Christiano
416, Fall 2005
Homework 4, due November 3

1. Consider a version of the model in the last question of the previous homework. Set $\gamma = 0$, so that there are no externalities. Replace the utility function by:

$$u(c_t, n_t) = \log(c_t - bc_{t-1}) + \psi \log(1 - n_t).$$

Replace the resource constraint by:

$$c_t + i_t \leq k_t^\alpha [\exp(z_t) n_t]^{1-\alpha},$$

$$k_{t+1} = (1-\delta) k_t + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t,$$

where

$$S(x_t) = \frac{\chi}{2}[\exp\{x_t - 1\} + \exp\{-(x_t - 1)\} - 2].$$

Notice here that $S(1) = S'(1) = 0$ and $S''(1) = \chi$. The time series representation for the technology is:

$$z_t = \rho z_{t-1} + \varepsilon_{t-p} + \xi_t,$$

where ε_t, ξ_t are iid random variables that are mutually uncorrelated at all leads and lags. Suppose agents observe ε_t, ξ_t at date t when they are realized. This time series representation captures the idea that there are shocks that operate on technology with long lags, and other shocks that operate quickly. These two may cancel. For example, a jump in ε_t drives up the forecast of z_{t+p}, but ex post if ξ_{t+p} happens to equal $-\varepsilon_t$, then the jump in z_{t+p} does not occur. Some argue that this sort of shock captures some stock market booms, in which people become convinced that a certain technology will have high returns in the future (say, they think fiber optic cable will be produce a huge return), but then ex post they are disappointed.

(a) Write this exogenous stochastic process in first order vector form:

$$s_t = P s_{t-1} + Q \epsilon_t,$$

where

$$\epsilon_t = \begin{pmatrix} \varepsilon_t \\ \xi_t \end{pmatrix}.$$

(b) Find a solution for this model. That is, determine a set of variables, Z_t, and find their solution,

$$Z_t = AZ_{t-1} + Bs_t.$$

(In finding this solution, it is fine to obtain the log-linearized first order conditions by numerically differentiating the Euler equations. The code, solvea.m, will solve for the A matrix given α_0, α_1, α_2, and the code, solveb.m, will solve for the B matrix given β_0, β_1. Also, tsemat.m can be used for the numerical derivatives. This and related code can be found by downloading and then executing the file in http://www.faculty.econ.northwestern.edu/faculty/christiano/research/Solve/solve00.exe.). To do this, set $p = 4$, $b = 4$, $\chi = 5$. Some guidance may be found in http://www.faculty.econ.northwestern.edu/faculty/christiano/workshop/overinvestment.pdf, which also provides some interpretation.

(c) Compute the impulse response in $Y_t = \left(\hat{c}_t, \hat{\imath}_t, \hat{y}_t, \hat{n}_t, \hat{P}_{k',t}\right)'$ to a one-time shock in ε_t. Here, $P_{k',t}$ is the period t price, in units of consumption goods, of a unit of k_{t+1}. Do the same for the case in which the expected jump in z_t is not realized. That is, put a shock in ε_t and then put a shock in ξ_{t+4} that is equal an opposite in sign. The latter experiment corresponds to a particular theory of a stock market boom bust cycle: this is an episode in which Y_t all rise during the period in which higher future z_t is anticipated, and then they fall when the realization sinks in that the higher z_t will not, in fact, materialize.

2. Invertibility.

(a) Do the shocks, ϵ_t, lie in the space of past Y_t?

(b) Replace z_t by the simpler process: $z_t = \varepsilon_t + \theta\varepsilon_{t-1}$. Does ε_t lie in the space of past Y_t when $\theta = 2, 1.5, 1.1,$ and 0.9?

3. Consider the following goods-producing sector:

$$Y = \left[\int_0^1 Y_i^{\frac{1}{\lambda}} di \right]^{\lambda}, \ 1 < \lambda.$$

Intermediate good firms in the interval $i \in (0, \theta)$ must set their price at the beginning of the period before they know the realization of period t shocks, and intermediate good firms in the interval $i \in (\theta, 1)$ set their price at the end of the period, when they know the realization of all period t shocks. The demand curve for intermediate good i is:

$$\left(\frac{P}{P_i} \right)^{\frac{\lambda}{\lambda - 1}} = \frac{Y_i}{Y}.$$

The production function for intermediate good firms is:

$$Y_i = z l_i,$$

where l_i is the amount of labor employed by firm i and z is the marginal product of labor.

(a) Derive the aggregate resource constraint:

$$c = \phi \left(P_f, P_s \right) z l, \ l = \int_0^1 l_i di,$$

where P_f is the price of flexible price firms and P_s is the price set by sticky price firms. Show that ϕ achieves its maximum of unity when $P_f = P_s$, and $\phi < 1$ when $P_f \neq P_s$.

(b) Consider P_f and P_s in the range $0.70 < P_f / P_s < 1.30$. What is the value of ϕ in this range, for $\lambda = 1.30, 1.40$ (these are big numbers)?

4. Recall the long-run identification for a single neutral technology shock discussed in the beginning of this course. Describe how identification can be achieved if there are *two* shocks: a neutral shock to technology and an embodied technology shock, which are the only *two* shocks that have an effect on labor productivity. In addition, assume that the shock to disembodied technology is the only shock that drives level of investment good prices at infinity. (Hint: the VAR vector, Y_t, will now have to include the growth rate of labor productivity and the growth rate of the investment good price.