• State at the beginning of the period:

\[ s = p_{-1}. \]

\( p_{-1} \) price set by those who set prices last period, scaled by current aggregate \( M \).

• State after Policy Maker Takes Current Action, \( \mu \):

\[ (s, \mu), \]

where

\[ \mu = \frac{M'}{M}. \]
• For Any Monetary Policy Function, $\sigma(s)$, Define Private Sector Equilibrium Functions:

$$c(s, \mu; \sigma), \ l(s, \mu; \sigma), \ p(s, \mu; \sigma).$$

Functions, $c, l, p$ conditioned on current monetary action and ‘expectation’ that future monetary actions determined by $\sigma$.

• Private Sector Period Utility:

$$u(s, \mu; \sigma) = U(c(s, \sigma(s); \sigma), l(s, \sigma(s)))$$
• Next period’s state:

\[ s' = H(s, \mu; \sigma) \equiv \frac{p(s, \mu; \sigma)}{\mu}. \]

• Value associated with Following Equilibrium Functions Today and Forever:

\[ V(s; \sigma) = u(s, \sigma(s); \sigma) + \beta V(H(s, \mu; \sigma); \sigma). \]
• Policy Problem:

\[ \tilde{\sigma}(s; \sigma) = \arg \max_{\mu} u(s, \mu; \sigma) + \beta V(H(s, \mu; \sigma); \sigma). \]

• A Markov equilibrium:

- \( \sigma^*(s) \) such that

\[ \sigma^*(s) = \tilde{\sigma}(s; \sigma^*). \]

- A Fixed Point of Operator, \( \tilde{\sigma} \).

• Hence:

\[ V(s) = \max_{\mu} u(s, \mu) + \beta V(H(s, \mu)), \]

Where Absence of \( \sigma \) Argument Signifies \( \sigma = \sigma^* \), i.e.,

\[ u(s, \mu) \equiv u(s, \mu; \sigma^*). \]
• FE Corresponds to the Following ‘Sequence Problem’:

\[
\max_{\{\mu_t, s_{t+1}\}_{t=0}^\infty} \sum_{t=0}^{\infty} \beta^t u(s_t, \mu_t),
\]

subject to:

\[s_{t+1} = H(s_t, \mu_t), \quad t = 0, 1, 2, \ldots\]

\[s_0 \text{ given.}\]

• Alternative Representation:

\[
\max_{(\mu_0, s_1), \{\mu_t, s_{t+1}\}_{t=1}^\infty} u(s_0, \mu_0) + \beta u(H(s_0, \mu_0), \mu_1) + \ldots
\]

\[s_{t+1} = H(s_t, \mu_t)\]
• First Order Condition for $\mu_t$ (Klein-Krusell-Rios Rull ‘Generalized Euler Equation’):

$$u_2(s, \sigma^*(s)) + \beta u_1(H(s, \sigma^*(s)), \sigma^*(H(s, \sigma^*(s)))) \times H_2(s, \sigma^*(s)) = 0$$
Finding Equation That Characterizes $p(p_1, \mu)$

Final Good Firms

- State:

  $$s_2 = (p_1, \mu)$$

- Technology:

  $$c(s_2) = \left[ \int_0^1 (y_i(s_2))^\lambda \, di \right]^{\frac{1}{\lambda}}$$

  $$= \left[ \frac{1}{2} y(s_2)^\lambda + \frac{1}{2} y_{-1}(s_2)^\lambda \right]^{\frac{1}{\lambda}}.$$

$y(s_2)$ output of intermediate good firms who set prices after $s_2$

$y_{-1}(s_2)$ output of intermediate good firms who set price in previous period
• Final Good Problem:

\[
\max_{c(s_2), \{y_i(s_2)\}} \bar{p}(s_2)c(s_2) - \int_0^1 p_i(s_2)y_i(s_2)di,
\]

\(\bar{p}(s_2)\) price of the final good
\(p_i(s_2)\) price of \(i^{th}\) intermediate good
Both prices scaled by Beginning-of-Period \(M\)

• First Order Conditions:

\[
y_i(s_2) = c(s_2) \left( \frac{\bar{p}(s_2)}{p_i(s_2)} \right) \frac{1}{1-\lambda}.
\]

• Aggregate Price Level:

\[
\bar{p}(s_2) = \left[ \int_0^1 p_i(s_2)^{\frac{\lambda}{\lambda-1}} di \right]^{\frac{\lambda-1}{\lambda}} = \left[ \frac{1}{2} p(s_2)^{\frac{\lambda}{\lambda-1}} + \frac{1}{2} p^{-\frac{\lambda}{\lambda-1}} \right]^{\frac{\lambda-1}{\lambda}}
\]
Intermediate Good Firms

- Technology:
  
  \[ y_i(s_2) = zl_i(s_2). \]

- Pricing:

  Each Firm Sets its Price for Two Periods
  \[ \frac{1}{2} \text{ Set Price in Even Periods} \]
  \[ \frac{1}{2} \text{ Set Price in Odd Periods} \]

- Period \( t \) Profits (Divided by Beginning of \( t \) Aggregate Money):

  \[ \pi_i(s_2; p_i) = p_i(s_2)y_i(s_2) - \left[ \frac{R(s_2)w(s_2)}{z} \right] y_i(s_2), \]
• From Demand Curve:

\[ p_i y_i = c \left( \bar{p} \right)^{1/(1-\lambda)} (p_i)^{\lambda/(\lambda-1)} , \]
\[ y_i = c \left( \bar{p} \right)^{1/(1-\lambda)} (p_i)^{-1/(1-\lambda)} \]

So, Period t Profits:

\[ \pi_i(s_2; p_i) = c(s_2) \left( \bar{p}(s_2) \right)^{1/(1-\lambda)} \]
\[ \times \left\{ (p_i(s_2))^{\lambda/(\lambda-1)} - \left[ \frac{R(s_2)w(s_2)}{z} \right] (p_i(s_2))^{1/(\lambda-1)} \right\} \]
• Present Value of a Dollar Paid to Households at the End of Current Period:

\[ \frac{\beta u_c(s'_2)}{M \bar{\mu} \bar{p}(s'_2)}. \]

• Value to Household, of Period \( t \) Profits:

\[
\frac{\beta u_c(s'_2)}{M \bar{\mu} \bar{p}(s'_2)} \times \pi_i(s_2; p_i) M \\
= \frac{\beta u_c(s'_2)}{\mu \bar{p}(s'_2)} \times \pi_i(s_2; p_i) \\
= q(s_2) \pi_i(s_2; p_i),
\]

where

\[
q(s_2) = \frac{\beta u_c(s'_2)}{\mu \bar{p}(s'_2)} = \frac{\beta u_c(H(s_2), \sigma(H(s_2)))}{\mu \bar{p}(H(s_2), \sigma(H(s_2)))}
\]
• Recall:
\[ s'_2 = (s', \mu') = (H(s_2), \sigma(H(s_2))) \]

• Then,
\[ q(s'_2) = q(s', \mu') = q(H(s_2), \sigma(H(s_2))) \]

• Objective of a Price-Setting Firm:
\[
\max_{p_i(s_2)} \{ \pi(s_2)q(s_2) \\
+ \beta q(H(s_2), \sigma(H(s_2)))\pi(H(s_2), \sigma(H(s_2))) \}
\]
Straightforward Differentiation yields:

\[ p(s_2) = \frac{a(s_2)}{b(s_2)}, \]

where

\[
a(s_2) = q(s_2)\bar{p}(s_2)\frac{2-\lambda}{1-\lambda}w(s_2)\frac{R(s_2)}{z}c(s_2) + \frac{\beta}{\mu}q(s_2')\left(\bar{p}(s_2')\mu\right)^{\frac{2-\lambda}{1-\lambda}}w(s_2')\frac{R(s_2')}{z'}c(s_2')
\]

\[
b(s_2) = \lambda[q(s_2)\bar{p}(s_2)\frac{1}{1-\lambda}c(s_2) + \frac{\beta}{\mu}q(s_2')\left(\bar{p}(s_2')\mu\right)^{\frac{1}{1-\lambda}}c(s_2')]
\]
Households

• Labor Euler Equation:

\[ \frac{w}{\bar{p}}(s_2) = -\frac{u_l(s_2)}{u_c(s_2)}, \]

• Intertemporal Euler Equation for Saving:

\[ \frac{u_c}{\bar{P}} = R\frac{\beta u'_c}{\bar{P}'}, \]

or

\[ \frac{u_{cc}}{c\bar{P}} = R\frac{\beta u'_c c'}{\bar{P}' c'}, \]

or

\[ \frac{u_{cc}}{\mu M} = R\frac{\beta u'_c c'}{\mu \sigma' M}, \]

or

\[ R(s_2) = \frac{c(s_2)u_c(s_2)}{\beta c(s'_2)u_c(s'_2)}. \]
Equilibrium Conditions

- Resource Constraint:

\[ c(s_2) = zl(s_2) \frac{g(s_2)}{h(s_2)}, \]

where

\[ g(s_2) = \left[ \frac{1}{2} + \frac{1}{2} \left( \frac{p(s_2)}{p_{-1}} \right)^{\frac{\lambda}{1-\lambda}} \right]^{\frac{1}{\lambda}} \]

\[ h(s_2) = \frac{1}{2} + \frac{1}{2} \left( \frac{p(s_2)}{p_{-1}} \right)^{\frac{1}{1-\lambda}}. \]