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416, Fall 2001
Homework 5.
Due: Monday, November 12

1. Consider the model outlined in section 1 of Woodford, 'On the Determination of the Public Debt' (http://www.faculty.econ.nwu.edu/ faculty/christiano/papers/w5684.pdf). Consider specifically the analysis of sticky prices that begins on page 8, and which is taken from Calvo (1983). In that analysis, a randomly selected fraction, $1-\alpha$, of firms can optimize their price in period t. They all select the same price, $\mathcal{P}_t$. Implicitly (I don't see it stated explicitly anywhere), equation (1.18) assumes that each of the α firms which do not get to change their price leave it fixed at its value from the previous period. There is a distribution of such prices, since some firms that cannot reoptimize price in the current period have not reoptimized for a long time, and some just reoptimized in the previous period. You should convince yourself that this distribution has been handled properly in equation (1.18). This equation is just (1.3) with the price of the firms that do not change prices replaced by $\mathcal{P}_t$. Also, the impact of past prices on the index is summarized by P_{t-1}. Make sure you understand that the single index, P_{t-1}, properly summarizes the impact on P_t of the heterogenous distribution of prices associated with the α firms that cannot reoptimize their price. Now, change the assumption that firms which cannot reoptimize their price must set their current price to the value it took on in the previous period. Let i denote the index of a firm that cannot reoptimize its price, and let $P_{i,t-1}$ denote the price it set in the previous period. In the model studied in Woodford, $P_{i,t} = P_{i,t-1}$. Suppose instead that $P_{i,t} = \pi_{t-1}P_{i,t-1}$, where $\pi_t = P_t/P_{t-1}$ and P_t is the aggregate price index. Deduce the implications of this change for the aggregate inflation expression, (2.14).

2. Consider the nonstochastic version of the Clarida-Gali-Gertler model. This is equations (2.14), (2.7) and a variant on (2.8) in the Woodford paper in question 1:

$$\delta\pi_{t+1} + \lambda y_t - \pi_t = 0$$

$$
\begin{aligned}
y_{t+1} - y_t - \frac{1}{\sigma}(r_t - \pi_{t+1}) &= 0 \\
\rho r_{t-1} + (1-\rho)\beta\pi_{t+1} + (1-\rho)\gamma y_t - r_t &= 0.
\end{aligned}
$$

(Some people call the first equation the 'supply curve' or 'Phillips curve', and the second equation an 'IS' curve. The third equation is a Taylor rule.) Write this system in the form:

$$AY_{t+1} + BY_t = 0,$$

for $t = 1, ...$ (note: in class the initial period is $t = 0$, but here I have changed it to $t = 1$). Here, $Y_t = (\pi_t, y_t, r_{t-1})$. Use the following parameter values, $\delta = 0.99$, $\sigma = 1$, $\lambda = 0.3$, $\gamma = 1$, $\rho = 0.5$, $\beta = 0.966$. Let the initial interest rate, r_0, have its steady state value of 0.

(a) Compute P and Λ in the eigenvector, eigenvalue decomposition of $-A^{-1}B$.

(b) Compute the set of minimal state variable solutions (i.e., the set of 2×3 matrices with the property $DY_1 = 0 \Longrightarrow DY_t = 0$ for all t).

(c) Suppose the first period is $t = 1$, and time evolves for $t = 1, 2, 3, ..., 20$. Construct the following 3 graphs. In each graph, display two lines of length 20 each, with $t = 1, ..., 20$ on the horizontal axis. One line, a line of zeros, corresponds to the nonstochastic, steady state equilibrium of the model. The other is a convergent solution to the Euler equations in which π_1 is a little greater than zero. Note how the high inflation equilibrium, while eventually returning to steady state, displays a period of high inflation and high output.

3. Now, construct a sunspot equilibrium for the model. To do this, note that in the stochastic version of the model, the equations are satisfied after application of the E_t operator. Write the state-space representation of the model,

$$AY_{t+1} + BY_t = \omega_{t+1},$$

where ω_{t+1} is an arbitrary random variable with $E_t\omega_{t+1} = 0$ for all t. Identify a convergent solution to this model in which ω_t is a non-trivial random variable. Draw ω_t from a random number generator in MATLAB and simulate the response of the three variables in Y_t to the sunspot. Display the results using the 3 graphs for the previous question. Also, display a simulation of the response of the system in a sunspot solution that does not display convergence.