1. Consider a version of the model in the last question of the previous homework. Set $\gamma = 0$, so that there are no externalities. Replace the utility function by:

$$u(c_t, n_t) = \log(c_t - bc_{t-1}) + \psi \log(1 - n_t).$$

Replace the resource constraint by:

$$c_t + i_t \leq k_t^\alpha [\exp (z_t) n_t]^{1-\alpha},$$

$$k_{t+1} = (1 - \delta) k_t + \left(1 - S \left(\frac{I_t}{I_{t-1}}\right)\right) I_t,$$

where

$$S(x_t) = \frac{\chi}{2} [\exp \{x_t - 1\} + \exp \{- (x_t - 1)\} - 2].$$

Notice here that $S(1) = S'(1) = 0$ and $S''(1) = \chi$. The time series representation for the technology is:

$$z_t = \rho z_{t-1} + \varepsilon_{t-p} + \xi_t,$$

where $\varepsilon_t, \xi_t$ are iid random variables that are mutually uncorrelated at all leads and lags. Suppose agents observe $\varepsilon_t, \xi_t$ at date $t$ when they are realized. This time series representation captures the idea that there are shocks that operate on technology with long lags, and other shocks that operate quickly. These two may cancel. For example, a jump in $\varepsilon_t$ drives up the forecast of $z_{t+p}$, but ex post if $\xi_{t+p}$ happens to equal $-\varepsilon_t$, then the jump in $z_{t+p}$ does not occur. Some argue that this sort of shock captures some stock market booms, in which people become convinced that a certain technology will have high returns in the future (say, they think fiber optic cable will be produce a huge return), but then ex post they are disappointed.

(a) Write this exogenous stochastic process in first order vector form:

$$s_t = Ps_{t-1} + Q\varepsilon_t,$$
where

\[ \epsilon_t = \begin{pmatrix} \varepsilon_t \\ \xi_t \end{pmatrix}. \]

(b) Find a solution for this model. That is, determine a set of variables, \( Z_t \), and find their solution,

\[ Z_t = AZ_{t-1} + Bs_t. \]

(In finding this solution, it is fine to obtain the log-linearized first order conditions by numerically differentiating the Euler equations. The code, solvea.m, will solve for the \( A \) matrix given \( \alpha_0, \alpha_1, \alpha_2 \), and the code, solveb.m, will solve for the \( B \) matrix given \( \beta_0, \beta_1 \). Also, tsemat.m can be used for the numerical derivatives. This and related code can be found by downloading and then executing the file in http://www.faculty.econ.northwestern.edu/faculty/christiano/research/Solve/solve00.exe.). To do this, set \( p = 4, b = 4, \chi = 5 \). Some guidance may be found in http://www.faculty.econ.northwestern.edu/faculty/christiano/workshop/overinvestment.pdf, which also provides some interpretation.

(c) Compute the impulse response in \( Y_t = (\hat{c}_t, \hat{h}_t, \hat{y}_t, \hat{n}_t, \hat{P}_k, t)' \) to a one-time shock in \( \varepsilon_t \). Here, \( P_{k', t} \) is the period \( t \) price, in units of consumption goods, of a unit of \( k_{t+1} \). Do the same for the case in which the expected jump in \( z_t \) is not realized. That is, put a shock in \( \varepsilon_t \) and then put a shock in \( \xi_{t+4} \) that is equal an opposite in sign. The latter experiment corresponds to a particular theory of a stock market boom bust cycle: this is an episode in which \( Y_t \) all rise during the period in which higher future \( z_t \) is anticipated, and then they fall when the realization sinks in that the higher \( z_t \) will not, in fact, materialize.

2. Invertibility.

(a) Do the shocks, \( \epsilon_t \), lie in the space of past \( Y_t \)?

(b) Replace \( z_t \) by the simpler process: \( z_t = \varepsilon_t + \theta \varepsilon_{t-1} \). Does \( \varepsilon_t \) lie in the space of past \( Y_t \) when \( \theta = 2, 1.5, 1.1, \) and \( 0.9 \)?
3. Consider the following goods-producing sector:

\[ Y = \left[ \int_0^{l_1} Y^i \lambda^i \, di \right]^\lambda, \quad 1 < \lambda. \]

Intermediate good firms in the interval \( i \in (0, \theta) \) must set their price at the beginning of the period before they know the realization of period \( t \) shocks, and intermediate good firms in the interval \( i \in (\theta, 1) \) set their price at the end of the period, when they know the realization of all period \( t \) shocks. The demand curve for intermediate good \( i \) is:

\[ \left( \frac{P_f}{P_s} \right)^{\frac{\lambda}{\lambda - 1}} = \frac{Y_i}{Y}. \]

The production function for intermediate good firms is:

\[ Y_i = zl_i, \]

where \( l_i \) is the amount of labor employed by firm \( i \) and \( z \) is the marginal product of labor.

(a) Derive the aggregate resource constraint:

\[ c = \phi(P_f, P_s) zl, \quad l = \int_0^{l_1} l_i \, di, \]

where \( P_f \) is the price of flexible price firms and \( P_s \) is the price set by sticky price firms. Show that \( \phi \) achieves its maximum of unity when \( P_f = P_s \), and \( \phi < 1 \) when \( P_f \neq P_s \).

(b) Consider \( P_f \) and \( P_s \) in the range \( 0.70 < P_f/P_s < 1.30 \). What is the value of \( \phi \) in this range, for \( \lambda = 1.30, 1.40 \) (these are big numbers)?

4. Recall the long-run identification for a single neutral technology shock discussed in the beginning of this course. Describe how identification can be achieved if there are two shocks: a neutral shock to technology and an embodied technology shock, which are the only two shocks that have an effect on labor productivity. In addition, assume that the shock to disembodied technology is the only shock that drives level of investment good prices at infinity. (Hint: the VAR vector, \( Y_t \), will now have to include the growth rate of labor productivity and the growth rate of the investment good price.)