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416, Fall 2005
Homework 6, Due November 18.

1. Prove the uniform taxation result for the three good static, nonmonetary model in the class notes on Ramsey policy.

2. Consider the function,

$$f(x) = \frac{1}{1+x^2},\ f : [5,5] \to R.$$

Approximate this function by an $n-$dimensional polynomial, for $n = 2, 8, 16, 40$.

   (a) Compute a sequence of $n^{th}$ order approximating polynomials, $\hat{g}^n(x)$, by choosing the $n+1$ parameters so that $\hat{g}^n$ and $f$ coincide at $n+1$ equally spaced points on the domain of $f$.

   (b) Do the same as above, but this time choose the points where $\hat{g}^n$ and $f$ match to be the ones associated with the zeros of the $n+1$ dimensional Chebyshev polynomial. Note how one polynomial approximation strategy converges and the other one does not.

3. Consider a model economy in which preferences are given by $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$, where

$$u(c,n) = \frac{\left[c(1-n)^{\psi}\right]^{1-\sigma}}{1-\sigma},$$

and technology is given by:

$$c_t + k_{t+1} - (1-\delta)k_t \le k_t^{\alpha}(\exp(x_t)n_t)^{1-\alpha}.$$

Here, $x_t$ is a zero-mean, two state Markov chain:

$$x_t \in [-x, x],$$

with

$$\begin{aligned} prob[x_{t+1} = x_t] &= \pi \\ prob[x_{t+1} \neq x_t] &= 1-\pi \end{aligned}$$

Suppose that the first order autocorrelation and standard deviation of $x_t$ are 0.90 and 0.10, respectively (that standard deviation is quite large!). Also, $\alpha = 1/3$, $\sigma = 2$, $\delta = 0.02$, $\beta = 1.03^{-0.25}$. Also, let $\psi$ be the value necessary to imply $n = 0.23$ in steady state.

(a) Compute the log-linear policy rule by first-order pertubation:

$$\log k_{t+1} = \log k + \lambda \log k_t + x_t,$$

where $k$ denotes the nonstochastic steady state value of the capital stock. (Hint: one way to go would be to log-linearize the intratemporal euler equation and use it to express the intertemporal euler equation entirely in terms of capital and the shocks, and then solve that.)

(b) Compute the same object using the Chebyshev approach described in class. Do a first order and second order Chebyshev approximation over a region of the capital stock that includes $k\,(1-3/4)\,, k\,(1+3/4)\,.$

(c) Graph the $R$ function corresponding to the two Chebyshev solutions and the first-order perturbation. Do you see an improvement over the first order perturbation?