Fitting a VAR to DATA
A Version of the Shapiro-Watson Approach

• $p^{th}$ Order VAR:

\[ Y_t = B_1 Y_{t-1} + \ldots + B_4 + u_t, \quad E u_t u_t' = V \]
VAR Analysis: General Background Comments

- My Focus Will Be On The Use of VARs to Learn About Construction and Parameterization of Dynamic, General Equilibrium Models of Money.
- A ‘Shock-Based Approach to Estimating Models’.
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  – Choose Parameters and Functional Forms for a Dynamic General Equilibrium Model to Match These Effects as Closely As Possible.
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  – Choose Parameters and Functional Forms for a Dynamic General Equilibrium Model to Match These Effects as Closely As Possible.

- Lucas Program for Model Selection:

  ‘Need to test them (models) as useful imitations of reality by subjecting them to shocks for which we are fairly certain how actual economies would react. The more dimensions on which the model mimics the answers actual economies give to simple questions, the more we trust its answers to harder questions.’
A Version of the Shapiro-Watson Approach ...

- Data \( (N = 10) \):

\[
\begin{pmatrix}
\Delta \ln \text{(relative price of investment}_t) \\
\Delta \ln \left( \frac{GDP_t}{\text{Hours}_t} \right) \\
\Delta \ln \left( \frac{GDP}{\text{deflator}}_t \right) \\
\text{capacity utilization}_t \\
\ln \left( \frac{\text{Hours}_t}{\text{Hours}_t} \right) \\
\ln \left( \frac{GDP_t}{\text{Hours}_t} \right) - \ln \left( \frac{W_t}{P_t} \right) \\
\ln \left( \frac{C_t}{GDP_t} \right) \\
\ln \left( \frac{I_t}{GDP_t} \right) \\
\text{Federal Funds Rate}_t \\
\ln \left( \frac{GDP}{\text{deflator}}_t \right) + \ln \left( \frac{GDP_t}{\text{deflator}}_t \right) - \ln \left( \frac{MZM_t}{\text{deflator}}_t \right)
\end{pmatrix}
\]

\( Y_t \) \( 10 \times 1 \)

\[
= \begin{pmatrix}
\Delta p_{It} \\
\Delta a_{It} \\
Y_{1t} \\
R_t \\
Y_{2t}
\end{pmatrix}
\]

\( 1 \times 1 \)

\( 1 \times 1 \)

\( 6 \times 1 \)

\( 1 \times 1 \)

\( 1 \times 1 \)
Figure 1: data used in the analysis
A Version of the Shapiro-Watson Approach ...

- Three Identified Shocks:
  - Two Technology Shocks: *Only* Shocks that Have a Long Run Impact on Labor Productivity
    - Neutral Shock to Technology.
    - Embodied Shock to Technology
      - only shock with long-run effect on investment good prices (Fisher)
  - Monetary Policy Shock
    - Disturbance in OLS Regression:

\[ R_t = f(\Omega_t) + \omega e_{R_t}, \]

\[ e_{R_t} \perp \Omega_t \]

- \( \Rightarrow \) Monetary Policy Has No Contemporaneous Impact on Prices and Aggregate Allocations.
- \( \Rightarrow \) Interest Rate Not Significantly Contemporaneously Affected by Money Demand Shocks, Other than Via \( \Omega_t \).
A Version of the Shapiro-Watson Approach ...

What Is A Monetary Policy Shock?

- Shocks to Preferences of Monetary Authority
- Technical Factors Like Measurement Error (Bernanke-Mihov):

\[ x_t(0) = x_t + v_t, \quad x_t(1) = x_t + u_t \]
\[ S_t = \beta_0 S_{t-1} + \beta_1 x_t(0) + \beta_2 x_{t-1}(1) \]

or

\[ S_t = \beta_0 S_{t-1} + \beta_1 x_t + \beta_2 x_{t-1} + \varepsilon_t \]
\[ \varepsilon_t = \beta_1 v_t + \beta_2 u_{t-1}. \]
A Version of the Shapiro-Watson Approach ...

What Is a Monetary Policy Rule?

- Combination of Structural Policy Rule and Other Stuff
  - Example (Clarida-Gertler):
    ‘True’ Policy Rule:
    \[ S_t = \alpha E_t x_{t+1} + \varepsilon_t \]
    \[ = f(\text{all time } t \text{ data used in } E_t x_{t+1}) + \varepsilon_t \]
A Version of the Shapiro-Watson Approach ...

- Restrictions on $A_0$ Implied by Monetary Policy Identification:

$$A_0 Y_t = A(L) Y_{t-1} + e_t$$

$$\begin{bmatrix}
A^{1,1}_0 & A^{1,2}_0 & A^{1,3}_0 & 0 & 0 \\
1 \times 1 & 1 \times 1 & 6 \times 6 & 1 \times 1 & 1 \times 1 \\
A^{2,1}_0 & A^{2,2}_0 & A^{2,3}_0 & 0 & 0 \\
1 \times 1 & 1 \times 1 & 1 \times 6 & 1 \times 1 & 1 \times 1 \\
A^{3,1}_0 & A^{3,2}_0 & A^{3,2}_0 & 0 & 0 \\
6 \times 1 & 6 \times 1 & 6 \times 6 & 6 \times 1 & 6 \times 1 \\
A^{4,1}_0 & A^{4,2}_0 & A^{4,3}_0 & A^{4,4}_0 & 0 \\
1 \times 1 & 1 \times 1 & 1 \times 6 & 1 \times 1 & 1 \times 1 \\
A^{5,1}_0 & A^{5,2}_0 & A^{5,3}_0 & A^{5,4}_0 & A^{5,5}_0 \\
1 \times 1 & 1 \times 1 & 1 \times 6 & 1 \times 1 & 1 \times 1 \\
\end{bmatrix}
\begin{bmatrix}
\Delta p_{It} \\
1 \times 1 \\
\Delta a_t \\
1 \times 1 \\
Y_{1t} \\
6 \times 1 \\
R_t \\
1 \times 1 \\
Y_{2t} \\
1 \times 1 \\
\end{bmatrix}
= A(L) Y_{t-1}
\begin{bmatrix}
e_{\Upsilon,t} \\
1 \times 1 \\
e_{z,t} \\
1 \times 1 \\
e_{1t} \\
6 \times 1 \\
e_{Rt} \\
1 \times 1 \\
e_{2t} \\
1 \times 1 \\
\end{bmatrix}
$$

$\Omega_t = \{ \Delta a_t, \Delta p_{It}, Y_{1t}, Y_{t-s}, s > 0 \}$

- $Y_{2t}$ Assumed Excluded in Monetary Policy Rule
- $Y_{2t}, R_t$ Must Not Affect First 8 Variables
A Version of the Shapiro-Watson Approach ...

- Analogous Restrictions on $A_0$ From Technology Identification Assumptions
- Instead, We Proceed as in Shapiro-Watson.
- Structural Form:

$$A_0Y_t = A(L)Y_{t-1} + e_t$$

- Scaled First Structural Equation:

$$\Delta p_{It} = a_{11}(L)\Delta p_{I_{t-1}} + a_{12}(L)\Delta^2 a_t + a_{13}(L)\Delta Y_{1t} + a_{14}(L)\Delta R_{t-1} + a_{15}(L)\Delta Y_{2,t-1} + \frac{e_{\gamma,t}}{A_0^{1,1}}$$

$e_{\gamma,t} \sim$ Shock to Price of Investment Goods

$$\Delta \equiv 1 - L$$
A Version of the Shapiro-Watson Approach ...

- Scaled First Structural Equation:

\[ \Delta p_{It} = a_{11}(L)\Delta p_{It-1} + a_{12}(L)\Delta^2 a_t + a_{13}(L)\Delta Y_{1t} + a_{14}(L)\Delta R_{t-1} + a_{15}(L)\Delta Y_{2,t-1} + \frac{e_{\Upsilon,t}}{A_{0,1}}, \]

\[ e_{\Upsilon,t} \sim \text{Shock to Price of Investment Goods} \]

\[ \Delta \equiv 1 - L \]

\[ a_{1j}(L) \sim \begin{cases} p - 1 \text{ unknown coefficients, } j = 4, 5 \\ p \text{ unknown coefficients, } j = 1, 2, 3 \end{cases} \]

- Presence of \( \Delta \) Reflects Identification Assumption that Shocks Other than \( e_{\Upsilon,t} \) Have Zero Impact on \( p_{It} \).
- Presence of \( R_{t-1}, Y_{2,t-1} \) Reflects Monetary Policy Identification.
- Ordinary Least Squares Not Consistent - Use Instrumental Variables.
  * Use \( Y_{t-1}, \ldots, Y_{t-p} \) (Have Testable Overidentifying Restrictions Here).
A Version of the Shapiro-Watson Approach ...

- Scaled Second Structural Equation:

\[
\Delta a_t = a_{22}(L) \Delta a_{t-1} + a_{21}(L) \Delta p_{It} \\
+ a_{23}(L) \Delta Y_{1t} + a_{24}(L) \Delta R_{t-1} + a_{25}(L) \Delta Y_{2,t-1} + \frac{e_{zt}}{A_{0,2}}
\]

\[
a_{2j}(L) \sim \begin{cases} 
  p + 1 \text{ unknown coefficients, } j = 1 \\
  p - 1 \text{ unknown coefficients, } j = 4, 5 \\
  p \text{ unknown coefficients, } j = 2, 3
\end{cases}
\]

- Absence of Any Extra \( \Delta \) on \( \Delta p_{It} \):
  * Reflects that Non-Technology Shocks Already Affect \( \Delta p_{It} \) With Unit Moving Average Root.
  * Implies One Extra Parameter to Instrument.

- Presence of \( R_{t-1}, Y_{2,t-1} \) Reflects Monetary Policy Identification.

- Ordinary Least Squares Not Consistent - Use Instrumental Variables.
  * Use \( \hat{e}_{\Upsilon,t}, Y_{t-1}, \ldots, Y_{t-p} \)
  * Note: Use Assumption that \( e_{zt} \) and \( e_{\Upsilon,t} \) Uncorrelated
A Version of the Shapiro-Watson Approach ...

- Scaled Ninth Structural Equation:

\[
R_t + \frac{A_{0}^{4,1}}{A_{0}^{4,4}} \Delta p_{It} + \frac{A_{0}^{4,2}}{A_{0}^{4,4}} \Delta a_t + \frac{A_{0}^{4,3}}{A_{0}^{4,4}} Y_{1t} = c(L)Y_{t-1} + \frac{e_{Rt}}{A_{0}^{4,4}}
\]

- OLS is Fine!

- Scaled Tenth Structural Equation:

\[
Y_{2t} + \frac{A_{0}^{5,1}}{A_{0}^{5,5}} \Delta a_t + \frac{A_{0}^{5,2}}{A_{0}^{5,5}} \Delta p_{It} + \frac{A_{0}^{5,3}}{A_{0}^{5,5}} Y_{1t} + \frac{A_{0}^{5,4}}{A_{0}^{5,5}} R_t = d(L)Y_{t-1} + \frac{e_{2t}}{A_{0}^{5,5}}
\]

- OLS is Fine! (\(Y_{2t}\) Does Not Enter Any Other Equation).
A Version of the Shapiro-Watson Approach ...

• Reminder: Structural Form

\[ A_0 Y_t = A(L) Y_{t-1} + e_t \]

• We Have Established that the 1\text{st}, 2\text{nd}, 9\text{th} and 10\text{th} Rows of Structural Form Are Identified.

• The Middle Six Rows of Structural Form Are Not Identified:

\[ A^3_{0,1} \Delta a_t + A^3_{0,32} \Delta p_{It} + A^3_{0,3} Y_{1t} = b(L) Y_{t-1} + e_{1t} \]

– For Any Setting of \( A_0 \) and \( A(L) \), Premultiplication of Middle Six Rows by an Orthonormal Matrix Leaves Reduced Form (And, Hence, Likelihood) Unaffected.

– Impulse Responses to Policy and Technology Shocks Invariant to Choice of Orthonormal Matrix.

– Without Loss of Generality, Restrict \( A^3_{0,3} \) To Be Lower Triangular.

– Estimate Coefficients of Middle 6 Equations by Instrumental Variables.
Confidence Intervals and the Bootstrap

• Estimation Produces:
  \[ Y_t = B(L)Y_{t-1} + \hat{A}_0^{-1}\hat{e}_t, \]
  \[ \hat{e}_t, \ t = 1, \ldots, T, \]
  where
  \[ B(L) = \hat{A}_0^{-1}\hat{A}(L). \]

• Bootstrap
  – Generate \( r = 1, \ldots, R \) Artificial Data Sets, Each of Length \( T \)
    * For \( r^{th} \) Dataset:
      \[ \lambda^r_t \in Uniform[0, 1], \ t = 1, \ldots, T \]
    * Draw Integers:
      \[ \tilde{\lambda}^r_t = \text{integer}(\lambda^r_t \times T), \ t = 1, \ldots, T \]
    * Draw Shocks:
      \[ \hat{e}_{\tilde{\lambda}_1^r}, \ldots, \hat{e}_{\tilde{\lambda}_T^r} \]
Confidence Intervals and the Bootstrap ...

* Generate Artificial Data:
\[ Y_t^r = B(L)Y_{t-1}^r + \hat{A}_0^{-1}\hat{\epsilon}_{\chi_t}, \ t = 1, ..., T. \]

– Suppose Statistic of Interest is \( \psi \) (could be vector of impulse response functions, serial correlation coefficients, etc.)
\[ \psi^r = f(Y_1^r, ..., Y_T^r), \ r = 1, ..., R \]

* Compute
\[ \sigma_\psi = \left\{ \frac{1}{T} \sum_{t=1}^{T} (\psi^r - \bar{\psi})^2 \right\}^{1/2} \]

* Report
\[ \hat{\psi} \pm 2 \times \sigma_\psi. \]

* Or, \( p - value \)
\[prob(\psi^r > \hat{\psi}).\]

* Impulse Response Functions, \( \psi = (\psi_1, ..., \psi_{600})\)

* \( \psi \) Measures of Serial Correlation, etc.
Results for Impulse Response Functions

• Show Data, VAR Lag Length: 4, Sample Period 1959Q1-2001Q3.
• Responses to Monetary Policy Shock
  – Impact on Money Growth and Interest Rate Over in 1 Year, Other Variables Keep Going
  – Significant Liquidity Effect
  – Inflation Peaks in Roughly Two Years
  – Output, consumption, investment, hours worked and capacity utilization are hump-shaped.
  – Velocity comoves with the interest rate
• Responses to a Positive, Neutral Technology Shock
  – Output, hours, investment, consumption display strong, positive, significant responses.
  – Strong, immediate drop in inflation
• Responses to a Negative, Embodied Shock to Technology
  – Rise in Output, Hours Worked, Interest Rate, Strong Rise in Investment.
Figure 3: Benchmark model – dynamic response to a monetary policy shock
Figure 4: Benchmark model – dynamic response to a neutral technology shock

Output

MZM Growth (Cash Balances, Q)

Inflation

Fed Funds

Capacity Util

Avg Hours

Real Wage

Consumption

Investment

Velocity

Avg Hours

Total money growth (M)
Figure 5: Benchmark model – dynamic response to an embodied technology shock
VAR Diagnostics

- Whether or not to First Difference Hours Worked Important
- Choosing VAR Lag Length

Akaike: \[ s(p) = \log(\text{det} \hat{V}_p) + (m + m^2p) \frac{2}{T} \]

Hannan-Quinn: \[ s(p) = \log(\text{det} \hat{V}_p) + (m + m^2p) \frac{2 \log(\log(T))}{T} \]

Schwarz: \[ s(p) = \log(\text{det} \hat{V}_p) + (m + m^2p) \frac{\log(T)}{Y} \]

\( T \) sample size, \( m \); Number of Variables (10); \( p \) Number of Lags

- Choice:

\[ \hat{p} = \min_p s(p). \]
VAR Diagnostics ...

- With $T = 170$ :

\[
\frac{2}{T} = 0.0118, \quad \frac{2 \log(\log(T))}{T} = 0.0192, \quad \frac{\log(T)}{Y} = 0.0302
\]

- Akaike Penalizes $p$ the Least
  
  * Known: In Population, Akaike Has Positive Probability of Overshooting True $p$

  * Hannan-Quinn and Schwarz are Consistent.
VAR Diagnostics ...

- Results (see picklag.m): HQ and SC Choose $p = 1$, AIC Chooses $p = 2$:

Table: Standard VAR Lag Length Selection Criteria

<table>
<thead>
<tr>
<th>$h$</th>
<th>AIC</th>
<th>HQ</th>
<th>SC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-101.24</td>
<td>-100.42</td>
<td>-99.21</td>
</tr>
<tr>
<td>2</td>
<td>-101.42</td>
<td>-99.84</td>
<td>-97.53</td>
</tr>
<tr>
<td>3</td>
<td>-101.28</td>
<td>-98.94</td>
<td>-95.52</td>
</tr>
<tr>
<td>4</td>
<td>-101.23</td>
<td>-98.13</td>
<td>-93.58</td>
</tr>
<tr>
<td>5</td>
<td>-101.02</td>
<td>-97.14</td>
<td>-91.46</td>
</tr>
<tr>
<td>6</td>
<td>-101.04</td>
<td>-96.37</td>
<td>-89.55</td>
</tr>
<tr>
<td>7</td>
<td>-101.02</td>
<td>-95.57</td>
<td>-87.60</td>
</tr>
<tr>
<td>8</td>
<td>-101.12</td>
<td>-94.88</td>
<td>-85.75</td>
</tr>
</tbody>
</table>
VAR Diagnostics ...

- **Multivariate $Q(s)$ Statistic**
  - Measure of Serial Correlation In Fitted Disturbances
  - Null Hypothesis: the First $s$ Autocorrelations Are Zero:
    \[
    Q(s) = T(T + 2) \sum_{j=1}^{s} \frac{1}{T-j} \text{trace} \left[ C_j C_0^{-1} C_j' C_0^{-1} \right],
    \]
    where
    \[
    C_j = \frac{1}{T} \sum_{t=j+1}^{T} \hat{e}_t \hat{e}_{t-j}'.
    \]
  - In the Scalar Case, It is the Weighted Sum of the Squares of the First $s$ Correlations.
  - Null Distribution:
    \[
    Q(s) \sim \chi_{m^2(s-p)}^2
    \]
VAR Diagnostics ...

– Results (see mkqmv.m):

<table>
<thead>
<tr>
<th>s</th>
<th>Q(s)</th>
<th>degrees of freedom</th>
<th>asymptotic p-value</th>
<th>bootstrap p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>166.81</td>
<td>0</td>
<td>NaN</td>
<td>0.89</td>
</tr>
<tr>
<td>6</td>
<td>350.41</td>
<td>200</td>
<td>0.00</td>
<td>0.92</td>
</tr>
<tr>
<td>8</td>
<td>551.56</td>
<td>400</td>
<td>0.00</td>
<td>0.83</td>
</tr>
<tr>
<td>10</td>
<td>795.67</td>
<td>600</td>
<td>0.00</td>
<td>0.39</td>
</tr>
</tbody>
</table>

– Strikingly Different Between Bootstrap and Asymptotics!

● Conclusion of Lag Length Diagnostics: Go With $p = 4$ Lags, But Redo Everything with $p = 6$, To be Safe
Variance Decompositions

- Forecast Error Variance Due to Various Shocks
- Business Cycle Variation Due to Various Shocks
  - In-Sample Measure
  - Population Measure
Variance Decompositions ...

- Forecast Errors
  - Model:
    \[ Y_t = \mu + B(L)Y_{t-1} + A^{-1}_0 e_t. \]
    \[ = \mu + BY_{t-1} + A^{-1}_0 e_t, \]
    for Simplicity (Note: I slipped the Constant Term Back In.)

- Conventional \( k \)-step Ahead Forecast Error Variance:
  \[ Y_{t+k} - E_{t-1}Y_{t+k} = A^{-1}_0 e_{t+k} + BA^{-1}_0 e_{t+k-1} + B^2 A^{-1}_0 e_{t+k-2} + \ldots + B^k A^{-1}_0 e_t \]

  so

  \[ \text{Var} = \text{Var} \left( Y_{t+k} - E_{t-1}Y_{t+k} \right) = \sum_{j=0}^{k} B^j A^{-1}_0 \left( A^{-1}_0 \right) \left( B^j \right)' \]
Variance Decompositions ...

• Portion of $k$-step Ahead Forecast Error Due to $i^{th}$ Shock:

$$Var(i) = Var_i(Y_{t+k} - E_{t-1}Y_{t+k}) = \sum_{j=0}^{k} B^j A_0^{-1} I_i (A_0^{-1}) (B^j)' ,$$

where $I_i$ is Identity Matrix With All Zeros on Diagonal Except $i^{th}$ Location.

• Obviously:

$$\sum_{i=1}^{m} Var(i) = Var$$

• Percent of Variance Due to $i^{th}$ Shock:

$$100 \times \frac{Var(i)}{Var}$$

• Obviously, Can Easily Bootstrap the Sampling Distribution of Variance Decomposition.
Variance Decompositions ... 

- Complication: Variables in $Y_t$ Are Not the Ones We are Necessarily Interested In!
  - Variables in $Y_t$ :

$$
Y_t \equiv \begin{pmatrix}
\Delta \ln \text{(relative price of investment}_t \\
\Delta \ln (GDP_t/\text{Hours}_t) \\
\Delta \ln (GDP \text{ deflator}_t) \\
\text{capacity utilization}_t \\
\ln (\text{Hours}_t) \\
\ln (GDP_t/\text{Hours}_t) - \ln (W_t/P_t) \\
\ln (C_t/GDP_t) \\
\ln (I_t/GDP_t) \\
\text{Federal Funds Rate}_t \\
\ln (GDP \text{ deflator}_t) + \ln (GDP_t) - \ln (MZM_t)
\end{pmatrix} \equiv \begin{pmatrix}
(1 - L)p_t^I \\
(1 - L)(y_t - h_t) \\
(1 - L)p_t \\
u_t \\
h_t \\
y_t - h_t - w_t \\
c_t - y_t \\
p_t^I + I_t - y_t \\
R_t \\
y_t + p_t - m_t
\end{pmatrix}
$$
Variance Decompositions ...

– We Are Interested in:

\[
\tilde{Y}_t = \begin{pmatrix}
  y_t \\
  4(1 - L)m_t \\
  4(1 - L)p_t \\
  R_t \\
  u_t \\
  h_t \\
  w_t \\
  c_t \\
  I_t \\
  p^I_t
\end{pmatrix}, \text{ not } Y_t = \begin{pmatrix}
  (1 - L)p^I_t \\
  (1 - L)(y_t - h_t) \\
  (1 - L)p_t \\
  u_t \\
  h_t \\
  y_t - h_t - w_t \\
  c_t - y_t \\
  p^I_t + I_t - y_t \\
  R_t \\
  y_t + p_t - m_t
\end{pmatrix}
\]

– Tedious Formulas Make It Possible to Compute the \( k \)-Percent Forecast Error in \( \tilde{Y}_t \).
Variance Decompositions ...

- Business Cycle Variance Decomposition

\[ Y_t = \mu + B Y_{t-1} + A_0^{-1} e_t \]

- In-Sample Variance Decomposition
  * Simulate \( \tilde{Y}_t \) Using All Historical Shocks, Then Apply HP Filter
  * Simulate \( \tilde{Y}_t \) Using Only \( i^{th} \) Shocks, Then Apply HP Filter
  * Ratio of Variance of Two Filtered Series is Business Variation Due to \( i^{th} \) Shock.
  * Graph All Data
  * To Straighten Out the Scale, Drop \( \mu \) In Simulation.

- Population Variance Decomposition
  * Simulate an Enormous Amount of Data, Not a Sample Same Length As Actual
  * Easily Done in Frequency Domain

- Business Cycle Variance Decomposition Can Be Bootstrapped to Obtain Sampling Uncertainty.
Figure 6: Historical decomposition – monetary policy shocks only
Figure 7: Historical decomposition – neutral technology shocks only

Output

Fed Funds

Real Wage

Capacity Util

Consumption

Avg Hours

Velocity

Price of Inv.
Figure 8: Historical decomposition – embodied technology shocks only

Output

MZM Growth

Inflation

Fed Funds

Capacity Util

Avg Hours

Real Wage

Consumption

Investment

Velocity

Price of Inv.

Price of Inv.
Figure 9: Historical decomposition – monetary policy and technology shocks
<table>
<thead>
<tr>
<th>Variable</th>
<th>Embodied Technology</th>
<th>Neutral Technology</th>
<th>Monetary Policy</th>
<th>All Three Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inflation</strong></td>
<td>15 (16) [14]</td>
<td>19 (13) [8]</td>
<td>29 (11) [7]</td>
<td>64 (40) [18]</td>
</tr>
<tr>
<td><strong>Capacity Util.</strong></td>
<td>25 (16) [12]</td>
<td>3 (9) [8]</td>
<td>38 (12) [8]</td>
<td>56 (37) [18]</td>
</tr>
<tr>
<td><strong>Consumption</strong></td>
<td>25 (16) [13]</td>
<td>17 (18) [12]</td>
<td>50 (12) [8]</td>
<td>95 (45) [21]</td>
</tr>
<tr>
<td><strong>Price of Inv.</strong></td>
<td>38 (42) [22]</td>
<td>1 (8) [7]</td>
<td>8 (6) [4]</td>
<td>56 (55) [21]</td>
</tr>
</tbody>
</table>

Notes: Numbers are point estimates, number in parentheses are mean of point estimates across bootstrap simulations; number in square brackets are standard deviation of point estimates across bootstrap simulations.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Embodied Technology</th>
<th>Neutral Technology</th>
<th>Monetary Policy</th>
<th>All Three Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>22 (16)</td>
<td>9 (14)</td>
<td>34 (15)</td>
<td>45 (41)</td>
</tr>
<tr>
<td>MZM Growth</td>
<td>6 (10)</td>
<td>4 (8)</td>
<td>30 (16)</td>
<td>31 (35)</td>
</tr>
<tr>
<td>Inflation</td>
<td>17 (15)</td>
<td>11 (13)</td>
<td>22 (12)</td>
<td>44 (40)</td>
</tr>
<tr>
<td>Fed Funds</td>
<td>17 (13)</td>
<td>7 (9)</td>
<td>47 (20)</td>
<td>47 (42)</td>
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<tr>
<td>Capacity Util.</td>
<td>21 (15)</td>
<td>6 (9)</td>
<td>32 (14)</td>
<td>42 (38)</td>
</tr>
<tr>
<td>Avg. Hours</td>
<td>22 (15)</td>
<td>13 (11)</td>
<td>33 (14)</td>
<td>41 (39)</td>
</tr>
<tr>
<td>Real Wage</td>
<td>9 (13)</td>
<td>4 (12)</td>
<td>6 (8)</td>
<td>20 (32)</td>
</tr>
<tr>
<td>Consumption</td>
<td>21 (15)</td>
<td>20 (17)</td>
<td>43 (17)</td>
<td>57 (49)</td>
</tr>
<tr>
<td>Investment</td>
<td>15 (15)</td>
<td>6 (10)</td>
<td>28 (14)</td>
<td>36 (38)</td>
</tr>
<tr>
<td>Velocity</td>
<td>14 (12)</td>
<td>6 (9)</td>
<td>45 (18)</td>
<td>41 (37)</td>
</tr>
<tr>
<td>Price of Inv.</td>
<td>22 (22)</td>
<td>8 (13)</td>
<td>8 (9)</td>
<td>48 (51)</td>
</tr>
</tbody>
</table>

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Conclusion from VAR Analysis

- Our Estimates Suggest that the Three Identified Shocks Account for A Substantial Fraction (Around 50%) of Business Cycle Variation.
- We have a set of Estimates of How Economy Responds to Three Types of Shocks
- Will use these to Estimate a DSGE Model.