Formulation and Estimation of a Standard General Equilibrium Macro Model: A Shock-Based Approach

- Shock-Based Strategy for Estimating a Dynamic GE Model

- Estimating the Dynamic Effects of Monetary Policy and Technology Shocks.

- Fit a General Equilibrium Model to these Effects
  (a) Nominal Part of Economy Important
      * Sticky Prices, Sticky Wages, etc.
  (b) Real Part of Economy Important
      * Adjustment Costs in Investment, Variable Capital Utilization, Habit Persistence in Preferences.
  (c) What Nominal Frictions and Real Features are Necessary for a Model to Conform Well with the Facts?
Shock-Based Strategy for Estimating a Dynamic GE Model

• Vector Autoregression (VAR):

\[ Y_t = B_1 Y_{t-1} + ... + B_p Y_{t-p} + u_t, \quad EU_t u_t' = V \]

\[ B_i \text{'s, } u_t \text{'s and } V \text{ Easily Obtained by OLS Regressions.} \]

• Fundamental Economic Shocks, \( e_t \):

\[ u_t = C e_t, \quad Ee_t e_t' = I, \quad CC' = V. \]
Shock-Based Strategy...

- Impulse Responses ($p = 2$):

\[
Y_t - E_{t-1}Y_t = C e_t
\]
\[
E_t Y_{t+1} - E_{t-1}Y_{t+1} = B_1 C e_t
\]
\[
E_t Y_{t+2} - E_{t-1}Y_{t+2} = B_1^2 C e_t + B_2 C e_t
\]
\[
E_t Y_{t+3} - E_{t-1}Y_{t+3} = [B_1 (B_1^2 + B_2) + B_1 B_2 + B_2 B_1] C e_t
\]

- Suppose Want Dynamic Response of $Y_t$ to $i^{th}$ Element of $e_t$.
  Need $B_i$’s and $i^{th}$ Column of $C$. 
Shock-Based Strategy...

• Problem:
  \( N^2 \) Unknown Elements in \( C \),
(a) Only \( N(N+1)/2 \) Equations in:

\[
CC'' = V
\]

(b) Need to Make (Identifying) Assumptions!
Shock-Based Strategy...

- Moving Average Representation:

\[ Y_t = c_1(L)e_{1t} + c_2(L)e_{2t} + \ldots + c_p(L)e_{pt} \]

\[ c_i(L) = c^0_i + c^1_i L + c^2_i L^2 \ldots \]

- A Dynamic GE Model Implies:

\[ c_i(L; \gamma), \text{ each } i. \]

- Estimate \( \gamma \) by making

\[ c_i(L; \gamma) - c_i(L) \]

small, \( i = 1, 2, 3 \ldots \)

- With Enough Shocks, Have a Model that Fits the Data As Well as a VAR.
Alternative (More Traditional) Strategy for Estimating A Model:

- Compute Unconditional Moments of Data
- Estimate Model Based on All Moments (Maximum Likelihood)
- Disadvantage of This Approach:
  - Need to Determine All Shocks in the Model

Advantages of Shock-Based Strategy

- Avoid Need to Specify All the Shocks Right Away
- The Economics of a Model Lie in its Impulse Responses to Shocks.
  - Analysis and Diagnosis of Models Made Transparent.
Advantages, cont’d...

- Sometimes, Interesting Puzzles and Questions Posed Directly in Terms of Shocks.
  (a) Monetary Policy Shock.
  * Why So Much Inflation Inertia, Output Persistence?
  * Other Issues (Price ‘Puzzle’)
  (b) Technology Shock.
  * How Important In Business Cycle?
  * Response of Employment and Other Variables to Technology?
  * What is Role of Monetary Policy in Propagation of Technology Shocks?
Outline of Remainder of Analysis

(1) The Vector Autoregression Used in the Analysis.

(2) Estimated Impulse Responses to:
   (a) Monetary Policy Shock.
   (b) Persistent Shock to Technology. Reconciliation With the Literature.

(3) Our GE Model.

(4) Fitting the GE Model to VAR Impulse Responses.
Vector Autoregression

- **VAR:**

\[ Y_t = B(L)Y_{t-1} + Ce_t, \]
\[ e_t \sim \text{fundamental shocks} \]

- Vector, \( Y_t \), Set Up to Satisfy Integration and Co-Integration Properties of Model.

\[
Y_t = \begin{bmatrix}
\Delta \ln(GDP_t/\text{Hours}_t) \\
\Delta \ln(GDP \text{ deflator}_t) \\
\text{capacity utilization}_t \\
\ln(GDP_t/\text{Hours}_t) - \ln(W_t/P_t) \\
\ln(\text{Hours}_t) \\
\ln(C_t/GDP_t) \\
\ln(I_t/GDP_t) \\
\text{Federal Funds Rate}_t \\
\ln(GDP \text{ deflator}_t) + \ln(GDP_t) - \ln(M2_t)
\end{bmatrix}
\]

- Estimation Period: 1960Q1 - 2001QIV
Figure 1
Data Used In VAR
Identification Assumptions for Monetary Policy Shock

- Monetary Policy Rule:

\[ R_t = f(\Omega_t) + \varepsilon_t \]
\[ \varepsilon_t \sim \text{Monetary Policy Shock} \]

- Identification Assumptions:
  - \( \Omega_t = \{\text{aggregate activity}_t, \text{aggregate prices and wages}_t, \text{lagged variables}\} \)
  - \( \varepsilon_t \) orthogonal with \( \Omega_t \)

- Our GE Model is Consistent with These Assumptions.
What Is A Monetary Policy Shock?

- Shocks to Preferences of Monetary Authority
- Technical Factors Like Measurement Error

(Bernanke-Mihov):

\[ x_t(0) = x_t + v_t, \quad x_t(1) = x_t + u_t \]

\[ S_t = \beta_0 S_{t-1} + \beta_1 x_t(0) + \beta_2 x_{t-1}(1) \]

or

\[ S_t = \beta_0 S_{t-1} + \beta_1 x_t + \beta_2 x_{t-1} + \varepsilon_t \]

\[ \varepsilon_t = \beta_1 v_t + \beta_2 u_{t-1}. \]

Recursiveness Assumption: \( \beta_0 = 0 \), or \( \beta_0 \neq 0 \), \( u_t = 0 \).
What Is a Monetary Policy Rule?

- Combination of Structural Policy Rule and Other Stuff
– Example 1:

‘True’ Policy Rule: $S_t = \alpha e_t + \varepsilon_t$,

where

$$e_t \sim \text{‘innovation’ in some variable} = \sum_{i=0}^{\infty} \beta_i x_{t-i}.$$

Then

Estimated Policy Rule: $S_t = \alpha \sum_{i=0}^{\infty} \beta_i x_{t-i} + \varepsilon_t$

Here, $\alpha$’s are not structural.

– Example 2 (Clarida-Gertler):

‘True’ Policy Rule: $S_t = \alpha E_t x_{t+1} + \varepsilon_t$
Dynamic Effects of a Monetary Policy Shock

- After a Positive Monetary Shock:
  (a) hump-shaped response of output, consumption, investment, labor with peak effect after roughly 1 year.
  (b) hump-shaped response in inflation, with peak response after about 2 years (‘Inflation Inertia’)
  (c) interest rate down for one year.
  (d) money growth up for 2-3 quarters.
  (e) real wage up slightly.

- Lots of Internal Propagation
- Strong Liquidity Effect
Figure: Impulse Responses to a Monetary Policy Shock
Identification Assumptions for Technology Shock

- Identification Assumption:
  Technology Shock is *Only* Shock that Has Long-Run Impact on (Forecast of) Level of Labor Productivity:

\[
\lim_{j \to \infty} [E_t y_{t+j} - E_{t-1} y_{t+j}] = f(\text{technology shock only})
\]

\[
y_t = \frac{\text{output}}{\text{hour}}
\]

- Blanchard-Quah/Jordi Gali:
  This Assumption Makes it Possible to Estimate Technology Shock, Even Without Direct Observations on Technology

- Our GE Model is Consistent with This Assumption.
Technology Identification

- Simple VAR:

\[
Y_t = BY_{t-1} + u_t, \quad Eu_tu_t' = V
\]

\[
u_t = Ce_t
\]

\[
Y_t = \begin{pmatrix} \Delta y_t \\ x_t \end{pmatrix}, \quad C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}, \quad e_t = \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}
\]

\[e_{1t} \sim \text{Technology Shock.} \]

- From Applying OLS to Both Equations in VAR, We Know:

\[B, V\]

- Problem: \(CC' = V\) Provides only Three Equations in Four Unknowns in C.

- Result: Assumption that \(e_{2t}\) Has No Long Run Impact on \(y_t\) Can Be Used to Identify All of \(C\)
Technology Identification

- Note #1:

\[ E_t [\Delta y_{t+1} + \Delta y_t] - E_{t-1} [\Delta y_{t+1} + \Delta y_t] \]
\[ = E_t [y_{t+1} - y_t + y_t - y_{t-1}] - E_{t-1} [y_{t+1} - y_t + y_t - y_{t-1}] \]
Technology Identification

- Note #1:

\[
E_t [\Delta y_{t+1} + \Delta y_t] - E_{t-1} [\Delta y_{t+1} + \Delta y_t] \\
= E_t [y_{t+1} - y_t + y_t - y_{t-1}] - E_{t-1} [y_{t+1} - y_t + y_t - y_{t-1}] \\
= E_t[y_{t+1}] - y_{t-1} - E_{t-1}[y_{t+1}] + y_{t-1}
\]
Technology Identification

• Note #1:

\[ E_t [\Delta y_{t+1} + \Delta y_t] - E_{t-1} [\Delta y_{t+1} + \Delta y_t] \]
\[ = E_t [y_{t+1} - y_t + y_t - y_{t-1}] - E_{t-1} [y_{t+1} - y_t + y_t - y_{t-1}] \]
\[ = E_t [y_{t+1}] - y_{t-1} - E_{t-1} [y_{t+1}] + y_{t-1} \]
\[ = E_t [y_{t+1}] - E_{t-1} [y_{t+1}] \]
Technology Identification

• Note #1:

\[ E_t [\Delta y_{t+1} + \Delta y_t] - E_{t-1} [\Delta y_{t+1} + \Delta y_t] = E_t [y_{t+1}] - E_{t-1} [y_{t+1}] \]

• Note #2:

\[ (1, 0) E_t [Y_{t+1} + Y_t] - (1, 0) E_{t-1} [Y_{t+1} + Y_t] = E_t [\Delta y_{t+1} + \Delta y_t] - E_{t-1} [\Delta y_{t+1} + \Delta y_t] \]
Technology Identification

- Note #1:

\[
E_t [\Delta y_{t+1} + \Delta y_t] - E_{t-1} [\Delta y_{t+1} + \Delta y_t] = E_t[y_{t+1}] - E_{t-1}[y_{t+1}]
\]

- Note #2:

\[
(1, 0)E_t [Y_{t+1} + Y_t] - (1, 0)E_{t-1} [Y_{t+1} + Y_t] = E_t [\Delta y_{t+1} + \Delta y_t] - E_{t-1} [\Delta y_{t+1} + \Delta y_t]
\]

- Note #3:

\[
(1, 0)E_t [Y_{t+1} + Y_t] - (1, 0)E_{t-1} [Y_{t+1} + Y_t] = (1, 0) \{ E_t [Y_{t+1}] - E_{t-1} [Y_{t+1}] + E_t [Y_t] - E_{t-1} [Y_t] \}
\]
Technology Identification

• Note #1:

\[ E_t [\Delta y_{t+1} + \Delta y_t] - E_{t-1} [\Delta y_{t+1} + \Delta y_t] = E_t[y_{t+1}] - E_{t-1}[y_{t+1}] \]

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• Note #3:

\[ (1, 0) E_t [Y_{t+1} + Y_t] - (1, 0) E_{t-1} [Y_{t+1} + Y_t] = (1, 0) \{ E_t [Y_{t+1}] - E_{t-1} [Y_{t+1}] + E_t [Y_t] - E_{t-1} [Y_t] \} = (1, 0) [B C e_t + C e_t] \]
Technology Identification

- Note #1:

\[
E_t [\Delta y_{t+1} + \Delta y_t] - E_{t-1} [\Delta y_{t+1} + \Delta y_t] \\
= E_t [y_{t+1}] - E_{t-1} [y_{t+1}]
\]

- Note #2:

\[
(1, 0) E_t [Y_{t+1} + Y_t] - (1, 0) E_{t-1} [Y_{t+1} + Y_t] \\
= E_t [\Delta y_{t+1} + \Delta y_t] - E_{t-1} [\Delta y_{t+1} + \Delta y_t]
\]

- Note #3:

\[
(1, 0) E_t [Y_{t+1} + Y_t] - (1, 0) E_{t-1} [Y_{t+1} + Y_t] \\
= (1, 0) [BC e_t + C e_t]
\]

- Conclude:

\[
E_t [y_{t+1}] - E_{t-1} [y_{t+1}] = (1, 0) [B + I] C e_t
\]
Technology Identification

- Result for Two Period-Ahead Forecast of $y_t$:

$$E_t[y_{t+2}] - E_{t-1}[y_{t+2}] = (1, 0) \left[ B^2 + B + I \right] C e_t$$
Technology Identification

- Result for Two Period-Ahead Forecast of \( y_t \):

\[
E_t[y_{t+2}] - E_{t-1}[y_{t+2}] = (1, 0) \left[ B^2 + B + I \right] Ce_t
\]

- Result for \( j \) Period-Ahead Forecast of \( y_t \):

\[
E_t[y_{t+j}] - E_{t-1}[y_{t+j}] = (1, 0) \left[ B^j + B^{j-1} + \ldots + B^2 + B + I \right] Ce_t
\]
Technology Identification

• Result for Two Period-Ahead Forecast of $y_t$:

$$E_t[y_{t+2}] - E_{t-1}[y_{t+2}] = (1, 0) \left[ B^2 + B + I \right] C e_t$$

• Result for $j$ Period-Ahead Forecast of $y_t$:

$$E_t[y_{t+j}] - E_{t-1}[y_{t+j}] = (1, 0) \left[ B^j + B^{j-1} + \ldots + B^2 + B + I \right] C e_t$$

• As $j \to \infty$:

$$\lim_{j \to \infty} E_t[y_{t+j}] - E_{t-1}[y_{t+j}] = (1, 0) \left[ \ldots + B^j + B^{j-1} + \ldots + B^2 + B + I \right] C e_t$$
Technology Identification

- Result for Two Period-Ahead Forecast of $y_t$:

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- As $j \to \infty$:

$$\lim_{j \to \infty} E_t[y_{t+j}] - E_{t-1}[y_{t+j}] = (1, 0) \left[ \ldots + B^j + B^{j-1} + \ldots + B^2 + B + I \right] C e_t$$

$$= (1, 0) \left[ I - B \right]^{-1} C e_t$$
Technology Identification

• As \( j \to \infty \):

\[
\lim_{j \to \infty} E_t[y_{t+j}] - E_{t-1}[y_{t+j}] = (1, 0) \left[ I - B \right]^{-1} C e_t
\]

• Identification Assumption About Technology:

\[
[I - B]^{-1} C e_t = \begin{bmatrix}
\text{number} & 0 \\
\text{number} & \text{number}
\end{bmatrix}
\begin{pmatrix}
e_{1t} \\
e_{2t}
\end{pmatrix}
\]
Technology Identification

- As $j \to \infty$:

$$\lim_{j \to \infty} E_t[y_{t+j}] - E_{t-1}[y_{t+j}] = (1, 0) [I - B]^{-1} C e_t$$

- Identification Assumption About Technology:

$$[I - B]^{-1} C e_t = \begin{bmatrix} \text{number} & 0 & \left( \begin{array}{c} e_{1t} \\ e_{2t} \end{array} \right) \\ \text{number} & \text{number} & \end{bmatrix} = \begin{bmatrix} \text{number} \times e_{1t} + 0 \times e_{2t} \\ \text{number} \times e_{1t} + \text{number} \times e_{2t} \end{bmatrix}$$
Technology Identification

• As $j \to \infty$:

$$\lim_{j \to \infty} E_t[y_{t+j}] - E_{t-1}[y_{t+j}] = (1, 0) [I - B]^{-1} C e_t$$

• Identification Assumption About Technology:

$$[I - B]^{-1} C = \begin{bmatrix} \text{number} & 0 \\ \text{number} & \text{number} \end{bmatrix}$$

• Final Result (?!)

Solve for $C$ Using

1, 2 element of $[I - B]^{-1} C$ is zero

$$CC'' = V$$
Conclusion of Technology Identification With Long-Run Restrictions

• Example (Taken from Blanchard-Quah) Gives Flavor of Identification When There are Long-Run Restrictions

• In Practice, An Alternative Computational Strategy, Based on Instrumental Variables Estimation, Also Useful.

• Other Applications of Long-Run Restrictions:
  – Labor Supply Shock Has Long-Run Impact on Hours Worked, No Long Run Impact on Productivity
  – Money Supply Shock Has No Impact on Real Variables in Long Run, Money Demand Shock Affects Real Balances in Long Run
Instrumental Variables (Shapiro-Watson) Approach

• IV Regression:

\[ \Delta y_t = a_{\Delta y}(L)\Delta y_{t-1} + a_1(L)Y_t + a_R(L)R_{t-1} \\
+ a_V(L)Velocity_{t-1} + \varepsilon_{xt} \]

• All Non-$\varepsilon_{xt}$ Shocks Affect $\Delta y_t$ Via $Y_t, R_t, Velocity_t$.

• To Have No Long-Run Effect of $y_t$
Requiring:

\[ a_1(L) = \tilde{a}_1(L)(1 - L), \]
\[ a_R(L) = \tilde{a}_R(L)(1 - L), \]
\[ a_V(L) = \tilde{a}_V(L)(1 - L). \]

• IV Regression With LR Restrictions:

\[ \Delta y_t = a_{\Delta y}(L)\Delta y_{t-1} + \tilde{a}_1(L)\Delta Y_t + \tilde{a}_R(L)\Delta R_{t-1} \\
+ \tilde{a}_V(L)\Delta Velocity_{t-1} + \varepsilon_{xt} \]
Shapiro-Watson Approach, Cont’d....

- First Stage: Estimate IV Regression (IV is Needed Because Technology Shocks, $\varepsilon^x_t$, Also Affect $Y_t$)

- Second Stage: Regress VAR Reduced-Form Disturbances, $u_t$, on $\hat{\varepsilon}_{xt}$, To Obtain Relevant $C_i$.

\[
Y_t = B(L)Y_{t-1} + u_t, \quad u_t = Ce_t,
\]

- Practical Problem: Method Does Not Impose Overindentifying Restrictions on $B(L)$ (see ACEL).
Results for Technology Identification

• (a) Labor and Capital Utilization Rise.
  (b) Inflation Low.
  (c) Money Growth Accommodates.
Figure: Impulse Responses to an Innovation in Technology
Relation to the Literature:

- Our Finding That Employment Rises Persistently After Technology Shock Contrasts with Results in Literature


- We Use Same Long-Run Identifying Restrictions Used in Literature.
Relation to the Literature, Cont’d...

- Our Analysis Suggests Results in Literature Reflect Two Forms of Specification Error:
  - Working With VAR’s With Too Few Variables.
  - Overdifferencing the Hours Data.

- How Do We Establish These Conclusions?
  - These Are The Implications of Our VAR Model
    * Model Passes a Basic Encompassing Test
  - Our VAR Model is More Plausible than Alternatives
    * ‘We can explain their results more easily than they can explain our results’ (Christiano-Ljungqvist, 1988.)
Analysis Conditional on Our VAR Being True

- Omitted Variables:
  - Drop $Cap. Ut., C_t, I_t, W_t/P_t, M^2_t$ from our system to a 4-variable model with levels hours.
  - This model implies: hours fall after positive technology shock.
  - This outcome is predicted by our (9-variable) model.

- Over-Differencing:
  - Also measure hours in first differences
  - With this further modification, hours fall a lot after positive technology shock.
  - This outcome is predicted by our (9-variable) model.
Figure 10: Analysis of Two Versions of Four-Variable VAR

4 Variable VAR, Hours in Levels

4 Variable VAR, Hours in Growth Rates
Our VAR Specification is More Plausible than Alternatives

- Alternative to Our VAR to Worry About: Version with First Difference In Hours.
- This Version Predicts Hours *Fall* After a Positive Shock To Technology.
- According to a Limited Information Bayesian Procedure:
  - Odds are 1.7 to 1 in Favor of Our VAR Over Alternative.
Effect of Differencing Hours Worked on our Benchmark Model
Limited Information Bayesian Analysis

- Define: \( Q = \{ \text{Event that Hours Rise After Positive Technology Shock in ‘Our’ Model And Hours Fall After Positive Technology Shock in ‘Their’ Model} \} \)

- Results of Simulation Analysis:

\[
P(Q|‘Our’) = 54\%
\]
\[
P(Q|‘Their’) = 32\%
\]

- Using Bayesian Updating Formula (With Flat Priors):

\[
\frac{P(‘Our’|Q)}{P(‘Their’|Q)} = \frac{54}{35} = 1.7
\]

- The Odds Favor ‘Our’ Model Nearly 2 to 1.
Computing $P(Q|\text{‘Our’})$  

Computing $P(Q|\text{‘Their’})$
Figure: Impulse Responses to a Monetary Policy Shock
Figure: Impulse Responses to an Innovation in Technology
Is there Other Useful Information We can Get From our VAR?

- Decomposition of Variance: What Fraction of Variance in Data is Due to this or That Identified Shock

- What Shocks Account Most for Various Episodes in the Data?

- Policy Analysis:
  - What Is the Impact on the Distribution of the Variables in the VAR, If Policy over the Next 6 Months Keeps the Interest Rate 25 Basis Points Lower than it is Today?
  - Formally:

Event : $Q = R \uparrow$ by 25 Basis Points for 6 Months

$E[\text{Variables of Interest}|Q]$

95 Confidence Interval on Variables of Interest$|Q$
Original Contributions on Policy Analysis in VAR: Doan-Litterman-Sims, 1980s; Sims, Minneapolis Fed Quarterly Review, 1980s (see RATS manual). See also recent work by Sims, Zha, Leeper.
Variance Decomposition Results

- Monetary Policy Shocks Account for Very Little Variance, Even in Money.
- Technology Shocks Account for Over 50 Percent of Overall Variance of Output Horizon.
- Technology Shocks Account for Only a Small Part of Business Cycle Fluctuations.
Percent of Overall Variance At Various Horizons, Due to Monetary Policy Shocks

Table 1: Contribution of Policy Shocks to Variance

<table>
<thead>
<tr>
<th>Variable</th>
<th>Forecast Variance at Indicated Horizons</th>
<th>B. C. Freq’s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Output</td>
<td>0.0</td>
<td>3.1</td>
</tr>
<tr>
<td>M2 Growth</td>
<td>6.2</td>
<td>6.5</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.0</td>
<td>1.7</td>
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<td>Fed Funds</td>
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<td>Capacity Util</td>
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<tr>
<td>Average Hours</td>
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<tr>
<td>Real Wage</td>
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<tr>
<td>Consumption</td>
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<tr>
<td>Investment</td>
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<tr>
<td>Velocity</td>
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<td>2.1</td>
</tr>
</tbody>
</table>
Percent of Overall Variance At Various Horizons, Due to Technology Shocks

Table 2: Contribution of Tech. Shocks to Variance

<table>
<thead>
<tr>
<th>Variable</th>
<th>Forecast Variance at Indicated Horizon</th>
<th>B. C. Freq’s</th>
</tr>
</thead>
<tbody>
<tr>
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<td>4</td>
</tr>
<tr>
<td>Output</td>
<td>48.4</td>
<td>48.8</td>
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<tr>
<td>M2 Growth</td>
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<tr>
<td>Inflation</td>
<td>41.1</td>
<td>32.1</td>
</tr>
<tr>
<td>Fed Funds</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>Capacity Util</td>
<td>1.4</td>
<td>9.7</td>
</tr>
<tr>
<td>Average Hours</td>
<td>4.1</td>
<td>15.6</td>
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<tr>
<td>Real Wage</td>
<td>27.7</td>
<td>32.1</td>
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<tr>
<td>Consumption</td>
<td>61.0</td>
<td>67.4</td>
</tr>
<tr>
<td>Investment</td>
<td>9.8</td>
<td>14.5</td>
</tr>
<tr>
<td>Velocity</td>
<td>11.4</td>
<td>3.0</td>
</tr>
</tbody>
</table>
Role of Technology Shocks: Big in Low Frequencies,
Small in Business Cycles

Thick Line - Actual Historical Data, Thin Line - Technology Only
Next Step:

- Construct a Model that is Consistent With Identifying Assumptions in VAR
- Estimate the Combination of Frictions Needed for Model Responses to Resemble Estimated Responses.
Key Features of the Model

• Consistent with Identifying Assumptions Used in Estimating Response of Economy to Shocks.

• Two Forms of Nominal Rigidities: Calvo-style Nominal Price and Wage Contracts.

• Real Side of Model - Three Departures from Standard Textbook Growth Model
  – Habit Persistence in Consumption
  – Adjustment Costs in Investment
  – Variable Capacity Utilization

• Model Will Do Well Empirically, with Reasonable Degree Price and Wage Stickiness.
Model

- Timing Assumptions.
- Firms.
- Households.
- Monetary Authority.
Timing

(1) Technology Shock Realized.


(3) Monetary Policy Shock Realized.

(4) Household Money Demand Decision Made.

(5) Production, Employment, Purchases Occur, and Markets Clear.

- Note: Wages, Prices and Output Predetermined Relative to Policy Shock.
Firms

Final Good Firms

• Technology:

\[ Y_t = \left[ \int_0^1 Y_{it}^{\frac{1}{\lambda_f}} di \right]^{\lambda_f}, \quad 1 \leq \lambda_f < \infty \]

• Objective:

\[ \max P_t Y_t - \int_0^1 P_{it} Y_{it} di \]

• Foms and Prices:

\[ \left( \frac{P_t}{P_{it}} \right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{it}}{Y_t}, \quad P_t = \left[ \int_0^1 P_{it}^{\frac{1}{1-\lambda_f}} di \right]^{(1-\lambda_f)} \]
Firm Sector

Final Good, Competitive Firms

Intermediate Good Producer 1

Intermediate Good Producer 2

Intermediate Good Producer infinity

Competitive Market For Homogeneous Capital

Competitive Market for Homogeneous Labor Input

Household 1

Household 2

Household infinity

1.
Intermediate Good Firms -

- Each $Y_{it}$ Produced by a Monopolist, With Demand Curve:

$$\left( \frac{P_t}{P_{it}} \right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{it}}{Y_t}.$$  

- Technology:

$$Y_{it} = \begin{cases} 
   k_{it}^\alpha (z_t L_{it})^{1-\alpha} - \phi z_t & \text{if } k_{it}^\alpha (z_t L_{it})^{1-\alpha} \geq \phi z_t \\
   0, & \text{otherwise}
\end{cases}.$$  

- Here, $z_t$ is a Technology Shock:

$$x_t = \log z_t - \log z_{t-1}, \quad x_t = (1-\rho_x)x + \rho_x x_{t-1} + \varepsilon_{xt}.$$  

- $\phi > 0$:
  - Ensures Zero Profits in Steady State
• Calvo Price Setting:
  - With Probability $1 - \xi_p$, \(i^{th}\) Firm Sets Price, \(P_{it}\), Optimally, to \(\tilde{P}_t\).
  - With Probability $\xi_p$, Do Not Optimize Current Price. Instead:

\[
P_{it} = \pi_{t-1}P_{i,t-1}, \quad \pi_t = \frac{P_t}{\bar{P}_{t-1}}.
\]
• Firms Setting Prices Optimally at $t$

Choose $\tilde{P}_t$ to max:

\[
v_t \left[ \tilde{P}_t Y_{it} - MC_t Y_{it} \right] \\
+ \beta \xi_p v_{t+1} \left[ \tilde{P}_t \pi_t Y_{i,t+1} - MC_{t+1} Y_{i,t+1} \right] \\
+ (\beta \xi_p)^2 v_{t+2} \left[ \tilde{P}_t \pi_t \pi_{t+1} Y_{i,t+2} - MC_{t+2} Y_{i,t+2} \right] \\
+ \ldots
\]

subject to:

\[
\left( \frac{P_t}{\tilde{P}_t} \right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{it}}{Y_t}.
\]

$v_t \sim$ value of a dividend at $t$

$MC_t \sim$ given
• Scaling:

\[ \tilde{p}_t = \frac{\tilde{P}_t}{P_t}, \quad w_t = \frac{W_t}{P_t} \]

\[ r_t^k = \frac{\text{rental rate on capital}}{P_t} \]

\[ s_t = \frac{MC_t}{P_t}. \]

• Real Marginal Cost:

\[ s_t = \left( \frac{1}{1 - \alpha} \right)^{(1-\alpha)} \left( \frac{1}{\alpha} \right)^\alpha (r_t^k)^\alpha (w_t R_t)^{1-\alpha} \frac{1}{z_t} \]

• Linear approximation:

\[ \hat{x}_t \equiv \frac{x_t - x}{x}. \]
- Approximate (Linearized) Solution:

\[
\hat{p}_t = \hat{s}_t + \sum_{l=1}^{\infty} (\beta \xi_p)^l (\hat{s}_{t+l} - \hat{s}_{t+l-1}) \\
+ \sum_{l=1}^{\infty} (\beta \xi_p)^l (\hat{\pi}_{t+l} - \hat{\pi}_{t+l-1})
\]

- Front-Loading:
  - \(\hat{p}_t > \hat{s}_t\) if \(\hat{s}_{t+l} > \hat{s}_t\) and/or \(\hat{\pi}_{t+l} > \hat{\pi}_t\).
Aggregate Price Level:

\[ P_t = \left[ \int_0^1 P_{it}^{\frac{1}{1-\lambda_f}} di \right]^{(1-\lambda_f)} \]

\[ = \left[ (1 - \xi_p) \tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p (\pi_{t-1} P_{t-1})^{\frac{1}{1-\lambda_f}} \right]^{1-\lambda_f} \]

Scale:

\[ 1 = \left[ (1 - \xi_p) \tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p \left( \frac{\pi_{t-1}}{\pi_t} \right)^{\frac{1}{1-\lambda_f}} \right]^{1-\lambda_f} \]

Approximately

\[ \tilde{P}_t = \frac{\xi_p}{1 - \xi_p} [\tilde{\pi}_t - \tilde{\pi}_{t-1}] . \]
Combining:

\[ \hat{\pi}_t = \frac{1}{1 + \beta} \hat{\pi}_{t-1} + \frac{\beta}{1 + \beta} E_{t-1} \hat{\pi}_{t+1} + \frac{(1 - \beta \xi_p)(1 - \xi_p)}{(1 + \beta) \xi_p} E_{t-1} \hat{s}_t, \]

Or:

\[ \hat{\pi}_t = \hat{\pi}_{t-1} + \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p} E_{t-1} \sum_{j=0}^{\infty} \beta^j \hat{s}_{t+j} \]

Damped Inflation Response Requires Damped Marginal Cost Response.

Econometric Estimates Likely to Emphasize Model Features That Mute Response of Marginal Cost to Shocks.
• Under Standard Price-Updating Scheme:

\[ P_{it} = \bar{\pi} P_{i,t-1}. \]

Associated Reduced Form:

\[ \hat{\pi}_t = \beta E_{t-1} \hat{\pi}_{t+1} + \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p} E_{t-1} \hat{s}_t. \]

  – Standard Approach Fits Data Badly.
  – Need Lagged Inflation.

• For Any Process Describing \( \hat{s}_t \), Inflation Will Be More Inertial with Dynamic Price-Updating.
Households

- Wage Decision.
- Consumption Decision.
- Investment Decision.
- Capital Utilization Decision.
- Portfolio Decision.
• State Contingent Securities
  – Allow Household to Insulate Consumption, Asset Holdings from Realization of Idiosyncratic Calvo Uncertainty.
  – This Simplifies Computation of Equilibrium.
  – Ignore State Contingent Securities in the Presentation.
  – Households Are Different With Respect to Wages and Employment.
• Preferences:

\[ E_t^h \sum_{l=0}^{\infty} \beta^{l-t} [u(c_{t+l} - b c_{t+l-1}) - z(h_{j,t+l}) + v(q_{t+l})] . \]

\[ b \sim \text{habit parameter} \]

\[ q = \frac{Q}{P} \]

\[ u(\cdot) = \log(\cdot) \]

\[ z(\cdot) = \frac{\psi_0}{2} (\cdot)^2 \]

\[ v(\cdot) = \psi_q (\cdot)^{1-\sigma_q} \]

\[ \frac{1}{1 - \sigma_q} \]
Habit Persistence and Response of Consumption

• Recall that after an expansionary monetary policy shock, we see
  – hump-shaped rise in consumption
  – decline in real interest rate.

• Euler equation in standard model:

\[
\frac{u_{c,t}}{\beta u_{c,t+1}} \approx \frac{c_{t+1}}{\beta c_t} = \frac{g_{t+1}}{\beta} = \frac{R_t}{\pi_{t+1}}, \quad \pi_{t+1} = \frac{P_{t+1}}{P_t}.
\]

• Problem: Can’t have \( g_t \) high and \( \frac{R_t}{\pi_{t+1}} \) simultaneously!
• Habit Persistence in Preferences (example):

\[ u(c_t - b\bar{c}_{t-1}), \quad \bar{c}_{t-1} \sim \text{aggregate consumption} \]

• Euler Equation:

\[
\frac{u_{c,t}}{\beta u_{c,t+1}} \approx \frac{c_{t+1} - bc_t}{\beta (c_t - b\bar{c}_{t-1})} = \frac{g_{t+1} - b}{\beta \left(1 - \frac{b}{g_t}\right)} \\
\approx \frac{g_{t+1} - bg_t}{\beta (1 - b)}
\]

• Result:
  – \( g_{t+1} \) and \( g_t \) Can Both be High, as Long as \( g_{t+1} < bg_t \).
  – Consistent with Simultaneous Hump-Shape \( c \) Response and Low Real Rate.

• Habit Persistence Also Helpful for Understanding Asset Prices
Flow Budget Constraint of $j^{th}$ Household (Ignoring Insurance Considerations):

$$M_{t+1} = R_t [M_t - Q_t + (\mu_t - 1)M_t^a]$$
$$+ Q_t + W_{jt}h_{jt} + P_t r_t^k u_t \bar{k}_t + D_t$$
$$- P_t (c_t + i_t + a(u_t)\bar{k}_t)$$

$$k_t = u_t \bar{k}_t, \text{ capital services}$$

$$\bar{k}_{t+1} = (1 - \delta)\bar{k}_t + F(i_t, i_{t-1})$$
Variables:

\( Q_t \sim \) cash balances
\( M_t \sim \) beginning-of-period \( t \) Household Money
\( M_t^a \sim \) beginning-of-period \( t \) Aggregate Money
\( D_t \sim \) profits
\( \mu_t \sim \) gross money growth rate
\( M_t - Q_t + (\mu_t - 1)M_t^a \sim \) deposits at financial intermediary
\( a(\cdot) \sim \) costs of utilizing capital more intensively
\( u_t \sim \) utilization rate of capital
\( F(i_t, i_{t-1}) \sim \) cost of adjusting investment
\( k_t \sim \) capital services
\( \bar{k}_t \sim \) physical capital.
Structure of the Labor Market

• Intermediate Good Firms Use Labor Aggregate:

\[ L_t = \left[ \int_0^1 h_{j,t}^{\lambda_w} \, dj \right]^{\lambda_w}. \]

• Price of \( L_t \):

\[ W_t = \left[ \int_0^1 W_{it}^{1-\lambda_w} \, di \right]^{1-\lambda_w}. \]

• Demand for Household Labor Service, \( h_{j,t} \):

\[ h_{j,t} = \left( \frac{W_t}{W_{jt}} \right)^{\frac{\lambda_w}{\lambda_w-1}} L_t, \; 1 \leq \lambda_w < \infty. \]

\( W_{jt} \sim \) wage set by household
\( L_t \sim \) homogeneous aggregate labor
\( W_t \sim \) wage rate of aggregate labor
Calvo-style Wage Setting:

• With Probability $1 - \xi_w$, $i^{th}$ Household Sets Wage, $W_{it}$, Optimally, to $\tilde{W}_t$.

• With Probability $\xi_w$,

$$W_{it} = \pi_{t-1}W_{i,t-1}, \quad \pi_t = \frac{P_t}{P_{t-1}}.$$

• First Order Condition:

$$E_{t-1} \sum_{l=0}^{\infty} (\xi_w\beta)^l h_{jt+l} \left[ \psi_{t+l} \frac{\tilde{W}_t X_{t,l}}{P_{t+l}} - \lambda_w z_{h,t+l} \right] = 0.$$

$\frac{\psi_t}{P_t}$ value of one dollar (Multiplier on Budget Constraint)
Cash Balance Decision, $Q_t$

- Households Set $Q_t$ To Maximize Utility

$$v' \left( \frac{Q_t}{P_t} \right) \frac{1}{P_t} + \frac{\psi_t}{P_t} = \frac{\psi_t}{P_t} R_t,$$

- $Q_t/P_t$ Decreasing in $R_t$.

- Liquidity Effect Lives In This Equation.
  - $c_t, i_t, Y_t, L_t, P_t, W_t$ Predetermined Relative to Monetary Shock
  - Loan Market Clearing:

  $$W_t L_t = \mu_t M_t - Q_t$$

  - $Q_t$ Must Absorb all Money Injections.
  - Can Only Happen With Fall in $R_t$.

- This is a ‘Limited Participation Story’
  - But With A Different Twist
Consumption Decision

\[ E_{t-1} \frac{u_{c,t}}{P_t} = \beta E_{t-1} \frac{u_{c,t+1}}{P_{t+1}} R_{t+1}. \]
Capital Utilization Decision

\[ E_{t-1} u_{c,t} \left[ r_t^k - a'(u_t) \right] = 0 \]

- Reduces Upward Pressure On Rental Rate of Capital and, hence, on Marginal Costs After Expansionary Monetary Policy Shock.

- Helps Account for Rise in \( Y/L \) After Expansionary Monetary Policy Shock.
  - Suppose Fixed Cost, \( \phi \), is Zero.
    - Standard Model: \( L \uparrow \Rightarrow \frac{Y}{L} = \left( \frac{k}{L} \right)^\alpha \downarrow \).
    - Our Model: \( L \uparrow \Rightarrow \frac{Y}{L} = \left( \frac{uk}{L} \right)^\alpha \uparrow \).
Investment and Adjustment Costs

• Rate of Return on Capital:

\[ R_t^k = \frac{r_{t+1}^k + P_{k',t+1}(1 - \delta)}{P_{k',t}}, \]

\[ r_{t+1}^k = s_{t+1} M P_{t+1}^k, \] rental rate on capital

\[ M P_t^k \sim \text{marginal product of capital} \]

\[ s_{t+1} = \frac{MC_t}{P_t} = \frac{1}{\text{markup}} \]

\[ \delta \in (0, 1) \sim \text{depreciation rate.} \]

\[ P_{k',t} \sim \text{consumption price of installed capital} \]
• Almost Any Model,

\[
\frac{R_t}{\pi_{t+1}} \approx R^k_t = \frac{\tau^k_{t+1} + P_{k',t+1}(1 - \delta)}{P_{k',t}}.
\]

• So, If a Positive Money Shock Drives Down Real Rate, Then

\[R^k_t \downarrow\]

• This is Trouble For Standard Models \((P_{k',t} = 1, s_t = 1)\):

\[R^k_t \text{ down requires } MP^k_t \text{ down}\]

• Problem:

\[MP^k \text{ down Requires Surge in Investment, especially with employment up.}\]
• With Adjustment Costs, No Surge in Investment

• Cost-of-Change Adjustment Costs:

\[ k' = (1 - \delta)k + F\left(\frac{\dot{i}}{i_{-1}}\right)i \]

Good for ‘Hump-shaped Investment Response’

• Other Reasons for Interest in Adjustment Costs:
  – Important for Understanding Asset Prices
  – Necessary for Movements in Price of Capital
Investment Decision

- Household Owns the Capital Stock and Carries Out Capital Accumulation.
- Technology for Capital Accumulation:

\[
\bar{k}_{t+1} = (1 - \delta)\bar{k}_t + F(i_t, i_{t-1}),
\]

\[
F(i_t, i_{t-1}) = (1 - S \left( \frac{i_t}{i_{t-1}} \right))i_t.
\]

- Euler Equation for \(\bar{k}_{t+1}\):

\[
E_{t-1} \psi_t = \beta E_{t-1} \psi_{t+1} \frac{u_{t+1} r^k_{t+1} - a(u_{t+1}) + P_{k',t+1}(1 - \delta)}{P_{k',t}}.
\]

\(P_{k',t}\) ~marginal cost, in units of consumption goods, of installed, physical capital
• Euler Equation for $i_t$:

$$E_{t-1} \psi_t = E_{t-1} [\psi_t P_{k',t} F_{1,t} + \beta \psi_{t+1} P_{k',t+1} F_{2,t+1}] .$$

• After linearization:

$$\hat{i}_t = \hat{i}_{t-1} + \frac{1}{S''} \sum_{j=0}^{\infty} \beta^j E_{t-1} \hat{P}_{k',t+j} .$$
• Monetary Growth, $\mu_t$:

$$\mu_t = \mu + \mu_{p,t} + \mu_{x,t},$$

‘Exogenous Component’ $\mu_{p,t} = \rho_{\mu_{p}} \mu_{p,t-1} + \varepsilon_{\mu_{p,t}}$

‘Endogenous Component’ $\mu_{x,t} = \rho_{\mu_{x}} \mu_{x,t-1} + c_{\mu_{x}} \varepsilon_{x,t}$
Econometric Estimation

- Two Types of Parameters:
  (a) Parameters Set Without Reference to VAR Data.
  (b) Parameters Estimated by Making Model Impulse Responses Look Like The Estimated Impulse Responses.
### Parameter Set 1: Parameters that Don’t Enter Formal Estimation Criterion

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>discount factor, $\beta$</td>
<td>$1.03^{-0.25}$</td>
</tr>
<tr>
<td>capital’s share, $\alpha$</td>
<td>0.36</td>
</tr>
<tr>
<td>capital depreciation rate, $\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>markup, labor suppliers, $\lambda_w$</td>
<td>1.05</td>
</tr>
<tr>
<td>mean, money growth, $\mu$</td>
<td>1.017</td>
</tr>
<tr>
<td>labor utility parameter, $\psi_0$</td>
<td>set to imply $L = 1$</td>
</tr>
<tr>
<td>real balance utility parameter, $\psi_q$</td>
<td>set to imply $Q/M = 0.44$</td>
</tr>
<tr>
<td>fixed cost of production, $\phi$</td>
<td>set to imply $\text{ss profits} = 0$</td>
</tr>
</tbody>
</table>
• $\gamma \sim 13$ free parameters to be estimated:

$$\gamma \equiv (\lambda_f, \xi_w, \xi_p, \sigma_q, S''', b, \sigma_a,$$

6 parameters governing exogenous shocks).

$\lambda_f$  Steady State Markup of Firms
$\xi_w$  Degree of Stickiness in Wages
$\xi_p$  Degree of Stickiness in Prices
$b$  Habit Persistence Parameter

• Estimation Criterion:

$$J = \min_{\gamma} (\hat{\psi} - \psi(\gamma))'V^{-1}(\hat{\psi} - \psi(\gamma)),$$

• $\psi(\gamma) \sim 353$ model impulse responses
• $\hat{\psi} \sim 353$ estimated VAR impulse responses
Estimation Results:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Policy and Technology</th>
<th>Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shocks Simultaneously</td>
<td>Shocks Only</td>
</tr>
<tr>
<td>$\lambda_f$</td>
<td>1.14 (0.10)</td>
<td>1.15 (0.13)</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>0.78 (0.04)</td>
<td>0.73 (0.04)</td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>0.42 (0.30)</td>
<td>0.45 (0.19)</td>
</tr>
<tr>
<td>$\sigma_q$</td>
<td>14.13 (4.25)</td>
<td>12.33 (3.19)</td>
</tr>
<tr>
<td>$S'''$</td>
<td>7.69 (3.04)</td>
<td>9.97 (3.90)</td>
</tr>
<tr>
<td>$b$</td>
<td>0.73 (0.07)</td>
<td>0.77 (0.05)</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.05 (0.007)</td>
<td>0.03 (0.005)</td>
</tr>
</tbody>
</table>

Parameters that Match Money Impulse Responses Work Well With Technology Too.
### Table: Exogenous Shock Processes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Policy and Technology</th>
<th>Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shocks Simultaneously</td>
<td>Shocks Only</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>0.80 (0.05)</td>
<td>na</td>
</tr>
<tr>
<td>$\sigma_{\varepsilon_x}$</td>
<td>0.12 (0.03)</td>
<td>na</td>
</tr>
<tr>
<td>$\rho_{\mu_x}$</td>
<td>0.47 (0.15)</td>
<td>na</td>
</tr>
<tr>
<td>$c_{\mu_x}$</td>
<td>2.07 (0.63)</td>
<td>na</td>
</tr>
<tr>
<td>$\rho_{\mu_p}$</td>
<td>0.27 (0.04)</td>
<td>0.27 (0.06)</td>
</tr>
<tr>
<td>$\sigma_{\varepsilon_{\mu_p}}$</td>
<td>0.11 (0.04)</td>
<td>0.13 (0.04)</td>
</tr>
</tbody>
</table>
Figure 1: Model and Data Impulse Response Functions to a Policy Shock
Model Diagnostics

• Employment Rises in Response to Technology Because Monetary Policy is Accommodative.

• Sticky Prices Not Important for Transmission of Monetary Policy or Technology Shocks.
Diagnostics: Monetary Accommodation Important for Transmission of
Technology Shocks
Diagnostics: Sticky Prices Not Important for Transmission of Technology Shocks
Conclusion

• We Presented Empirical Estimates of the Dynamic Effects of Monetary Policy and Technology Shocks.
  – A Model Was Found that Can Roughly Reproduce These Effects.
Finding: Technology Shocks Seem to Have Little to do with Business Cycles
- Need to Bring Other Technology Shocks Into the Analysis.
- Stationary Shocks to Disembodied Technology.
- Stationary Investment-Specific Shocks.
- These Shocks May Account for Business Cycle Fluctuations.
- Paper Describes a ‘Model-Based’ Identification Strategy For Doing This.

Hope: Model with Additional Shocks Capable of Accounting for Details of Quarterly Data.

Such a Model is Ready for Policy Analysis!