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 416, Fall 2005
 Homework 3, due October 21.

1. Consider the following model:

$$\begin{aligned}\delta\pi_{t+1} + \lambda y_t - \pi_t &= 0 \\ y_{t+1} - y_t - \frac{1}{\sigma}(r_t - \pi_{t+1}) &= 0 \\ \rho r_{t-1} + (1 - \rho)\beta\pi_{t+1} + (1 - \rho)\gamma y_t - r_t &= 0.\end{aligned}$$

Here, π_t denotes inflation, y_t denotes output and r_t denotes a nominal rate of interest, all expressed in percent deviation from steady state. (The second equation will later be interpreted as the log-linear expansion of an intertemporal Euler equation, the third equation is a monetary policy rule, and the first equation is the reduced form representation of a model with price frictions.) Write this system in the form:

$$AY_{t+1} + BY_t = 0,$$

for $t = 1, \dots$ (note: in class the initial period is $t = 0$, but here I have changed it to $t = 1$). Here, $Y_t = (\pi_t, y_t, r_{t-1})$. Use the following parameter values, $\delta = 0.99$, $\sigma = 1$, $\lambda = 0.3$, $\gamma = 1$, $\rho = 0.5$, $\beta = 0.8$. Let the initial interest rate, r_0 , have its steady state value of 0.

- (a) Compute P and Λ in the eigenvector, eigenvalue decomposition of $-A^{-1}B$.
- (b) Compute the set of minimal state variable solutions (i.e., the set of 2×3 matrices with the property $DY_1 = 0 \implies DY_t = 0$ for all t).
- (c) Suppose the first period is $t = 1$, and time evolves for $t = 1, 2, 3, \dots, 20$. Construct the following 3 graphs, one corresponding to each MSV solution. In each graph, display two lines of length 20 each, with $t = 1, \dots, 20$ on the horizontal axis. One line, a line of zeros, corresponds to the nonstochastic, steady state equilibrium of the model. The other is a convergent solution to the Euler equations in which π_1 is a little greater than zero. Note how the high inflation equilibrium, while eventually returning to steady state, displays a period of high inflation and high output.

2. Consider an economy in which household preferences have the following form:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t), \beta = .99, \text{ and } u(c_t, n_t) = \log(c_t) + \psi \log(1 - n_t).$$

The household budget constraint is:

$$c_t + k_{t+1} - (1 - \delta)k_t = r_t k_t + w_t n_t + \pi_t, \delta = .028,$$

where r_t , w_t , π_t denote the rental rate on capital, the wage rate, and profits, respectively. Firms operate the following technology:

$$y_t = A_t k_t^\alpha n_t^{1-\alpha}, \alpha = 0.36.$$

Here,

$$A_t = Y_t^\gamma,$$

where Y_t denotes the economy-wide level of output, in per capita terms. Firms are competitive, and maximize profits, which are zero in equilibrium.

- (a) Derive the household static and dynamic Euler equations. Derive the firm Euler equation.
- (b) In the household Euler equations, substitute out the rental rate on capital and the wage rate for labor, using the firm first order conditions. You should have a static and dynamic Euler equation. You should focus on symmetric equilibria, in which the economy-wide average stock of capital and level of employment coincide with the firm's stock of capital and employment. After making this identification, the static and dynamic Euler equations should have the following arguments:

$$\begin{aligned} \text{'static'} & : v_h(n_t, k_t, k_{t+1}) = 0 \\ \text{'dynamic'} & : v_k(k_t, k_{t+1}, k_{t+2}, n_t, n_{t+1}) = 0. \end{aligned}$$

Log-linearize these equations about the steady state values of employment and the firm's stock of capital. In computing the steady states, fix steady state employment at 1/3 and choose the value of ψ that rationalizes that. Initially, fix $\gamma = 0.01$.

- (c) Substitute out for labor in the dynamic Euler equation, using the static Euler equation. This will give you one dynamic Euler equation in k_t, k_{t+1}, k_{t+2} .
- (d) Set the Euler equation up as a first order difference equation:

$$aY_{t+1} + bY_t = 0, t = 0, 1, 2, \dots \quad (1)$$

Show the values of a , and b . What are the eigenvalues of $\Pi = -a^{-1}b$? Find a value of γ such that, for values larger than that, both eigenvalues of Π are less than unity in absolute value. Display the left eigenvectors of Π associated with each of these two eigenvalues. Hint: to find out how to compute eigenvalues and eigenvectors in MATLAB, type ‘help eig’, in MATLAB. Display the policy rules corresponding to each eigenvector, i.e., the representations, $\tilde{k}_{t+1} = d\tilde{k}_t$, $\tilde{n}_t = q\tilde{k}_t$, where a tilde denotes deviation from steady state.