 2.5 years.
(c) hump-shaped response of consumption, with peak response after 1.5-2 years.
(d) interest rate down for one year.
(e) profits up.
(f) real wage up.
(g) labor productivity up.

(2) These ‘Facts’ Do Rest on Assumptions: Policy Shock Has Zero Impact On -

(a) Current Aggregate Quantities.
(b) Current Aggregate Prices.
• Frictions Analyzed in Monetary Economics Literature

(a) Monetary Misperception:
   Lucas, Cooley-Hansen

(b) Sticky Prices:
   Blanchard-Kiyotaki

(c) Sticky Wages.
   Traditional Keynesian IS-LM Model With Supply-Side.

(d) Limited Participation (‘Sticky Portfolios’)
   Lucas, Fuerst, Christiano-Eichenbaum-Evans.
STICKY PRICE MODEL

Price

Demand

Demand'

Supply

Demand
Limited Participation Model

Labor Market

Goods Market
• ‘Pure Frictions’ Have Problems
  (a) Misperception:
      Counterfactual Price Implications
      (Requires Immediate Price Response).
  (b) Pure Limited Participation
      * Consistent, Qualitatively, with Some Features of Facts.
         (Delayed Price Response.)
      * Quantitatively, Has Problems
  (c) Pure Sticky Price
      * Excessive Wage Response
      * Counterfactual Profits Implication.
      * Not enough persistence in inflation.
      * Not enough persistence in output.
      * Counterfactual implications for interest rate.
      * Strong Incentive for Price-Setting Firms to Not Be Sticky.
Finding #1:

- With Flexible Factor Prices, Aggregate Price Level May Not be Very ‘Sticky’ Even if There are a Lot of Sticky Price Firms.
Finding #2

- With Sticky Prices, there May Not Be Much Persistence after Demand Jump

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• What is Needed: A Mixture of Frictions
  (a) Need Limited Participation For Interest Rate
  (b) Need Sticky Price Setting For Sluggish Prices
  (c) Need Sticky Wages to Fix Problems With Sticky Price Model
      * Wage Problems
      * Persistence in Inflation and Output
      * Profits Problem.
• Other Challenges:

(a) Consumption.

Hump-Shape Response of Consumption and Fall in Real Rate Hard to Reconcile in Standard Models.

(b) Investment.

Standard Models Tend to Imply Counterfactually Large Surge in Investment and Output Following a Monetary Shock.

(c) Productivity.

Standard Models Imply that if Monetary Policy Stimulates the Economy, then Labor Productivity Falls.

(d) VAR Identification.

Coherence of Analysis Requires that Model Be Consistent With Identifying Assumptions Underlying VAR Facts.
Consumption Challenge

- Euler Equation in Standard Model:

\[
\frac{u_{c,t}}{\beta u_{c,t+1}} \approx \frac{c_{t+1}}{\beta c_t} = \frac{g_{t+1}}{\beta} = \frac{R_t}{\pi_{t+1}}, \quad \pi_{t+1} = \frac{P_{t+1}}{P_t}.
\]

- Problem: Can’t Have \( g_t \) High and \( \frac{R_t}{\pi_{t+1}} \) Simultaneously!
• Habit Persistence in Preferences:

\[ u = u(c_t - H_t), \quad H_t = f(c_t, c_{t-1}, \ldots) = \text{`habit stock'} \]

• Simple Example: \( u(c_t - b\bar{c}_{t-1}), \quad \bar{c}_{t-1} \sim \text{aggregate consumption} \)

• Euler Equation:

\[
\frac{u_{c,t}}{\beta u_{c,t+1}} \approx \frac{c_{t+1} - bc_t}{\beta (c_t - bc_{t-1})} = \frac{g_{t+1} - b}{\beta \left(1 - \frac{b}{g_t}\right)} \\
\approx \frac{g_{t+1} - bg_t}{\beta (1 - b)}
\]

• Result:

– \( g_{t+1} \) and \( g_t \) Can Both be High, as Long as \( g_{t+1} < bg_t \).

– Consistent with Simultaneous Hump-Shape \( c \) Response and Low Real Rate.

• Habit Persistence Also Helpful for Understanding Asset Prices
Investment Challenge

• Rate of Return on Capital:

\[ R^k_t = \frac{MP^k_{t+1} + P_{k',t+1}(1 - \delta)}{P_{k',t}}, \]

\( P_{k',t} \sim \) consumption price of installed capital
\( MP^k_t \sim \) marginal product of capital
\( \delta \in (0, 1) \sim \) depreciation rate.

• Almost Any Model,

\[ \frac{R_t}{\pi_{t+1}} \approx R^k_t. \]

• So, If a Positive Money Shock Drives Down Real Rate, Then

\[ R^k_t \downarrow \]

• This is Trouble For Standard Models.
• Standard Model \((P_{k',t} = 1)\):

\[ R_t^k \text{ down requires } MP_t^k \text{ down} \]

• Problem:

\[ MP_t^k \text{ down Requires Surge in Investment, especially with employment up.} \]

• Model With Adjustment Costs:

\[ P_{k',t} \neq 1, \quad R_t^k \downarrow \text{ may not require } MP_t^k \downarrow \]

• Capital Losses, \(P_{k',t+1} < P_{k',t}\) Can Drive Down \(R_t^k\):

\[ R_t^k = \frac{MP_{t+1}^k + P_{k',t+1}(1 - \delta)}{P_{k',t}}. \]
• What Type of Adjustment Costs to Work With:
  – Lucas-Prescott (Standard in Tobin’s Q Literature):

\[ k' = (1 - \delta)k + F\left(\frac{I}{k}\right)I \]

Not Good For ‘Humps’
Period of Rising Investment Produces Anticipated Capital Gains,

\[ \frac{P_{k',t+1}}{P_{k',t}} > 1 \]

This increases the fall in \( MP^k \) needed to produce a fall in \( R^k_t \)

– Cost-of-Change:

\[ k' = (1 - \delta)k + F\left(\frac{I}{I_{-1}}\right)I \]

Good for ‘Humps’.
Period of Rising Investment Consistent
with Anticipated *Capital Losses*,

\[
\frac{P_{k',t+1}}{P_{k',t}} < 1
\]

This makes it possible to have a hump in investment, a fall in \( R^k \) and a rise in \( MP^k \) simultaneously.
• Other Reasons for Interest in Adjustment Costs:
  – Important for Understanding Asset Prices.
  – Necessary for Movements in Price of Capital.
Productivity Challenge

• Problem:

\[ L \uparrow \Rightarrow \frac{Y}{L} \downarrow, \text{ with standard models, } Y = K^{\alpha}L^{1-\alpha} \]

• Possible Resolution:

Distinguish Between Physical Stock of Capital, \( \bar{K} \) and Services from Capital, \( u\bar{K} \).

\[
\text{if } u \uparrow \text{ when } L \uparrow, \quad \text{then maybe } \frac{Y}{L} = \left( \frac{u\bar{K}}{L} \right)^{\alpha} \uparrow
\]
VAR Identification Challenge

• VAR ‘Facts’ Presume Money Shocks Have No Contemporaneous Impact on:
  – Prices
  – Aggregate Quantities

• Timing Assumption in the Analysis:
  (a) Price-Setting, Consumption, Investment, Capital Utilization Decisions Made
  (b) Money Action Realized
  (c) Money Demand Decision Made
  (d) Production, Employment, Purchases Occur.

• Interpretation of Timing:
  Monetary Policy Occurs At Higher Frequency in Time than Other Economic Activities.
Bottom Line Of Overview

In Construction of a Good Monetary Model, The Following Frictions Could Play a Positive Role:

- Sticky Prices.
- Sticky Wages.
- Sticky Portfolios.
- Habit Persistence in Preferences.
- ‘Cost-of-Change’ Adjustment Costs in Capital.
- Capital Utilization.
Model

- Firms.
- Households.
- Equilibrium.
- Parameter Values
- Properties of the Model.
Firms

Final Good Firms

- Technology:

\[ Y_t = \left[ \int_0^1 Y_{it} \frac{1}{\lambda_f} di \right]^{\lambda_f} , \ 1 \leq \lambda_f < \infty \]

- Objective:

\[ \max P_t Y_t - \int_0^1 P_{it} Y_{it} di \]

- Firms and Prices:

\[ \left( \frac{P_t}{P_{it}} \right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{it}}{Y_t} , \ P_t = \left[ \int_0^1 P_{it}^{\frac{1}{1-\lambda_f}} di \right]^{(1-\lambda_f)} . \]
Intermediate Good Firms -

- Each $Y_{it}$ Produced by a Monopolist, With Demand Curve:

$$\left( \frac{P_t}{P_{it}} \right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{it}}{Y_t}.$$ 

- Technology:

$$Y_{it} = K_{it}^\alpha L_{it}^{1-\alpha}, \ 0 < \alpha < 1.$$
Calvo Price Setting:

- With Probability $1 - \xi_p$, $i^{th}$ Firm Sets Price, $P_{it}$, Optimally, to $\bar{P}_t$.
- With Probability $\xi_p$,

$$P_{it} = \pi_{t-1} P_{i,t-1}, \quad \pi_t = \frac{P_t}{P_{t-1}}.$$

- Standard Approach in Literature:

$$P_{it} = \bar{\pi} P_{i,t-1}, \text{ or } \quad P_{it} = P_{i,t-1}.$$

- Stand on Indexing Matters
  Determines Extent of ‘Front-Loading’
Firms Setting Prices Optimally at $t$ Choose $\tilde{P}_t$ to max:

\[
v_t \left[ \tilde{P}_t Y_{it} - MC_t Y_{it} \right] \\
+ \beta \xi_p v_{t+1} \left[ \tilde{P}_t \pi_t Y_{i,t+1} - MC_{t+1} Y_{i,t+1} \right] \\
+ (\beta \xi_p)^2 v_{t+2} \left[ \tilde{P}_t \pi_t \pi_{t+1} Y_{i,t+2} - MC_{t+2} Y_{i,t+2} \right] \\
+ \ldots
\]

subject to:

\[
\left( \frac{P_t}{\tilde{P}_t} \right)^{\lambda_f \lambda_{f-1}} = \frac{Y_{it}}{Y_t}.
\]

\[
s_t = \frac{MC_t}{P_t} = \left( \frac{1}{1 - \alpha} \right)^{(1-\alpha)} \left( \frac{1}{\alpha} \right)^\alpha \left( r_t^k \right)^\alpha (w_t R_t)^{1-\alpha}
\]

$v_t \sim$ value of a dividend at $t$

\[
w_t \sim \text{real wage}
\]

$r_t^k \sim \text{real rental rate on capital}$
• Normally ($\xi_p = 0$) :

Price = Markup ($\lambda_f$) $\times$ Marginal Cost ($MC$)

or

$$\tilde{p}_t - \lambda_f s_t,$$

where

$$\tilde{p}_t = \frac{\tilde{P}_t}{P_t}, \ s_t = \frac{MC_t}{P_t}.$$
With Calvo ($\xi_p > 0$):

$$E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j v_{t+j} Y_{t+j} (X_{t,j})^{\lambda_f - 1} [\tilde{p}_t X_{t,j} - \lambda f s_{t+j}] .$$

where

$$X_{t,j} = \frac{\pi_t}{\pi_{t+j}} ,$$

Note Potential Incentive to ‘Front Load’

Linearization: $\hat{x}_t \equiv (x_t - x) / x$.

Approximate (Linearized) Solution:

$$\hat{p}_t = (1 - \beta \xi_p) \sum_{j=0}^{\infty} (\beta \xi_p)^j \hat{s}_{t+j} - \beta \xi_p \hat{\pi}_t$$

$$+ (1 - \beta \xi_p) \sum_{l=1}^{\infty} (\beta \xi_p)^l \hat{\pi}_{t+l}$$
• Aggregate Price Level:

\[
P_t = \left[ \int_0^1 P_{it}^{\frac{1}{1-\lambda_f}} \, di \right]^{1-\lambda_f}
\]

\[
= \left[ (1 - \xi_p) \tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p (\pi_{t-1} P_{t-1})^{\frac{1}{1-\lambda_f}} \right]^{1-\lambda_f}
\]

• Scale:

\[
1 = \left[ (1 - \xi_p) \tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p \left( \frac{\pi_{t-1}}{\pi_t} \right)^{\frac{1}{1-\lambda_f}} \right]^{1-\lambda_f}
\]

• Approximately

\[
\tilde{P}_t = \frac{\xi_p}{1 - \xi_p} \left[ \hat{\pi}_t - \hat{\pi}_{t-1} \right].
\]
• Combining:

\[ \hat{\pi}_t = \frac{1}{1 + \beta} \hat{\pi}_{t-1} + \frac{\beta}{1 + \beta} E_t \hat{\pi}_{t+1} + \frac{(1 - \beta \xi_p)(1 - \xi_p)}{(1 + \beta) \xi_p} \hat{s}_t, \]

• Under Normal (No-Indexing) Approach to Indexing:

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p} \hat{s}_t. \]

• Note: With Backward Indexing, Weight Shifts Towards Past in Inflation Equation.
• Resource Constraint:

\[ c_t + I_t \leq Y_t, \]

where

\[ Y_t \sim \text{output of Final Goods Firms} \]
Households

- Preferences:

\[ E_{t-1} \sum_{j=0}^{\infty} \beta^{j-t} [u(c_{t+j} - bc_{t+j-1}) - z(l_{t+j}) + v(q_{t+j})]. \]

\( b \) ~ habit parameter

\[ q = \frac{Q}{P} \]

\[ u(\cdot) = \log(\cdot) \]

\[ z(\cdot) = \frac{\psi_0}{2} (\cdot)^2 \]

\[ v(\cdot) = \psi_q \frac{(\cdot)^{1-\sigma_q}}{1 - \sigma_q} \]
Flow Budget Constraint (Ignoring Insurance Considerations):

\[ M_{t+1} = R_t [M_t - Q_t + (\mu_t - 1) M^a_t] + Q_t + P_t w_i l_t + P_t r_i^k k_t + D_t - P_t \left( c_t + i_t + \tilde{\delta}(u_t) \bar{k}_t \right). \]

\( Q_t \sim \text{deposits} \)
\( M_t \sim \text{beginning-of-period } t \text{ Household Money} \)
\( M^a_t \sim \text{beginning-of-period } t \text{ Aggregate Money} \)
\( D_t \sim \text{profits} \)
\( \mu_t \sim \text{gross money growth rate} \)
\( M_t - Q_t + (\mu_t - 1) M^a_t \sim \text{deposits at financial intermediary} \)
\( \tilde{\delta}(\cdot) \sim \text{costs of utilizing capital more intensively} \)
\( u_t \sim \text{utilization rate of capital.} \)
\( k_t \sim u_t \bar{k}_t, \text{ capital services.} \)
Household Problem:

(a) Labor Decision.
   Monopoly Supplier of Differentiated Labor Service
   Sets Wage, Calvo-style

(b) Portfolio Decision.
   Makes Money Demand Decision, Calvo-Style

(c) Consumption Decision.

(d) Investment Decision.

(e) Capital Utilization Decision.

(f) Insurance Decision.
Labor Decision

- Demand for Household Labor Service, $l_t$:

$$l_t = \left( \frac{W_t}{W_{it}} \right)^{\frac{\lambda_w}{\lambda_w - 1}} L_t, \ 1 \leq \lambda_w < \infty.$$  

$W_{i,t}$ ~ wage set by household  
$L_t$ ~ homogeneous aggregate labor  
$W_t$ ~ wage rate of aggregate labor

- Source of demand:

$$L_t = \left[ \int_0^1 (l_{it})^{\frac{1}{\lambda_w}} \, di \right]^{\lambda_w} , \ 1 \leq \lambda_w < \infty.$$  

- Households Setting Wage Do So To Maximize Utility
• Household Does Calvo Wage Setting:
  – With Probability $1 - \xi_w$, $i^{th}$ Household
    Sets Wage, $W_{it}$, Optimally, to $\tilde{W}_t$.
  – With Probability $\xi_w$,

    \[ W_{it} = \pi_{t-1} W_{i,t-1}, \quad \pi_t = \frac{P_t}{P_{t-1}}. \]

  – Existing Approach (Erceg, Henderson, Levin):

    \[ W_{it} = \bar{\pi} W_{i,t-1}. \]

  – Stand on Indexing Matters
    Determines Extent of ‘Front-Loading’
- Normally \((\xi_w = 0)\) :

\[
\text{Price} = \text{Markup} (\lambda_w) \times \text{Marginal Cost} (MC)
\]

or

\[
\bar{W}_t = \lambda_w P_t \frac{z'_t}{u_{c,t}}.
\]

\(z'_t \sim \text{marginal disutility of labor}\)

\(u_{c,t} \sim \text{marginal utility of consumption}\)

- With Calvo \((\xi_w > 0)\) :

\[
0 = E_{t-1} \sum_{j=0}^{\infty} (\xi_w/\beta)^j l_{t+j} u_{c,t+j} \left[ \bar{W}_t \frac{\pi_t}{\pi_{t+j}} - \lambda_w P_t \frac{z'_{t+j}}{u_{c,t+j}} \right].
\]

- Note Potential Incentive to ‘Front Load’
- Note Timing in Information Set.
Money Decision, $Q_t$

- Households Set $Q_t$ To Maximize Utility
- Household Follows Calvo in Setting $Q_t$:
  - With Probability $1 - \xi_q$, $i^{th}$ Household Sets $Q_{it}$ Optimally, to $\bar{Q}_t$.
  - With Probability $\xi_q$,

$$Q_{it} = \pi_{t-1}Q_{i,t-1}, \quad \pi_t = \frac{P_t}{P_{t-1}}.$$

- Stand on Indexing Matters
  Determines Extent of ‘Front-Loading’
• Normally ($\xi_q = 0$) : (standard money demand)

$$v' \left( \frac{\tilde{Q}_t}{P_t} \right) \frac{1}{P_t} + \frac{u_{c,t}}{P_t} = \frac{u_{c,t}}{P_t} R_t,$$

• With Calvo ($\xi_q > 0$):

$$E_t \sum_{j=0}^{\infty} (\beta \xi_q)^j \left\{ v' \left( \frac{\tilde{Q}_t \pi_{t+j}}{P_t \pi_{t+j}} \right) - u_{c,t+j} [R_{t+j} - 1] \right\} \frac{\pi_t}{\pi_{t+j}} = 0$$

• Here, ‘Front Load’ means:
  – Short Run Money Demand Elasticity Small
Consumption Decision

\[ E_{t-1} \frac{u_{c,t}}{P_t} = \beta E_{t-1} \frac{u_{c,t+1}}{P_{t+1}} R_{t+1}. \]
Investment Decision

- Household Owns the Capital Stock and Carries Out Capital Accumulation.

- Technology for Capital Accumulation:

\[ \ddot{k}' = (1 - \delta) \dot{k} + F(I, I_{-1}), \]

- Household Euler Equation:

\[ E_{t-1}u_{c,t} = \beta E_{t-1}u_{c,t+1} \frac{[r_{t+1}^k + P_{k',t+1}(1 - \delta)]}{P_{k',t}}. \]

\( P_{k',t} \sim \text{marginal cost, in units of consumption goods, of installed capital} \)

- Technology for Capital Accumulation:

\[ E_{t-1} \hat{P}_{k',t} = S'' E_{t-1} \left\{ \hat{I}_t - \hat{I}_{t-1} - \beta \left[ \hat{I}_{t+1} - \hat{I}_t \right] \right\}. \]
Capital Utilization Decision

- Equate Marginal Gain With Marginal Cost of Additional Utilization:

\[ E_{t-1}u_{c,t} \left[ r^k_t - \tilde{\delta}'(u_t) \right] = 0 \]
Insurance

• Notation:

\[ s_t \sim \text{realization of idiosyncratic uncertainty in period } t \]
\[ s^t = (s_0, s_1, s_2, \ldots, s_t) \sim \text{history of realizations up to period } t \]

(Forget about aggregate uncertainty for now)

• Arrow Securities:

\[ B_{t,\tau}(s^{t+\tau}) \] quantity of bonds purchased in history \( s^t \), which pay $1 if \( s^{t+\tau} \) occurs, 0 otherwise

• Household Pays for Arrow Securities in the beginning of the Period when they Pay off. A household in history \( \tilde{s}^t \) owes:

\[
\sum_{j=0}^{t} \sum_{s^t|\tilde{s}^{t-j-1}} \delta_{t-j,j}(s^t) B_{t-j,j}(s^t).
\]
• Household with history $\tilde{s}^t$ receives the following payments:

$$\sum_{j=0}^{t} B_{t-j,j}(\tilde{s}^t).$$
• Household budget constraint:

\[
0 \leq R(\mu) \left[ M(s^{t-1}) - Q(s^t) + (\mu - 1)\bar{M} \right] + D(\mu) + \sum_{j=0}^{t} B_{t-j,j}(\tilde{s}^t) - \sum_{j=0}^{t} \delta_{t-j,j}(s^t) B_{t-j,j}(s^t) + \left[ Q(s^t) + W(s^t)L(s^t) - P(\mu) \left( c(s^{t-1}) + I(s^{t-1}) \right) \right] + R^k(\mu)\bar{k}(s^{t-2}) - M(s^t), \quad t = 0, 1, 2, \ldots
\]

• Lagrangian problem:

\[
\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t F(s^t) \left\{ U \left( c(s^t) - bc(s^{t-1}) \right) + f(L(s^t)) + \nu \left( \frac{Q(s^t)}{P(\mu)} \right) \right\} + \lambda(s^t) \left\{ R(\mu) \left[ M(s^{t-1}) - Q(s^t) + (\mu - 1)\bar{M} \right] + D(\mu) + \sum_{j=0}^{t} B_{t-j,j}(s^t) - \sum_{j=0}^{t} \delta_{t-j,j}(\tilde{s}^t) B_{t-j,j}(\tilde{s}^t) + \left[ Q(s^t) + W(s^t)L(s^t) - P(\mu) \left( c(s^t) + I(s^t) \right) \right] + R^k(\mu)\bar{k}(s^{t-1}) - M(s^t) \right\}
\]
• First order conditions:

\[
\sum_{s_t} F(s^{t-1}, s_t) \lambda(s^{t-1}, s_t) = \lambda(s^{t-1}, s_t^0)
\]

\[
\Rightarrow \lambda(s^{t-1}, s_t^0) = \lambda(s^{t-1})
\]

\[
\sum_{s_{t-1}, s_t} F(s^{t-2}, s_{t-1}, s_t) \lambda(s^{t-2}, s_{t-1}, s_t) = \lambda(s^{t-2}, s_{t-1}^0, s_t^0)
\]

\[
\Rightarrow \lambda(s^{t-2}, s_{t-1}^0, s_t^0) = \lambda(s^{t-2})
\]

• From this and the fact, \( \lambda_t = \frac{u_{c,t}}{P_t} \), infer:
  
  \( c \) not a function of \( s_t \).
### Parameter Values

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<td>Average Money Growth, $\mu$</td>
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<tr>
<td>Habit, $b$</td>
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<tr>
<td>Capital Depreciation Rate, $\delta$</td>
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<td>Curvature on Investment Adjustment Costs</td>
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<td>Share of Income to Capital, $\alpha$</td>
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<td>Markups (11%), $\lambda_f$, $\lambda_w$</td>
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<td>Money Utility Parameter, $\sigma_q$</td>
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<td>Weight on Money Utility, $\psi_q$</td>
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<td>Discount factor, $\beta$</td>
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<td>Curvature on Capital Utilization Cost, $\sigma_\delta$</td>
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<td>Probabilities of Not Optimizing</td>
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<td>Price, $\xi_p$</td>
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<td>Money, $\xi_q$</td>
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<tr>
<td>Wage, $\xi_w$</td>
<td>0.75</td>
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</tbody>
</table>

- Note: The adjustment cost probabilities were informally chosen to optimize the model’s ability to reproduce the VAR facts.
• Estimation Method:

\[
J = \min_{\gamma} (\hat{\psi} - \psi(\gamma))'V^{-1}(\hat{\psi} - \psi(\gamma)),
\]

where \( \gamma \) denotes economic model parameters. They are estimated by optimizing the above criterion. Under null hypothesis,

\[
J \sim \chi_m
\]

\( m \) dimension of \( \psi \) minus no. of elements in \( \gamma \)

\( \gamma \sim \) parameters in economic model

\( \psi(\gamma) \sim \) model impulse responses associated with \( \gamma \)

\( \hat{\psi} \sim \) VAR parameters

\( V \sim \) estimated sample variance-covariance of \( \hat{\psi} \).
• Basic Econometric Idea:
  
  Suppose the model is ‘true’ and the true parameter values are known, so that \( \psi(\gamma) \) is the true population impulse response function

\[
* \ (\hat{\psi} - \psi(\gamma)) \sim N(0, V) \text{ (actually, } V = \frac{V}{T})
\]

\[
* \text{Let } CVC' = I, \ y = C(\hat{\psi} - \psi(\gamma)), \ C \sim \text{lower triangular. Then,}
\]

\[
(\hat{\psi} - \psi(\gamma))'V^{-1}(\hat{\psi} - \psi(\gamma))
\]

\[
= (\hat{\psi} - \psi(\gamma))'C'(C')^{-1}V^{-1}(C)^{-1}C(\hat{\psi} - \psi(\gamma))
\]

\[
= y'(C')^{-1}V^{-1}(C)^{-1}y
\]

\[
= y'[CVC']^{-1}y
\]

\[
= y'y \sim \chi_p,
\]

\[p \sim \text{no. of elements in } \hat{\psi}\]
– Suppose model is ‘true’, but $n$ elements in $\gamma$ are not known. Optimized $\gamma$ sets $n$ linear combinations of $y$ to zero.

\[ m = p - n. \]
Figure 1: Benchmark Model, $\psi_p=0$, $\psi_w=.75$, $\psi_q=.5$, $\sigma_{mad}=.01$, $\sigma_{mac}=\sigma_{ma}=1$, $\sigma_{mq}=7$, cost-of-change, $s''=8$, $\lambda_m=\lambda_f=1.05$, $L=1$, $b=.72$, $\mu=1.01$, $\phi_M=1$. 5 money growth model, internal habit, W and Q indexed on past actual inflation.
Figure 2: Benchmark model with $k_{SIP} = 0.75$, Firm menu cost = 3.6% of output.
Figure 3: Benchmark with $ksip=0.75$, $ksiw=0$, Menu costs = 219% of output.
Figure 4: Benchmark with ksip=.95, ksiw=0, menu costs = 291% of output.
Figure 5: Benchmark model with ksip=.95, ksiw=0, kspi=0, Menu cost = 296% of output.
Figure 5: Benchmark model with $ksip=.95$, $ksiw=0$, $ksiq=0$, Menu cost = 296% of output.
Figure 6: Benchmark with ksiq=0.
Figure 7: Benchmark with P and W set after M shock

- Inflation (APR)
- Real wage
- Interest rate (APR)
- Output
- Investment
- Consumption
- Profits
- Price of capital
- Real rate and MRS (APR)
- Rental rate on k
- Marginal cost
- Productivity
- Utilization rate
Figure 8: Benchmark with Indexing to steady state inflation.
Figure 9: Benchmark with External Habit.
Figure 10: Benchmark model with large cost of changing capital utilization rate.
Figure 11: Benchmark model with large cost of changing capital utilization, and Lucas-Prescott Model of Adjustment Costs
Figure 12: Benchmark model with no habit.