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416, Fall 2001
Homework 3.
Due: Monday, October 29.

The typical household's utility function is:

$$\sum_{t=0}^{\infty} \beta^t \left[\log c_{1t} + \alpha \log c_{2t} - n_t\right],$$

where c_{1t} denotes cash goods, c_{2t} denotes credit goods, and n_t denotes employment. Total financial wealth in the period t asset market is A_t, which the household allocates between money M_t and bonds B_t. The household holds non-interest bearing money because it must do so to satisfy a cash requirement in the period t goods market:

$$P_t c_{1t} \leq M_t,$$

The household's financial wealth in next period's asset market reflects activities in the current period asset market and in the current period goods market:

$$A_{t+1} = M_t - P_{1t} c_{1t} - P_{2t} c_{2t} + B_t R_t + W_t n_t + T_t,$$

where R_t is the gross nominal interest rate and T_t is a government transfer. Here, $T = (g_t - 1)\bar{M}_t$, where g_t is the gross money growth rate and $\bar{M}_t$ is the aggregate, economy-wide stock of money.

Production of cash and credit goods occurs in different sectors, indexed by $i = 1, 2$, respectively. Producers in these sectors are perfectly competitive and have access to the following production technology:

$$y_i = \left[\int_0^1 y_i(x)^{\lambda} dx\right]^{\frac{1}{\lambda}}, \ i = 1, 2,$$

where $0 < \lambda < 1$ and $y_i(x)$ is intermediate good x is sector i, with $0 < x < 1$.

Intermediate goods producers' technology is given by:

$$y_i(x) = n_i(x), \text{ for } i = 1, 2,$$

where $n_i(x)$ is employment of labor by intermediate producer x in sector i. Intermediate good producers are perfectly competitive in the market for labor but are monopolists in their own output market. A fraction $0 < \mu < 1$ of intermediate goods producers in each sector sets their price before the monetary authority

chooses policy, g_t. These are 'sticky price producers'. The remaining $(1-\mu)$ producers are the 'flexible price producers' and set their prices after current period monetary policy has been set.

1. Write out the households' first order conditions.

2. Write out the necessary conditions for optimality for final goods producers and for sticky- and flexible-price intermediate goods producers in each sector. Consider symmetric equilibria in which all advance price-setters set the same price, P^e, and all flexible-price producers set the same price, $\hat{P}$. Derive an expression for the equilibrium final goods price level, P_i, as a function of $\hat{P}$ and P^e, for $i = 1, 2$. Explain why the final goods price for cash and credit goods coincide with each other, i.e., $P_1 = P_2$. (Throughout, let all prices be scaled by the beginning of period aggregate money stock.)

3. You may proceed as though the monetary authority can, by suitable choice of g, achieve whatever value of $\hat{P}$ that it wishes. That is, may suppose that the choice variable of the policymaker is actually $\hat{P}$, rather than g. Derive the mapping from $\hat{P}$ and P^e to private sector equilibrium objects, $\{c_1, c_2, n, R, P\}$. Use this to define the 'reduced form' utility function of the benevolent monetary authority:

$$U(\hat{P}, P^e) = \log c_1(\hat{P}, P^e) + \alpha \log c_2(\hat{P}, P^e) - n(\hat{P}, P^e).$$

   Show how one can use the household's intertemporal utility function to deduce the value of g needed to support the given $\hat{P}$ (here, you may assume that the cash in advance constraint holds as an equality in the current and next periods).

4. Show that when

$$1 - \mu < \alpha\mu < \frac{1-\mu}{1-\lambda}, \tag{0.1}$$

   then there is a unique price, P^*, that satisfies $U_{\hat{P}}(P^*, P^*) = 0$. For this value of P, when $\hat{P} = P^e = P^*$, then $R > 1$. (Here, $U_{\hat{P}}$ is the derivative of U with respect to its first argument.) Although this value of P satisfies the necessary condition for monetary policy authority optimization, we do not know if it is sufficient. To do so, assign parameter values consistent with (0.1) and graph $U(\hat{P}, P^*)$ over a range of values of $\hat{P}$, to verify that $U(P^*, P^*) = \max_{\hat{P}} U(\hat{P}, P^*)$, i.e., it is indeed the solution to the monetary authority's problem.

5. Show that $U_{\hat{P}}(P, P)$ can be decomposed into the sum of a term that corresponds to an inflation distortion and a term that corresponds to a monopoly distortion (as in Albanesi, Chari and Christiano 'Expectation Traps and Monetary Policy' (ACC)). In this example, these distortions are a function of c_1, rather than c_1/c_2, since $c_2 = \alpha\lambda$ when $\hat{P} = P^e$. What is the relevant domain of values for c_1? Show that the inflation and monopoly distortions are both non-negative over this domain.

6. Show that if (0.1) holds, then a Ramsey equilibrium with $R = 1$ and the cash in advance constraint holding as an equality, is not a Markov equilibrium.

7. Show that if $\alpha\mu < 1 - \mu$, then a private economy equilibrium in which $R = 1$ and the cash in advance constraint holds as an equality *is* a Markov equilibrium. Do this by separately considering the left and right derivatives of U with respect to $\hat{P}$, evaluated at $\hat{P} = P^e = P$, where P is the price in the posited private economy equilibrium. (The equations characterizing the private economy equilibrium are different for the left and right derivatives, since in one case the cash constraint is binding and in the other it is not.) After this, graph $U(\hat{P}, P)$ over a range of $\hat{P}$, to verify that $\hat{P} = P$ is the global optimum. Are there other Markov equilibria in this case? Again, you may assume that the monetary authority has direct control over $\hat{P}$.

8. Explain why it is that you find only one Markov equilibrium in this model, while ACC find at least two. Explain how you would pin down the parameters of the model, to determine its prediction for the magnitude of the inflation bias. Note how there exist parameter values under which this model predicts no inflation bias. That is, if we take the model with these parameter values seriously, then lack of commitment plays no role explaining the poor inflation outcomes that have been observed.