Formulating and Estimating a Dynamic, General Equilibrium Model Useable for Policy Analysis

based on work by Altig, Christiano, Eichenbaum, Linde

Objectives

• Constructing a DSGE Model
  – Model Features
  – Estimation of Model using VAR’s

• Resolve Apparent Conflict Between Macro and Micro Data
  – Macro Evidence:
    • Inflation is Inertial
  – Micro Evidence:
    • Prices Change Frequently

Example: Analysis with Calvo-Sticky Prices

• Analysis with Aggregate European and US Data (see Smets-Wouters, Gali-Gertler):
  – Prices Re-optimized Every 6 Quarters

• Micro Evidence:
  – Prices ‘Re-optimized’ Every 1.7 Quarters

Proposed Resolution of Conflict

• Firms Re-optimize Frequently (As in Micro)

• When Firms Re-optimize, They Change Price By a Small Amount
  – Firms’ Short Run Marginal Cost Increasing in Own Output
  – Firm-Specific Factors of Production (Capital)
  – Build on Sbordone, Woodford, others
Standard Model

- Capital is homogeneous
- Traded in perfectly competitive markets
  - Firm marginal cost independent of own output
- Assumptions unrealistic
  - Made for computational simplicity
  - Hope: It doesn’t matter
  - In fact: It matters a lot!

Intuition: Rising Marginal Cost and Incentive to Raise Price

- A firm contemplates raising price
  - This implies output falls
  - Marginal cost falls
  - Incentive to raise price falls
- Effect quantitatively important when:
  - Demand elastic
  - Marginal cost steep

Strategy for Evaluating Proposed Resolution of Conflict

- Incorporate idea into otherwise standard equilibrium model
- Estimate model parameters using macro data (elasticity of demand and slope of marginal cost particularly important)
- Ask: Is model consistent with
  - Macro evidence on inflation inertia?
  - Micro evidence on price changes?
Key results

• Make Progress On Macro/Micro Conflict
  – Account for Macro Evidence of Inflation Inertia
  – Prices re-optimized on average once every 1.6 quarters.
  – This finding depends on the assumption that capital is firm specific.

• Wage-setting Frictions play Important Role.
  – Wage contracts re-optimized on average once every 3 quarters.

• Monetary Policy Crucial In Transmission of Technology Shocks

• According to our model, in absence of monetary accommodation,
  – Output and hours would fall in the wake of a positive neutral technology shock;
  – Output and hours worked would rise by much less than they actually do after a positive capital embodied technology shock.

• Consistent with findings in Gali, Lopez-Salido and Valles (2002).

Outline

• Model
  • Econometric Estimation of Model
    – Fitting Model to Impulse Response Functions
  • Model Estimation Results
  • Implications for Micro Data on Prices
  • Evaluate the Reliability of VAR Analysis

Model...

• Two Versions of Model
  – Homogeneous Capital
  – Firm-specific Capital

• Describe Model Under Homogeneous Capital Assumption

• What to Change to Obtain Firm-Specific Capital Version

Description of Model

• Timing Assumptions
• Firms
• Households
• Monetary Authority
• Goods Market Clearing and Equilibrium
Timing

• Technology Shocks Realized.
• Agents Make Price/Wage Setting, Consumption, Investment, Capital Utilization Decisions.
• Monetary Policy Shock Realized.
• Household Money Demand Decision Made.
• Production, Employment, Purchases Occur, and Markets Clear.
• Note: Wages, Prices and Output Predetermined Relative to Policy Shock.

Firm Sector

Final Good, Competitive Firms

Intermediate Good, Producer 1

Intermediate Good, Producer 2

Intermediate Good, Producer infinity

Competitive Market for Homogeneous Capital

Competitive Market for Homogeneous Labor

Household 1

Household 2

Household infinity

Intermediate Good Firms -
• Each $Y_d$ Produced by a Monopolist, With Demand Curve:
  \[
  \left( \frac{P_i}{P} \right) \frac{Y_d}{Y} = Y_d \left( \int_0^1 y d y \right)^{\lambda-1}.
  \]
  * Technology:
    \[
    Y_d = \left( \int_0^1 y d y \right)^{\lambda}, \quad 1 \leq \lambda < \infty
    \]
  * Objective:
    \[
    \max_{Y_d} \left( \frac{P_i}{P} \right) \frac{Y_d}{Y}, \quad P = \int_0^1 p_d y d y
    \]
  * Firms and Prices:
    \[
    \left( \frac{P_i}{P} \right) \frac{Y_d}{Y} = \frac{Y_d}{Y}, \quad P = \int_0^1 p_d y d y
    \]

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    \]

\[\text{86}\]
• Calvo Price Setting:
  - With Probability $1 - \zeta$, $t^{th}$ Firm Sets Price, $P_t$, Optimally, to $\bar{P}$.
  - With Probability $\zeta$, Do Not Optimize Current Price. Instead:
    \[
    P_t = \pi \cdot \frac{1}{1 - \pi} \cdot \frac{1}{P_{t-1}}
    \]

• Firms Setting Prices Optimally at $t$ Choose $\hat{P}$ to max:
  \[
  v_t \left[ P Y_0 - MC Y_0 \right] + \beta \gamma Y_{t-1} \left[ P \pi Y_{t-1} - MC Y_{t-1} \right] + \ldots
  \]
  subject to:
  \[
  \left( \frac{P}{P_t} \right)^{\gamma} = \frac{Y_{t+1}}{Y_t}
  \]
  $\pi_t$ value of a dividend at $t$
  $MC_t$ given

• Scaling:
  \[
  \hat{P}_t = \frac{P_t}{P'} \quad \hat{w}_t = \frac{w_t}{P'}
  \]
  \[
  \gamma = \frac{MC_t}{P_t}
  \]

• Real Marginal Cost:
  \[
  s_t = \left( \frac{1}{1 - \alpha} \right)^{(1+\alpha)} \left( \frac{1}{\alpha} \right)^{(1+\alpha)} \gamma (\alpha R_t) t^{\alpha - 1} \frac{1}{z_t}
  \]

• Linear approximation:
  \[
  z_t = \frac{\pi_t - x_t}{x_t}
  \]

• Price Optimization Leads to:
  \[
  \hat{P}_t = s_t + \sum_{i=1}^\infty \left( \left( \beta \gamma \right)^i (s_{i+1} - s_{i+1}) \right)
  \]
  \[
  + \sum_{i=1}^\infty \left( \left( \beta \gamma \right)^i (\pi_{i+1} - \pi_{i+1}) \right)
  \]
  \* Reaction Function:
  \* $\hat{P}_t > s_t$ if $s_{i+1} > s_i$ and/or $\pi_{i+1} > \pi_i$. 
  \*
Households: Sequence of Events

- Technology shock realized.

- Decisions: Consumption, Capital accumulation, Capital Utilization.

- Insurance markets on wage-setting open.

- Wage rate set.

- Monetary policy shock realized.

- Household allocates beginning of period cash between deposits at financial intermediary and cash to be used in consumption transactions.

Households...

- Monopoly supplier of differentiated labor
  - Sets wage subject to Calvo style frictions like firms

- Preferences of $j^{th}$ household
  \[
  E_t^j \sum_{t=0}^{\infty} \beta^t \left[ \log (C_{t+1} - \Delta C_{t+1}) - \psi_t \frac{h_{j,t+1}^t}{2} \right]
  \]

  - $E_t^j$: expectation operator, conditional on aggregate and household $j$ idiosyncratic information.
  - $C_t$: consumption
  - $h_{j,t}$: hours worked.
Habit Persistence and Response of Consumption

- Recall that an Expansionary Monetary Policy Shock, we see
  - houseprice rise in consumption
  - decline in real interest rate.
- Euler Equation in Standard Model:
  \[ \frac{\beta}{\frac{\partial u_{j+1}}{\partial c_{j+1}} - \beta} = \frac{\beta}{\frac{\partial u_{j}}{\partial c_{j}}} \]
  \[ \frac{\beta}{\frac{\partial u_{j+1}}{\partial c_{j+1}} - \beta} = \frac{\beta}{\frac{\partial u_{j}}{\partial c_{j}}} \]
  \[ \pi_{j+1} = \frac{\pi_{j}}{\beta} \]
- Problem: Can't Have \( g_j \) High and \( \frac{\beta}{\beta} \) Simultaneously!

- Habit Persistence in Preferences (example):
  - aggregate consumption
  - Euler Equation:
    \[ \frac{\beta}{\frac{\partial u_{j+1}}{\partial c_{j+1}} - \beta} = \frac{\beta}{\frac{\partial u_{j}}{\partial c_{j}}} \]
    \[ \frac{\beta}{\frac{\partial u_{j+1}}{\partial c_{j+1}} - \beta} = \frac{\beta}{\frac{\partial u_{j}}{\partial c_{j}}} \]
  - Result:
    - \( g_{j+1} \) and \( g_j \) Can Both be High, as Long as \( g_{j+1} < \beta g_j \).
  - Consistent with Simultaneous Hemp-Shape \( \pi \) Response and Low Real Rate.
- Habit Persistence Also Helpful for Understanding Asset Prices

Households...

- Asset Evolution Equation:
  \[ M_{t+1} = R_t \left[ M_t - Q_t + (z_t - 1)M_t^\prime \right] + A_{t+1} + Q_t + W_{t+j}b_{t+j} \]
  \[ + \rho_\beta \beta - \alpha(n)K_t + D_t - R_t \left[ (1 + \eta(V(t)))C_t + \gamma \right] \]
  \[ (I_t + \alpha(n)K_t) \]
  - \( M_t \): Beginning of Period Base Money; \( Q_t \): Transactions Balances
  - \( z_t \): Growth Rate of Base; \( n_t \): Utilization Rate of Capital
  - \( I_t \): (Real) Price of investment goods, \( \rho_\beta \beta - \alpha(n)K_t \)
  - \( \gamma \): Velocity:
    \[ V_t = \frac{PC_t}{Q_t} \]

Money Demand

- Asset Evolution Equation:
  \[ M_{t+1} = R_t \left[ M_t - Q_t + (z_t - 1)M_t^\prime \right] + A_{t+1} + Q_t + W_{t+j}b_{t+j} \]
  \[ + \rho_\beta \beta - \alpha(n)K_t + D_t - R_t \left[ (1 + \eta(V(t)))C_t + \gamma \right] \]
  \[ (I_t + \alpha(n)K_t) \]
  - Increase in \( Q_t \):
    - Marginal Cost of Interest Foresone: \( B_t \)
    - Marginal Benefit:
      \[ 1 - \rho_\beta \left( \frac{V_t}{Q_t} \right) \frac{\partial V_t}{\partial Q_t} \]
      \[ \text{Additionally cash available at end of period} \]
      \[ = \frac{\rho_\beta \left( \frac{V_t}{Q_t} \right) \left( \frac{V_t}{Q_t} \right)}{Q_t} \]
Money Demand...

- Money Demand: Equate Marginal Benefits and Costs of $Q_t$:
  \[ R_t = \frac{1 + \eta}{1 - \eta} \left( \frac{P_t C_t}{Q_t} \right) \left( \frac{P_t C_t}{Q_t} \right)^2 \]

- Properties of Money Demand:
  - Unit Consumption Elasticity of Money Demand
    - Increase $C_t$ by 1 percent and hold $R_t$, $P_t$ fixed \( \Rightarrow \) Desired $Q_t$ increases 1 percent
  - $R_t$ \( \uparrow \) implies $Q_t$ \( \downarrow \)
    - To induce households to hold additional $Q$, must have lower $R$
    - Money Demand Elasticity is Bigger, the Bigger is $\eta$

Money Demand...

- Quantitative Analysis of Money Demand
  - Consider the following Parametric Function for $\eta$
    \[ \eta = \frac{AV_t}{V_t} - 2\sqrt{AB} \]

  \[ \Rightarrow \]
  \[ R = 1 + \eta(V) \times V^2 = 1 + [A - BV^{-3}] V^2 = 1 - B + AV^3 \]

- Data:
  - Money - St. Louis Fed's M3M, 1974-2004
  - Consumption - NIPA Consumption of Services and Nondurables
  - Interest Rate - One Year T-Bills
  - OLS Regression of $V^2$ on $R$ \( \Rightarrow A = 0.0174 \) and $B = 0.0187$

Money Demand...

- Top Graph: Velocity of Money
- Bottom Graph: Actual and Predicted Interest Rate

Households...

- Capital Evolution:
  \[ K_{t+1} = (1 - \delta)K_t + F(I_t, I_{t-1}); \]
  \[ F(I_t, I_{t-1}) = (1 - S \left( \frac{L_t}{L_{t-1}} \right)) I_t, \]
  \[ S = S^* = 0, \quad S^* > 0 \quad \text{in steady state} \]

Findings: Static Money Demand Equation Fits the Data Well!
Wage Decisions

- Households supply differentiated labor.
- Standard Calvo set up as in Erceg, Henderson and Levin and CEE.

Structure of the Labor Market

- Intermediate Good Firms Use Labor Aggregate:
  \[ L_i = \left[ \sum h_{j,i} \right]^{1-\lambda_j} \]
- Price of \( L_i \):
  \[ W_i = \left[ \int_0^1 W_{it}^{1-\lambda_i} d\lambda_i \right]^{1-\lambda_i} \]
- Demand for Household Labor Service, \( h_{jt} \):
  \[ h_{jt} = \left( \frac{W_{jt}}{W_{it}} \right)^{\lambda_j} \]
  where \( 1 \leq \lambda_j < \infty \).
  \( W_{jt} \) - wage set by household
  \( L_t \) - homogeneous aggregate labor
  \( W_i \) - wage rate of aggregate labor

Household Wage Decision

- Demand for Household’s Specialized Labor:
  \[ h = D(w) = w^{1-\delta} \]

- Household Choice of real wage:
  \[ \max_{w} w(\epsilon) - z(h) \]
  subject to \( \epsilon \leq wh, h = D(w) \)

- Substituting out for budget constraint and demand curve:
  \[ \max_{x'} x'(wD(w)) - z(D(w)) \]

- First order condition:
  \[ n_{c} \times [wD'(w) + D'(w)] = \lambda' \times D'(w) \]

Household Wage Decision

- First order condition:
  \[ n_{c} \times [wD'(w) + D'(w)] = \lambda' \times D'(w) \]
  or
  \[ w_{n} \times [wD'(w) + D'(w)] = \lambda' \times wD'(w) \]
  or
  \[ w_{n} = \frac{\lambda'}{\lambda_0} \times \frac{wD'(w)}{wD'(w) + D'(w)} \]
Household Wage Decision

- First Order Condition:
  \[ w = \frac{\ell}{u_c} \times \frac{w D'(w)}{w D'(w) + D(w)} \]

  Note: Household Marginal Cost (consumption value of a unit of leisure):
  \[ \frac{\ell}{u_c} = \frac{\frac{\partial u_c}{\partial \ell}}{\frac{\partial u_c}{\partial u_c}} = \frac{\text{consumption}}{\text{leisure}} \]

- Note: With Constant Elasticity Utility Function:
  \[ \frac{w D'(w)}{w D'(w) + D(w)} = \lambda_c \]

- Conclude ‘Wage Equals Markup Times Marginal Cost’:
  \[ w = \lambda_c \frac{\ell}{u_c} \]

Calvo-style Wage Setting:

- With Probability \(1 - \xi_c\), \(\phi^0\) Household Sets Wage, \(W_0\), Optimally, to \(\bar{W}_c\):
  \[ W_0 = \bar{W}_c \]  
  \[ \mu_c \sim \text{steady state growth rate of economy} \]

- With Probability \(\xi_c\):
  \[ W_{c,t} = \pi_{c,t-1} \mu_c W_{c,t-1} \]

  First Order Condition:
  \[ E_{c,1} \sum_{t=0}^{\infty} (\xi_c)^t [h_{c,t} \psi_{c,t}] \left[ \frac{\bar{W}_c X_{c,t}}{P_{c,t}} - \lambda_c \frac{\psi_{c,t}}{\psi_{c,t}} \right] = 0. \]

- Households Attempt to Set Price (the wage) as a markup over marginal cost.

| \(\psi_{c,t}\) | utility value of consumption (Multiplier on Budget Constraint) |
| \(h_{c,t}\) | Household Marginal utility of Leisure |
| \(\psi_{c,t}\) | Marginal Cost (in Consumption Units) of a Unit of Leisure |

Monetary and Fiscal Policy

- Monetary Policy:
  \[ x_t = M_t / M_{t-1} \]

- Fiscal Policy:
  \[ \tilde{x}_{x,t} = \rho_x \tilde{x}_{x,t-1} + \varepsilon_{x,t} \]
  \[ \tilde{x}_{z,t} = \rho_z \tilde{x}_{z,t-1} + \varepsilon_{z,t} + \varepsilon_{x,z,t-1} \]
  \[ \tilde{x}_{y,t} = \rho_y \tilde{x}_{y,t-1} + \varepsilon_{y,t} + \varepsilon_{x,y,t-1} + \varepsilon_{z,y,t-1} \]

- \(\tilde{x}_{x,t}\): response of monetary policy to a monetary policy shock, \(\varepsilon_{x,t}\).
- \(\tilde{x}_{z,t}\): response of monetary policy to an innovation in neutral technology, \(\varepsilon_{z,t}\).
- \(\tilde{x}_{y,t}\): response of monetary policy to an innovation in capital embodied technology, \(\varepsilon_{y,t}\).
- Government has access to lump sum taxes, pursues a Ricardian fiscal policy.

Loan Market and Final Good Market Clearing Conditions, Equilibrium

- Financial intermediaries receive \(M_t - Q_t + (x_t - 1) M_t\) from the household.
  - Lend all of their money to intermediate good firms, which use the funds to pay for \(R_k\).
- Loan market clearing:
  \[ W_c H_c = x_t M_t - Q_t \]

- The aggregate resource constraint is:
  \[ (1 + \eta(V_c)C_t + T_{c,t}^{-1} [L_t - a(n_t)R_t] \leq Y_t \]

- We adopt a standard sequence-of-markets equilibrium concept.
The Firm - Specific Capital Model

- Firms own their own capital which can’t be adjusted during the period.
- Can only be increased or decreased over time by varying rate of investment.
- In all other respects, problem of intermediate good firm is same as before.
- Technology for accumulating physical capital by intermediate good firm $i$:
  \[ F(L^i, L\rightarrow(i)) = (1-S) \left( \frac{F(i)}{L\rightarrow(i)} \right) + u(i) \]
  \[ K_{i+1}(i) = (1-\delta)K_i(i) + FU(i), L\rightarrow(i)) \]
- Present discounted value of $F^{th}$ intermediate good’s cash flow:
  \[ E \sum_{j=0}^{\infty} \frac{\beta^j}{\lambda} \left\{ P_{i+j}(i) - P_{i+j}R_{i+j}(i) \right\} \]

Implications for Inflation

- Equations which characterize equilibrium for homogeneous and firm specific capital model are identical:
  \[ \Delta x_i = \beta \Delta x_{i+1} + \gamma \Delta t + \Omega \]
  where
  \[ \gamma = \frac{(1-\xi)(1-\beta)}{\xi} \]
  and
  \[ \chi = \begin{cases} 1 & \text{homogeneous capital model} \\ < 1 & \text{firm-specific capital model} \end{cases} \]
- In the firm specific capital model, $\chi$ is a particular non-linear function of the parameters of the model.
- Given $\gamma$, the two models are observationally equivalent with respect to aggregate prices and quantities.

Firm Specific Capital Model...

- Timing:
  - Firms see technology shock.
  - Sets $P(i)$, subject to Calvo frictions.
  - Also depends on $L(i)$ and $\omega(i)$.
- Time $t$ monetary policy shock occurs, demand for the firm’s product is realized.
  - Firm must satisfy demand.

Implications for Inflation...

\[ \gamma = \frac{(1-\xi)(1-\beta)}{\xi} \]
- For example, our estimated $\gamma$ is
  \[ \gamma = .035 \]
- Under homogeneous capital model ($\chi = 1$), this implies
  \[ \xi_p = .63 \text{ and } 1/(1-\xi_p) = 6 \]
- For the firm specific capital model, $\chi = .031$, and
  \[ \xi_p = .38 \text{ and } 1/(1-\xi_p) = 1.6 \]
Econometric Methodology

- Variant of limited information strategy used in CEE (2004).
  - Impose a subset of assumptions made in equilibrium model to estimate impulse response functions of key macroeconomic variables to the three shocks in our model.
  - Neutral technology shocks, capital embodied technology shocks and monetary policy shocks.
- Choose values for key parameters of structural model to minimize difference between estimated impulse response functions and analogous objects in model.

Identifying Assumptions

- Shocks to technology
  - Innovations to technology (both neutral and capital embodied) are only shocks that affect level of labor productivity in the long run.
  - Capital embodied technology shocks are the only shocks that affect price of investment goods relative to consumption goods in the long run. (Fisher (2005)).
- Our equilibrium model is consistent with these assumptions
- Shocks to monetary policy
  - CEE (2004), there's exactly one shock, the monetary policy shock that affects interest rate contemporaneously, over and above shocks that drive aggregate prices and quantities.
  - This assumption is satisfied in our equilibrium

Identification...

\[
\begin{align*}
\gamma_{10} &= \begin{pmatrix}
\Delta \ln(\text{relative price of investment}) \\
\Delta \ln(GDP/Hours) \\
\Delta \ln(GDP \text{ deflator}) \\
(\text{capacity utilization}) \\
\ln(\text{Hours}) \\
\ln(GDP/Hours) - \ln(W_i/P) \\
\ln(C_i/GDP) \\
\ln(I_i/GDP) \\
\text{Federal Funds Rate} \\
\ln(GDP \text{ deflator}) + \ln(GDP) - \ln(MZ_i)
\end{pmatrix}_{10} \\
\gamma_{10} &= \begin{pmatrix}
\Delta \ln(\text{relative price of investment}) \\
\Delta \ln(GDP/Hours) \\
\Delta \ln(GDP \text{ deflator}) \\
(\text{capacity utilization}) \\
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\ln(GDP/Hours) - \ln(W_i/P) \\
\ln(C_i/GDP) \\
\ln(I_i/GDP) \\
\text{Federal Funds Rate} \\
\ln(GDP \text{ deflator}) + \ln(GDP) - \ln(MZ_i)
\end{pmatrix}_{10}
\end{align*}
\]

- Monetary Policy
  \[R_t = f(\Omega_t) + \epsilon_M,\]
  \[f(\Omega_t) \text{ is linear, } \Omega_t \text{ contains } Y_{t-1}, Y_{t-2}, Y_{t-3}, Y_{t-4} \text{ and the only date } t \text{ variables in } \Omega_t \text{ are } \{\Delta \delta_t, \Delta \phi_t, Y_t\} \text{ and } \epsilon_M \text{ is orthogonal with } \Omega_t.

Estimating Parameters in the Model

- Partition Parameters into Three Groups.
  - Parameters set a priori (e.g., \(\beta, \delta, \lambda\)).
  - \(\zeta_1\) remaining parameters pertaining to the nonstochastic part of model.
  - \(\zeta_2\) parameters pertaining to stochastic part of the model.
  - Number of parameters, \(\zeta = (\zeta_1, \zeta_2)\), to be estimated - 18
- Estimation Criterion
  - \(\Psi(\zeta)\): mapping from \(\zeta\) to model impulse responses.
  - \(\Psi\): 592 impulse responses estimated using VAR.
- Estimation Strategy
  - \(\hat{\zeta} = \arg \min_{\zeta} \left\{ \Psi(\zeta) - (\Psi(\zeta))^\top V^{-1}(\Psi(\zeta)) \right\} \).
  - \(V\): diagonal matrix with sample variances of \(\Psi\) along the diagonal.
Implications for Wage and Price Re-Optimization

- Our benchmark estimates imply that wage decisions are re-optimized on average 3.6 quarters.
- The implication of our estimate of gamma for how frequently firms re-optimize prices depends critically on whether we assume capital is firm specific or homogeneous.
  - If capital is homogeneous, firms re-optimize prices on average once every 6 quarters.
  - If capital is firm specific, firms re-optimize prices once every 1.6 quarters.
- At a broad level, this is consistent with micro evidence from Bils and Klenow, Lucas and Golosov and Klenow and Kryvtsov.
- I'll provide intuition for this in a moment.

**Estimated Parameter Values, \( \zeta_1 \)**

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( \xi )</th>
<th>( \gamma )</th>
<th>( \sigma_\pi )</th>
<th>( \beta )</th>
<th>( \sigma_\epsilon )</th>
<th>( \epsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.63</td>
<td>.72</td>
<td>.035</td>
<td>2.01</td>
<td>.05</td>
<td>2.22</td>
<td>1.06</td>
</tr>
</tbody>
</table>

- \( \epsilon \) is a little low...
- \( \beta \) similar to other estimates in literature
- \( \xi \), wages reoptimized on average every 3.6 quarters
- Parameters important in subsequent discussion...
  - \( \gamma \): costly to vary utilization of capital
  - \( \lambda \): close to perfect competition
  - \( \gamma \): amazingly low! (similar to estimates reported in literature)

**Estimated Parameters of Exogenous Shocks, \( \zeta_2 \)**

<table>
<thead>
<tr>
<th>( \beta_{\phi} )</th>
<th>( \sigma_{\phi} )</th>
<th>( \beta_{\pi} )</th>
<th>( \sigma_{\pi} )</th>
<th>( \beta_{\epsilon} )</th>
<th>( \sigma_{\epsilon} )</th>
<th>( \beta_{\gamma} )</th>
<th>( \sigma_{\gamma} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.24</td>
<td>.26</td>
<td>.92</td>
<td>.06</td>
<td>.37</td>
<td>3.36</td>
<td>1.19</td>
<td>.21</td>
</tr>
</tbody>
</table>

**Figure 10: Benchmark model - dynamic response to a monetary policy shock**

**Figure 11: Benchmark model - dynamic response to a real technology shock**
Monetary Policy and Technology Shocks

• How would the economy have responded to technology shocks if monetary policy had not been accommodative?
Understanding the Microeconomic Price Implications of the Model

- Reduced Form Parameter, $\gamma$:
  - Model: $\gamma = 0.005$  
  A 1 percent temporary rise in marginal cost leads to a 0.005\% rise in price level
  
  $\tilde{\pi}_t = \beta \tilde{\pi}_{t-1} + \gamma \tilde{s}_t$

- Direct Analysis of Macroe Data Supports Low Estimate of $\gamma$

- Inflation inertia:
  - Conventional Def: Sluggish Response of Inflation to (Monetary) Shocks
  - Conventional Solution: Identify Model Features that Slow Rise in Marginal Cost After Shock
  - Does not help with low $\gamma$

Microeconomic Price Implications of the Model...

- Homogeneous Capital Model:
  
  $\gamma = \frac{(1 - \xi_p) (1 - \beta \xi_p)}{\xi_p}$

  $\gamma = 0.005 \rightarrow \xi_p = 0.83, \frac{1}{1 - \xi_p} = 5.8$

- Homogeneous Capital Model:
  - Price Sems Not to Respond Much to Marginal Cost - Prices Must be VERY Sticky!
  - Striking Conflict Between Macro ($\gamma$) and Micro Evidence

Analysis of relation, $\Delta \pi_t = \gamma s_{t-1} + \beta \Delta \pi_{t+1}$

Microeconomic Price Implications of the Model...

- Firm-specific Capital Model:
  
  $\gamma = \frac{(1 - \xi_p) (1 - \beta \xi_p)}{\xi_p}$

  $0.001 \rightarrow \chi(\sigma_{s1}, \lambda_1, \xi_s) = 0.36$

- Firm-specific capital breaks Micro/Macro by introducing endogenous price stickiness
- How does it do it?
The Experiment

- Begin at steady state and assume there is an expansionary monetary policy shock in period 1.
- Period 1
  - Prices and output is the same for all firms.
- Period 2
  - \((1 - \xi)\) firms re-optimize and implement new price, \(\xi\) do not.
- Period 3: there are 4 types of firms.
  - \((1 - \xi)\xi^3\) re-optimize in period 2 and 3.
  - \(\xi^2\) don’t re-optimize in either period 2 or period 3.
  - \((1 - \xi)\xi^2\) re-optimize in period 2 but not in period 3.
  - \(\xi(1 - \xi)\) did not re-optimize in period 2 but did re-optimize in period 3
- In period \(s\) there are \(2^{s-1}\) different firms.
- For each period \(s\) we calculated the distribution of output and prices across firms.

Micro Findings

- Homogeneous and Firm-Specific Capital Models are Indistinguishable from the Point of View of Aggregate Data
- Different Implications for
  - Degree of Price Stickiness in Micro Data
  - Dispersion of Prices and Output Across Firms
    - Homogenous Capital Model – Some Firms Reduce Output In Wake of Positive Technology Shock
  - Firm-Specific Capital Model Seems to Have Better Micro Implications

A Check on the Econometric Procedure

- CKM Have Used an Example to Question Whether Estimated VARs are a Reliable Estimator of Impulse Response Functions to a Shock
- We Did an Experiment to Investigate Whether We Have the Problems They Describe
Basic Idea

• Generate Artificial Data from Economic Model, then Feed it to 10 Variable VAR Program Which Was Applied to Actual Data

• Wait!
  – Economic Model Only Has Three Shocks
  – Can’t Fit 10 Variable VAR to Data From Model

• Solution
  – Empirical Procedure Recognizes We’re Short on Shocks
  – Offers a Natural Solution

Background

• Recall, Structural Form of VAR:
  \[ A_0 Y_t = A(L) Y_{t-1} + \epsilon_t \]

• Reduced Form:
  \[ Y_t = B(L) Y_{t-1} + C \epsilon_t \]

where
\[ B(L) = A_0^{-1} A(L), \quad C = A_0^{-1}, \]

and
\[ \epsilon_t = \begin{pmatrix} \epsilon_{1,t}^{(1)} \\ \epsilon_{1,t}^{(2)} \\ \epsilon_{1,t}^{(3)} \\ \epsilon_{1,t}^{(4)} \end{pmatrix} \]

Background ...

• Can Write:
  \[ C \epsilon_t = C_1 \begin{pmatrix} \epsilon_{1,t}^{(1)} \\ \epsilon_{2,t} \end{pmatrix} + C_2 \begin{pmatrix} \epsilon_{1,t}^{(4)} \\ \epsilon_{2,t} \end{pmatrix} \]

• So
  \[ Y_t = B(L) Y_{t-1} + C_1 \begin{pmatrix} \epsilon_{1,t}^{(1)} \\ \epsilon_{2,t} \end{pmatrix} + C_2 \begin{pmatrix} \epsilon_{1,t}^{(4)} \\ \epsilon_{2,t} \end{pmatrix} \]

• Stochastic Process for \( Y_t \) Can be Decomposed into Two Orthogonal Processes:
  \[ Y_t = Y_t^{Model} + Y_t^{Other} \]

  \[ Y_t^{Model} = B(L) Y_{t-1}^{Model} + C_1 \begin{pmatrix} \epsilon_{1,t}^{(1)} \\ \epsilon_{2,t} \end{pmatrix} \]

  \[ Y_t^{Other} = B(L) Y_{t-1}^{Other} + C_2 \begin{pmatrix} \epsilon_{1,t}^{(4)} \\ \epsilon_{2,t} \end{pmatrix} \]
Experiment

• Generate Artificial Data
  – Extremely Long Data Set to Get Plim (20,000 Observations)
  – Many Data Sets of Length 170 Each
• Feed Each Data Set to Same VAR Fit to US Data
• Compute Impulse Response Functions
  – Dotted Lines: Small Sample Means
  – Dashed Lines: Plims
Summary

- We constructed a dynamic GE model of cyclical fluctuations.
- Given assumptions satisfied by our model, we identified dynamic response of key US economic aggregates to 3 shocks:
  - Monetary Policy Shocks
  - Neutral Technology Shocks
  - Capital Embodied Technology Shocks
- These shocks account for substantial cyclical variation in output.
- Estimated GE model does a good job of accounting for response functions (However, Misses on Inflation Response to Neutral Shock)
- Have Made Progress on Micro/Macro Conflict

Summary...

- Calvo Sticky Prices and Wages Seems Like Good Reduced Form
  - What is the Underlying Structure?