Expectation Traps and Monetary Policy

by

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Countries Have Experienced Destructive Periods of High and Variable Inflation (‘Great Inflation’ of 1970s).

Can Absence of Commitment in Monetary Policy Account for This?
Absence of Commitment and Variable Inflation

² Kydland-Prescott, Barro-Gordon: Variability Reflects Movements in Fundamentals

² Possibility Explored Here: Variability Reflects Movements in Expectations.
Expectation Traps

Low Inflation

Private Agents Expect Low Inflation → Private Actions → Monetary Authority Supplies low Inflation

High Inflation

Private Agents Expect High Inflation → Private Actions → Monetary Authority Supplies High Inflation
Objective:

² This Paper:
   – Study the Nature of Equilibria in Standard Models
   – Are there Expectation Trap Equilibria?

² Longer-Term:
   Quantitative, Empirical Assessment of Expectation Trap Hypothesis.

² What’s At Stake?
Outline:

(1) Version of Lucas-Stokey Cash-Credit Good Model With
    – Some Preset Prices.
    – Svensson Timing ($P c \cdot M_{i_1}$).
    – Endogeneity of Cash/Credit Good Distinction.

(2) Findings

(3) Conclusion.
Preview of Findings

² Expectation Traps Can Occur.

² Financial Variables More Variable When Inflation is High.

² Money Demand Implications of the Model Promising.
The Model

² Households, Firms, Monetary Authority.
² Continuum of Goods.
² Infinite Horizon.
Timing

- Private Agents Expect High Inflation
  - $P^e$ Set High
  - Number of Goods Bought With Cash Reduced
- Monetary Authority May Produce High Inflation
  - Monopoly Distortion
  - Inflation Distortion
Basic Idea

Drive Towards a ‘Best Response Function’. Will Do So By Constructing a Mapping from $P^e$ to $\hat{P}$ for each possible $\theta, g, z$. 
State of The Economy At Various Points in the Period

- Shocks Realized, After Which the State is:

  \[ \theta, g, z \]

  \( z \) is a ‘money demand shock’ which is later endogenized

- Sticky Price Firms Select \( P^e \). After this the State is:

  \[ S = (\theta, g, z, P^e) \]

- Monetary Authority Selects Money Growth Rate, \( x \). After this the State is:

  \[ S_1 = (S, x). \]
Firms

• Each Good Produced by a Monopolist:

\[ y(\omega) = \theta n(\omega), \; \omega \in (0, 1). \]

• Wage Rate:

\[ W(S, x). \]

• 1 – \( \mu \) ‘flexible price firms’ set \( \hat{P}(S, x) \)

\[ \hat{P}(S, x) = \frac{W(S, x)}{\theta \rho}, \; 0 < \rho < 1 \]

• \( \mu \) ‘sticky price firms’ set \( P^e \) Before Observing \( x \). They ‘Conjecture’ \( x = X(S) \)

\[
\begin{align*}
P^e(\theta, g, z) &= W(\theta, g, z, P^e(\theta, g, z), X(\theta, g, z, P^e(\theta, g, z))) \\
&= \frac{P^e(\theta, g, z)}{\theta \rho}
\end{align*}
\]
Representative Household

Preferences:

\[ \sum_{t=0}^{\infty} \beta^t u(c_t, n_t), \quad c_t = \left[ \int_0^1 c_t(\omega)^\rho d\omega \right]^{1/\rho}, \]

\[ c_t(\omega) \sim \text{consumption of type } \omega \text{ good} \]

\[ \omega > z \sim \text{credit goods} \]

\[ \omega < z \sim \text{cash goods} \]

\[ n_t \sim \text{labor time} \]
• Asset Allocation Constraint:

\[ M + B \leq A. \]

All Nominal Quantities Scaled by Aggregate Stock of Money.

• Cash In Advance Constraint:

\[ M - \left[ P^e \mu z c_{11} + \hat{P}(S, x)(1 - \mu) z c_{12} \right] \geq 0 \]

\( c_{11} \sim \) cash goods from sticky price producers  
\( c_{12} \sim \) cash goods from flexible price producers
• Asset Evolution Equation:

\[ 0 \leq W(S, x)n + (1 - R(S, x))M \]
\[ -z \left[ P^e \mu c_{11} + \hat{P}(S, x)(1 - \mu)c_{12} \right] \]
\[ -(1 - z) \left[ P^e \mu c_{21} + \hat{P}(S, x)(1 - \mu)c_{22} \right] \]
\[ + R(S, x)A + (x - 1) + D(S, x) - xA' \].

\[ c_{21} \sim \text{credit goods from sticky price producers} \]
\[ c_{22} \sim \text{credit goods from flexible price producers} \]
Recursive Representation of Household Problem

\[ v(A, S, x) = \max_{n, M, A', c_{ij}; i, j = 1,2} \{ u(c, n) \]
\[ + \beta E_{\theta', g', z'} [v(A', S', X(S'))|\theta, g, z] \}

with:

\[ c = [z \mu c_{11}^\rho + z(1 - \mu)c_{12}^\rho \]
\[ + (1 - z)\mu c_{21}^\rho + (1 - z)(1 - \mu)c_{22}^\rho]^{\frac{1}{\rho}}. \]

\[ S' = (\theta', g', z', P^e(\theta', g', z')). \]
Solution to Household Problem

\[ n(A, S, x), \ M(A, S, x), \ v(A, S, x), \ A'(A, S, x), \ c_{ij}(A, S, x), \ i, j = 1, 2 \]
Private Sector Equilibrium

Definition: Given a monetary policy rule, $X(S)$, and a current money growth rate, $x$, a Private Sector Equilibrium is a collection of functions $P^e(\theta, g, z)$, $\hat{P}(S_1)$, $W(S_1)$, $v(A, S_1)$, $c_{ij}(A, S_1)$, $n(A, S_1)$, $M(A, S_1)$, $A'(A, S_1)$, $R(S_1)$, where $S_1 = (\theta, g, z, P^e(\theta, g, z), x)$, such that:

1. Functions $v$, $c_{ij}$, $n$, $M$, $A'$ solve household problem,

2. Firm optimization conditions satisfied,

3. Asset markets clear:

   $$A'(1, S_1) = 1 \text{ and } M(1, S_1) = 1,$$

4. Resource constraint satisfied: $\theta n(1, S_1) = g + z [\mu c_{11} + (1 - \mu) c_{12}] + (1-z) [\mu c_{21} + (1 - \mu) c_{22}]$. 
Monetary Authority Problem

\[
\max_x u(c(1, S, x), n(1, S, x)) + \beta E_{\theta', g', z'}[v(1, S', X(S'))|\theta, g, z],
\]

where

\[S' = (\theta', g', z', P^e(\theta', g', z'))\]

Definition A Markov equilibrium is a private sector equilibrium and a monetary policy rule such that \(X(S')\) solves Monetary Authority’s Problem.
Monetary Authority

² Problem:

\[ \max_{\hat{P}} U (\hat{P}; P^e; \mu; g; z) \]

Equilibrium

² (off R₁ corner):

\[ U_{\hat{P}} = 0; \hat{P} = P^e, \]
\[ c_{\text{cash; preset price}} = c_{\text{cash; flex price}} + c_{\text{cash}} \]
\[ c_{\text{credit; preset price}} = c_{\text{credit; flex price}} + c_{\text{credit}} \]
Findings

\[ U_p \Rightarrow \ddot{A}(\frac{c_{\text{cash}}}{c_{\text{credit}}}; z) = i \ddot{A}_{\text{ID}}(\frac{c_{\text{cash}}}{c_{\text{credit}}}; z) + \ddot{A}_{\text{MD}}(\frac{c_{\text{cash}}}{c_{\text{credit}}}; z): \]

Inflation Distortion:

\[ \ddot{A}_{\text{ID}} \frac{\mu}{c_{\text{credit}}}; z \Rightarrow (R \cdot 1) \frac{M}{P} \]

Monopoly Distortion:

\[ \ddot{A}_{\text{MD}} \frac{\mu}{c_{\text{credit}}}; z = [u_n + \mu u_{\text{credit}}] n_P \]
Two Examples

² Cash-Credit Distinction Exogenous
Calibration:

‘Money Demand Regression’ \( z = 0.182; \frac{1}{2}z = 0.643; \)
Parks \( 1 = 0.01 \)
Christiano-Eichenbaum \( \tilde{\alpha} = 4 \)
\( \mu = 1; g = 0.05 \)
Two Markov Equilibria : \( R = 1:20; 1:60; \)

² Cash-Credit Distinction Endogenous
Marginal Costs and Benefits of Unexpected Inflation: Calibrated Parameter Values

\[ \theta = 1, \mu = 0.1, \rho = 0.643, \psi = 4, z = 0.182, g = 0.05 \]
Money Demand Implications of Endogenous z Model

Money Demand Equation \((u_1 = u_2 = R)\)

\[
\frac{\text{consumption}}{M = P} = 1 + \frac{1}{z} \frac{i}{R} \frac{1}{z^{1/2}}
\]

Has potential to resolve money demand puzzles:

(1) ‘Short Run Elasticity of Demand Lower Than Long Run’.

(2) Money Demand Disturbances Highly Persistent.

(3) Upward Drift in Velocity.
Numerical Example

2 Non-Shock Parameters:

\[ \bar{\gamma} = 1:03; \; \gamma' = 0.63; \; \tilde{\bar{A}} = 1:64; \; \frac{1}{2} = 0.83; \]
\[ \gamma = 0:1; \; \dot{z} = 0:3; \; \frac{3}{4} = 1:01: \]

Shock Parameters, \( g; \mu; \gamma' \):

means : 0.55; 1; 0.01
std deviations : 0.001, 0.05, 0.0005
autocorrelations : 0.9, 0:9; 0:9:

2 Simulation Results:

<table>
<thead>
<tr>
<th></th>
<th>High Inflation</th>
<th>Low Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{4} )</td>
<td>0.020</td>
<td>0.020</td>
</tr>
<tr>
<td>( \frac{3}{4}_h )</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>( \frac{3}{4}_r )</td>
<td>0.002</td>
<td>0.00</td>
</tr>
<tr>
<td>( \frac{3}{4}_q )</td>
<td>0.025</td>
<td>0.017</td>
</tr>
</tbody>
</table>
Expected Money Growth, $g$

Actual Money Growth, $G$

A: For $g$ Above This, $Z$ Falls
Conclusion

² Expectation Traps Equilibria Occur in Simple Monetary Models.

² They are More Likely, the More Elastic is Money Demand.

² There is Reason to Expect that Models with Expectation Trap Equilibria Can Account for Other Key Features of the Data:
   – Classic Money Demand Puzzles.
   – Properties of High and Low Inflation Economies.

² The Expectation Trap Hypothesis About Variable Inflation Deserves Further Consideration.