Formulating and Estimating a Dynamic, General Equilibrium Model Useable for Policy Analysis

based on work by

Altig, Christiano, Eichenbaum, Linde

Objectives

- Constructing a DSGE Model
  - Model Features
  - Estimation of Model using VAR's
- Resolve Apparent Conflict Between Macro and Micro Data
  - Macro Evidence:
    - Inflation is Inertial
  - Micro Evidence:
    - Prices Change Frequently

Example: Analysis with Calvo-Sticky Prices

- Analysis with Aggregate European and US Data (see Smets-Wouters, Gali-Gertler):
  - Prices Re-optimized Every 6 Quarters
- Micro Evidence:
  - Prices 'Re-optimized' Every 1.7 Quarters

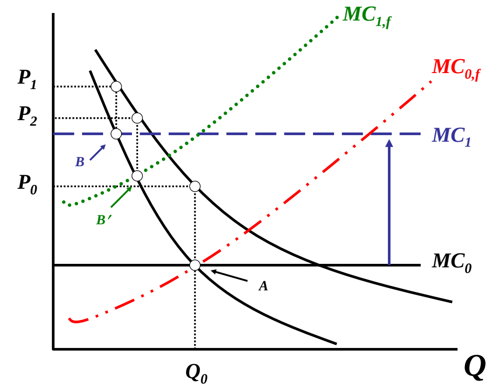
Proposed Resolution of Conflict

- Firms Re-optimize Frequently (As in Micro)
- When Firms Re-optimize, They Change Price By a Small Amount
  - Firms' Short Run Marginal Cost Increasing in Own Output
  - Firm-Specific Factors of Production (Capital)
  - Build on Sbordone, Woodford, others

Standard Model

- Capital Is Homogeneous
- Traded in Perfectly Competitive Markets
  - Firm Marginal Cost Independent of Own Output
- Assumptions Unrealistic
  - Made for Computational Simplicity
  - Hope: It Doesn't Matter
  - In Fact: It Matters A Lot!

Intuition: Rising Marginal Cost and Incentive to Raise Price



More Intuition: Rising Marginal Cost and Incentive to Raise Price

- A Firm Contemplates Raising Price
  - This Implies Output Falls
  - Marginal Cost Falls
  - Incentive to Raise Price Falls
- Effect Quantitatively Important When:
  - Demand Elastic
  - Marginal Cost Steep

Strategy for Evaluating Proposed Resolution of Conflict

- Incorporate Idea Into Otherwise Standard Equilibrium Model
- Estimate Model Parameters Using Macro Data (Elasticity of Demand and Slope of Marginal Cost Particularly Important)
- Ask: Is Model Consistent With
  - Macro Evidence on Inflation Inertia?
  - Micro Evidence on Price Changes?

Key results

- Make Progress On Macro/Micro Conflict
  - Account for Macro Evidence of Inflation Inertia
  - Prices re-optimized on average once every 1.6 quarters.
  - This finding depends on the assumption that capital is firm specific.
- Wage-setting Frictions play Important Role.
  - Wage contracts re-optimized on average once every 3 quarters.
- Monetary Policy Crucial In Transmission of Technology Shocks
- According to our model, in absence of monetary accommodation,
  - Output and hours would fall in the wake of a positive neutral technology shock;
  - Output and hours worked would rise by much less than they actually do after a positive capital embodied technology shock.
- Consistent with findings in Gali, Lopez-Salido and Valles (2002).

Outline

- Model
- Econometric Estimation of Model
  - Fitting Model to Impulse Response Functions
- Model Estimation Results
- Implications for Micro Data on Prices
- Evaluate the Reliability of VAR Analysis

Model…

- Two Versions of Model
  - Homogeneous Capital
  - Firm-specific Capital
- Describe Model Under Homogeneous Capital Assumption
- What to Change to Obtain Firm-Specific Capital Version

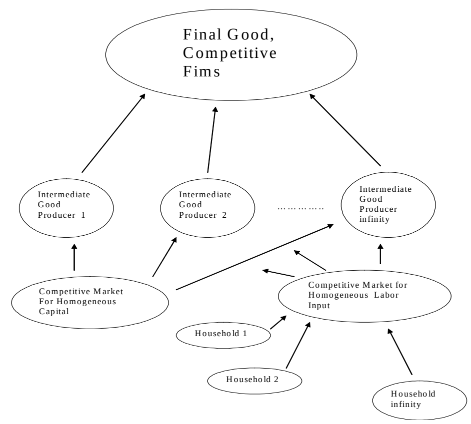
Description of Model

- Timing Assumptions
- Firms
- Households
- Monetary Authority
- Goods Market Clearing and Equilibrium

Timing

- Technology Shocks Realized.
- Agents Make Price/Wage Setting, Consumption, Investment, Capital Utilization Decisions.
- Monetary Policy Shock Realized.
- Household Money Demand Decision Made.
- Production, Employment, Purchases Occur, and Markets Clear.
- Note: Wages, Prices and Output Predetermined Relative to Policy Shock.

Firm Sector



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Firms

Final Good Firms

- Technology:

$$Y_t = \left[\int_0^1 Y_{it}^{\frac{1}{\lambda_f}} di \right]^{\lambda_f}, \; 1 \leq \lambda_f < \infty$$

- Objective:

$$\max P_t Y_t - \int_0^1 P_{it} Y_{it} di$$

- Foncs and Prices:

$$\left(\frac{P_t}{P_{it}} \right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{it}}{Y_t}, \; P_t = \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_f}} di \right]^{(1-\lambda_f)}.$$

Intermediate Good Firms -

- Each Y_{it} Produced by a Monopolist, With Demand Curve:

$$\left(\frac{P_t}{P_{it}} \right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{it}}{Y_t}.$$

- Technology:

$$Y_{it} = k_{it}^{\alpha} \left(z_t L_{it} \right)^{1-\alpha}.$$

- Here, z_t is a Technology Shock:

$$\mu_{zt} = \log z_t - \log z_{t-1}, \; \hat{\mu}_{zt} = \rho_{\mu_z} \hat{\mu}_{zt-1} + \varepsilon_{\mu_z, t}.$$

- Calvo Price Setting:
  - With Probability $1-\xi_p$, i^{th} Firm Sets Price, P_{it}, Optimally, to $\tilde{P}_t$.
  - With Probability ξ_p, Do Not Optimize Current Price. Instead:

$$P_{it} = \pi_{t-1} P_{i,t-1},\ \pi_t = \frac{P_t}{P_{t-1}}.$$

- Firms Setting Prices Optimally at t Choose $\tilde{P}_t$ to max:

$$\begin{aligned} &\upsilon_t \left[\tilde{P}_t Y_{it} - MC_t Y_{it}\right] \\ &+\beta\xi_p \upsilon_{t+1}\left[\tilde{P}_t \pi_t Y_{i,t+1} - MC_{t+1} Y_{i,t+1}\right] \\ &+\left(\beta\xi_p\right)^2 \upsilon_{t+2}\left[\tilde{P}_t \pi_t \pi_{t+1} Y_{i,t+2} - MC_{t+2} Y_{i,t+2}\right] \\ &+\ldots \end{aligned}$$

subject to:

$$\left(\frac{P_t}{\tilde{P}_t}\right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{it}}{Y_t}.$$

υ_t ~ value of a dividend at t

MC_t ~given

...

- Scaling:

$$\begin{aligned} \tilde{p}_t &= \frac{\tilde{P}_t}{P_t},\ w_t = \frac{W_t}{P_t} \\ r_t^k &= \frac{\text{rental rate on capital}}{P_t} \\ s_t &= \frac{MC_t}{P_t}. \end{aligned}$$

- Real Marginal Cost:

$$s_t = \left(\frac{1}{1-\alpha}\right)^{(1-\alpha)}\left(\frac{1}{\alpha}\right)^{\alpha}(r_t^k)^{\alpha}(w_t R_t)^{1-\alpha}\frac{1}{z_t}$$

- Linear approximation:

$$\hat{x}_t \equiv \frac{x_t - x}{x}.$$

- Price Optimization Leads to:

$$\begin{aligned} \widehat{\tilde{p}}_t \ &= \ \hat{s}_t + \sum_{l=1}^{\infty}\left(\beta\xi_p\right)^l\left(\hat{s}_{t+l} - \hat{s}_{t+l-1}\right) \\ &+\sum_{l=1}^{\infty}\left(\beta\xi_p\right)^l\left(\hat{\pi}_{t+l} - \hat{\pi}_{t+l-1}\right) \end{aligned}$$

- Front-Loading:
  - $\widehat{\tilde{p}}_t > \hat{s}_t$ if $\hat{s}_{t+l} > \hat{s}_t$ and/or $\hat{\pi}_{t+l} > \hat{\pi}_t$.

...

- Aggregate Price Level:

$$P_t = \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_f}} di\right]^{(1-\lambda_f)}$$
$$= \left[(1-\xi_p)\tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p \left(\pi_{t-1} P_{t-1}\right)^{\frac{1}{1-\lambda_f}}\right]^{1-\lambda_f}$$

- Scale:

$$1 = \left[(1-\xi_p)\tilde{p}_t^{\frac{1}{1-\lambda_f}} + \xi_p \left(\frac{\pi_{t-1}}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}\right]^{1-\lambda_f}$$

- Approximately

$$\widehat{\tilde{p}}_t = \frac{\xi_p}{1-\xi_p}\left[\hat{\pi}_t - \hat{\pi}_{t-1}\right].$$

...

- Combining Optimal Price and Aggregate Price Relation:

$$\Delta\hat{\pi}_t = \beta E_t \Delta\hat{\pi}_{t+1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p} E_t \hat{s}_t,$$

- Under Standard Price-Updating Scheme:

$$P_{it} = \bar{\pi} P_{i,t-1}.$$

Associated Reduced Form:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p} E_t \hat{s}_t.$$

Households: Sequence of Events

- Technology shock realized.
- Decisions: Consumption, Capital accumulation, Capital Utilization.
- Insurance markets on wage-setting open.
- Wage rate set.
- Monetary policy shock realized.
- Household allocates beginning of period cash between deposits at financial intermediary and cash to be used in consumption transactions.

Households...

- Monopoly supplier of differentiated labor
  - Sets wage subject to Calvo style frictions like firms
- Preferences of j^{th} household

$$E_t^j \sum_{l=0}^{\infty} \beta^{l-t}\left[\log\left(C_{t+l} - bC_{t+l-1}\right) - \psi_L \frac{h_{j,t+l}^2}{2}\right]$$

- E_t^j : expectation operator, conditional on aggregate and household j idiosyncratic information.
- C_t : consumption
- h_{jt} : hours worked.

...

Habit Persistence and Response of Consumption

- Recall that After an Expansionary Monetary Policy Shock, we see
  - hump-shaped rise in consumption
  - decline in real interest rate.
- Euler Equation in Standard Model:

$$\frac{u_{c,t}}{\beta u_{c,t+1}} \approx \frac{c_{t+1}}{\beta c_t} = \frac{g_{t+1}}{\beta} = \frac{R_t}{\pi_{t+1}},\ \pi_{t+1} = \frac{P_{t+1}}{P_t}.$$

- Problem: Can't Have g_t High and $\frac{R_t}{\pi_{t+1}}$ Simultaneously!

...

- Habit Persistence in Preferences (example):

$u(c_t - b\bar{c}_{t-1}),\ \bar{c}_{t-1}$ ~ aggregate consumption

- Euler Equation:

$$\frac{u_{c,t}}{\beta u_{c,t+1}} \approx \frac{c_{t+1} - bc_t}{\beta\left(c_t - bc_{t-1}\right)} = \frac{g_{t+1} - b}{\beta\left(1 - \frac{b}{g_t}\right)}$$

$$\approx \frac{g_{t+1} - bg_t}{\beta(1-b)}$$

- Result:
  - g_{t+1} and g_t Can Both be High, as Long as $g_{t+1} < bg_t$.
  - Consistent with Simultaneous Hump-Shape c Response and Low Real Rate.
- Habit Persistence Also Helpful for Understanding Asset Prices

Households...

- Asset Evolution Equation:

$$\begin{aligned} M_{t+1} \;=\; & R_t\left[M_t - Q_t + (x_t - 1)M_t^a\right] + A_{j,t} + Q_t + W_{j,t}h_{j,t} \\ & + P_t r_t^k u_t \bar{K}_t + D_t - P_t\left[(1+\eta(V_t))\,C_t + \Upsilon_t^{-1}\left(I_t + a(u_t)\bar{K}_t\right)\right] \end{aligned}$$

  - M_t : Beginning of Period Base Money; Q_t : Transactions Balances
  - x_t : Growth Rate of Base; u_t : Utilization Rate of Capital
    - $u_t = 1$ in steady state, $a(1) = 0$, $a'(1) > 0$, $\sigma_a = a''(1)/a'(1)$.
  - Υ_t^{-1}: (Real) Price of investment goods, $\mu_{\Upsilon,t} = \Upsilon_t/\Upsilon_{t-1}$,

$$\hat{\mu}_{\Upsilon,t} \;=\; \rho_{\mu_\Upsilon}\hat{\mu}_{\Upsilon,t-1} + \varepsilon_{\mu_\Upsilon,t}$$

- Velocity:

$$V_t = \frac{P_t C_t}{Q_t},$$

Money Demand

- Asset Evolution Equation:

$$\begin{aligned} M_{t+1} \;=\; & R_t\left[M_t - Q_t + (x_t - 1)M_t^a\right] + A_{j,t} + Q_t + W_{j,t}h_{j,t} \\ & + P_t r_t^k u_t \bar{K}_t + D_t - P_t\left[(1+\eta(V_t))\,C_t + \Upsilon_t^{-1}\left(I_t + a(u_t)\bar{K}_t\right)\right] \end{aligned}$$

- Increase in Q_t :
  - Marginal Cost of Interest Foregone: R_t
  - Marginal Benefit:

$$1 - P_t\eta'(V_t)\,C_t\frac{dV_t}{dQ_t}$$

$$= \overbrace{1}^{\text{addititional cash available at end of period}} + \overbrace{\eta'\left(\frac{P_tC_t}{Q_t}\right)\left(\frac{P_tC_t}{Q_t}\right)^2}^{\text{reduction in transactions costs due to extra cash}}$$

Money Demand ...

- Money Demand: Equate Marginal Benefits and Costs of Q_t–

$$R_t = 1 + \eta' \left(\frac{P_t C_t}{Q_t}\right) \left(\frac{P_t C_t}{Q_t}\right)^2.$$

- Properties of Money Demand:
  - Unit Consumption Elasticity of Money Demand
    - Increase C_t 1 percent and Hold R_t, P_t Fixed $\Rightarrow$ Desired Q_t increases 1 percent
  - $R_t \uparrow$ Implies $Q_t \downarrow$
    - To Induce Households to Hold Additional Q, Must Have Lower R
    - Money Demand Elasticity is Bigger, the Bigger is η''

Money Demand ...

- Quantitative Analysis of Money Demand
  - Consider the Following Parametric Function for η

$$\eta = AV_t + \frac{B}{V_t} - 2\sqrt{AB}$$

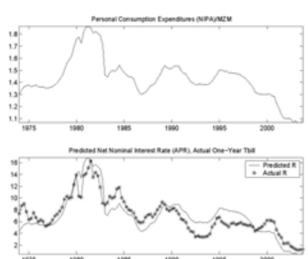
$$\Rightarrow$$

$$R = 1 + \eta'(V) \times V^2 = 1 + \left[A - BV^{-2}\right] V^2 = 1 - B + AV^2$$

  - Data:
    - Money - St. Louis Fed's MZM, 1974-2004
    - Consumption - NIPA Consumption of Services and Nondurables
    - Interest Rate - One Year T-Bills.
    - OLS Regression of V^2 on $R \Rightarrow A = 0.0174$ and $B = 0.0187$

Money Demand ...

- Top Graph: Velocity of Money
- Bottom Graph: Actual and Predicted Interest Rate



- Findings: Static Money Demand Equation Fits the Data Well!

Households...

- Capital Evolution:

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + F(I_t, I_{t-1}),$$

$$F(I_t, I_{t-1}) = (1 - S\left(\frac{I_t}{I_{t-1}}\right))I_t,$$

$$S = S' = 0,\ S'' > 0 \text{ in steady state}$$

Wage Decisions

- Households supply differentiated labor.
- Standard Calvo set up as in Erceg, Henderson and Levin and CEE.

...

Structure of the Labor Market

- Intermediate Good Firms Use Labor Aggregate:

$$L_t = \left[\int_0^1 h_{j,t}^{\frac{1}{\lambda_w}} dj\right]^{\lambda_w}.$$

- Price of L_t :

$$W_t = \left[\int_0^1 W_{it}^{\frac{1}{1-\lambda_w}} di\right]^{1-\lambda_w}.$$

- Demand for Household Labor Service, $h_{j,t}$:

$$h_{jt} = \left(\frac{W_t}{W_{jt}}\right)^{\frac{\lambda_w}{\lambda_w - 1}} L_t,\ 1 \leq \lambda_w < \infty.$$

W_{jt} ~wage set by household
L_t ~homogeneous aggregate labor
W_t ~wage rate of aggregate labor

Household Wage Decision

- Demand for Household's Specialized Labor:

$$h = D(w) = w^{-\frac{\lambda_w}{\lambda_w - 1}}$$

- Household Choice of real wage:

$$\max_{w,c} u(c) - z(h)$$
$$\text{subject to } c \leq wh,\ h = D(w)$$

- Substitute out for budget constraint and demand curve:

$$\max_{w,c} u(wD(w)) - z(D(w))$$

- First order condition:

$$u_c \times [wD'(w) + D(w)] = z' \times D'(w)$$

Household Wage Decision ...

- First order condition:

$$u_c \times [wD'(w) + D(w)] = z' \times D'(w)$$

or

$$wu_c \times [wD'(w) + D(w)] = z' \times wD'(w)$$

or

$$w = \frac{z'}{u_c} \times \frac{wD'(w)}{wD'(w) + D(w)}$$

Household Wage Decision ...

- First Order Condition:

$$w = \frac{z'}{u_c} \times \frac{wD'(w)}{wD'(w) + D(w)}$$

- Note: Household Marginal Cost (consumption value of a unit of leisure):

$$\frac{z'}{u_c} = \frac{\frac{d\text{utility}}{d\text{leisure}}}{\frac{d\text{utility}}{d\text{consumpion}}} = \frac{d\text{consumpion}}{d\text{leisure}}$$

- Note: With Constant Elasticity Utility Function:

$$\frac{wD'(w)}{wD'(w) + D(w)} = \lambda_w$$

- Conclude 'Wage Equals Markup Times Marginal Cost':

$$w = \lambda_w \frac{z'}{u_c}$$

...

Calvo-style Wage Setting:

- With Probability $1 - \xi_w$, i^{th} Household Sets Wage, W_{it}, Optimally, to $\tilde{W}_t$.
- With Probability ξ_w,

$$W_{i,t} = \pi_{t-1}\mu_{z^*}W_{i,t-1},$$

$$\mu_{z^*} \sim \text{ steady state growth rate of economy}$$

$$\mu_{z^*,1} = \frac{z_t^*}{z_{t-1}^*}, \; z_t^* = \Upsilon_t^{\frac{\alpha}{1-\alpha}} z_t$$

- First Order Condition:

$$E_{t-1}\sum_{l=0}^{\infty} (\xi_w\beta)^l \, h_{jt+l}\psi_{t+l}\left[\frac{\tilde{W}_t X_{t,l}}{P_{t+l}} - \lambda_w \frac{z_{h,t+l}}{\psi_{t+l}}\right] = 0.$$

- Households Attempt to Set Price (the wage) as a markup over marginal cost.

ψ_{t+l} : utility value of consumption (Multiplier on Budget Constraint)

$z_{h,t+l}$: Household Marginal utility of Leisure

$\frac{z_{h,t+l}}{\psi_{t+l}}$: Marginal Cost (in Consumption Units) of a Unit of Leisure

Monetary and Fiscal Policy

$$x_t = M_t/M_{t-1}$$

$$\begin{aligned}\hat{x}_{M,t} &= \rho_M \hat{x}_{M,t-1} + \varepsilon_{M,t} \\ \hat{x}_{z,t} &= \rho_{xz}\hat{x}_{z,t-1} + c_z\varepsilon_{z,t} + c_z^p\varepsilon_{z,t-1} \\ \hat{x}_{\Upsilon,t} &= \rho_{x\Upsilon}\hat{x}_{\Upsilon,t-1} + c_\Upsilon\varepsilon_{\Upsilon,t} + c_\Upsilon^p\varepsilon_{\Upsilon,t-1}\end{aligned}$$

- $\hat{x}_{M,t}$: response of monetary policy to a monetary policy shock, $\varepsilon_{M,t}/$
- $\hat{x}_{z,t}$: response of monetary policy to an innovation in neutral technology, $\varepsilon_{z,t}$.
- $\hat{x}_{\Upsilon,t}$: response of monetary policy to an innovation in capital embodied technology, $\varepsilon_{\Upsilon,t}$.
- Government has access to lump sum taxes, pursues a Ricardian fiscal policy.

Loan Market and Final Good Market Clearing Conditions, Equilibrium

- Financial intermediaries receive $M_t - Q_t + (x_t - 1)\, M_t$ from the household.
  - Lend all of their money to intermediate good firms, which use the funds to pay for H_t.
- Loan market clearing

$$W_t H_t = x_t M_t - Q_t.$$

- The aggregate resource constraint is

$$(1 + \eta(V_t))C_t + \Upsilon_t^{-1}\left[I_t + a(u_t)\bar{K}_t\right] \leq Y_t.$$

- We adopt a standard sequence-of-markets equilibrium concept.

The Firm - Specific Capital Model

- Firms own their own capital which can't be adjusted during the period.
  - Can only be increased or decreased over time by varying rate of investment.
- In all other respects, problem of intermediate good firm is same as before.
- Technology for accumulating physical capital by intermediate good firm i :

$$F(I_t(i), I_{t-1}(i)) = (1 - S\left(\frac{I_t(i)}{I_{t-1}(i)}\right))I_t(i),$$

$$\bar{K}_{t+1}(i) = (1-\delta)\bar{K}_t(i) + F(I_t(i), I_{t-1}(i)).$$

- Present discounted value of i^{th} intermediate good's cash flow:

$$E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t+j} \left\{ \begin{array}{c} P_{t+j}(i) y_{t+j}(i) - P_{t+j} R_{t+j} w_{t+j}(i) h_t(i) \\ -P_{t+j} \Upsilon_{t+j}^{-1} \left[I_{t+j}(i) + a\left(u_{t+j}(i)\right) \bar{K}(i)_{t+j} \right] \end{array} \right\}.$$

Firm Specific Capital Model...

- Timing:
  - Firms sees technology shock
  - Sets $P_t(i)$, subject to the Calvo frictions.
  - Also decides on $I_t(i)$ and $u_t(i)$.
- Time t monetary policy shock occurs, demand for the firm's product is realized.
  - Firm must satisfy demand.

Implications for Inflation

- Equations which characterize equilibrium for homogeneous and firm specific capital model are identical:

$$\Delta\hat{\pi}_t = E\left[\beta\Delta\hat{\pi}_{t+1} + \gamma\hat{s}_t|\Omega_t\right]$$

where

$$\gamma = \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}\chi$$

and

$$\chi = \left\{ \begin{array}{ll} 1 & \text{homogeneous capital model} \\ <1 & \text{firm-specific capital model} \end{array} \right.$$

- In the firm specific capita model, χ is a particular non-linear function of the parameters of the model
- Given γ, the two models are observationally equivalent with respect to aggregate prices and quantities.

Implications for Inflation...

$$\gamma = \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}\chi$$

- For example, our estimated of γ is

$$\gamma = .035$$

- Under homogeneous capital model ($\chi = 1$), this implies

$$\xi_p = .83 \text{ and } 1/(1-\xi_p) = 6$$

- For the firm specific capital model, $\chi = .031$, and

$$\xi_p = .36 \text{ and } 1/(1-\xi_p) = 1.6$$

Econometric Methodology

- Variant of limited information strategy used in CEE (2004).
  - Impose a subset of assumptions made in equilibrium model to estimate impulse response functions of ten key macroeconomic variables to the three shocks in our model.
  - Neutral technology shocks, capital embodied technology shocks and monetary policy shocks.
- Choose values for key parameters of structural model to minimize difference between estimated impulse response functions and analogous objects in model.

Identifying Assumptions

- Shocks to technology
  - Innovations to technology (both neutral and capital embodied) are only shocks that affect level of labor productivity in the long run.
  - Capital embodied technology shocks are the only shocks that affect price of investment goods relative to consumption goods in the long run. (Fisher (2003)).
  - Our equilibrium model is consistent with these assumptions
- Shocks to monetary policy
  - CEE (2004): there's exactly one shock - the monetary policy shock - that affects interest rate contemporaneously, over and above shocks that drive aggregate prices and quantities.
  - This assumption is satisfied in our equilibrium

Identification...

$$\underbrace{Y_t}_{10\times 1} = \begin{pmatrix} \Delta\ln(\text{relative price of investment}_t) \\ \Delta\ln(GDP_t/\text{Hours}_t) \\ \Delta\ln(GDP \text{ deflator}_t) \\ (\text{capacity utilization}_t) \\ \ln(\text{Hours}_t) \\ \ln(GDP_t/\text{Hours}_t) - \ln(W_t/P_t) \\ \ln(C_t/GDP_t) \\ \ln(I_t/GDP_t) \\ \text{Federal Funds Rate}_t \\ \ln(GDP \text{ deflator}_t) + \ln(GDP_t) - \ln(MZM_t) \end{pmatrix} = \begin{pmatrix} \underbrace{\Delta p_{It}}_{1\times 1} \\ \underbrace{\Delta a_t}_{1\times 1} \\ \underbrace{Y_{1t}}_{6\times 1} \\ \underbrace{R_t}_{1\times 1} \\ \underbrace{Y_{2t}}_{1\times 1} \end{pmatrix}$$

- Monetary Policy

$$R_t = f(\Omega_t) + \varepsilon_{Mt}.$$

- Function f is linear, Ω_t contains Y_{t-1}, Y_{t-2}, Y_{t-3}, Y_{t-4} and the only date t variables in Ω_t are $\{\Delta a_t, \Delta p_{It}, Y_{1t}\}$ and ε_{Mt} is orthogonal with Ω_t.

Estimating Parameters in the Model

- Partition Parameters into Three Groups.
  - Parameters set a priori (e.g., β, δ,...)
  - ζ_1: remaining parameters pertaining to the nonstochastic part of model

$$\zeta_2 = [\xi_w, \gamma, \sigma_a, b, S'', \epsilon]$$

  - ζ_2: parameters pertaining to stochastic part of the model
- Number of parameters, $\zeta = (\zeta_1, \zeta_2)$, to be estimated - 18
- Estimation Criterion
  - $\Psi(\zeta)$: mapping from ζ to model impulse responses
  - $\hat{\Psi}$: 592 impulse responses estimated using VAR
  - Estimation Strategy:

$$\hat{\zeta} = \arg\min_{\zeta} \left(\hat{\Psi} - \Psi(\zeta)\right)' V^{-1} \left(\hat{\Psi} - \Psi(\zeta)\right).$$

  - V: diagonal matrix with sample variances of $\hat{\Psi}$ along the diagonal.

Estimated Parameter Values, ζ_1

λ_f	ξ_w	γ	σ_a	b	S''	ϵ
1.01	.72	.035	2.01	.65	2.22	1.06

- ϵ a little low....
- b similar to other estimates in literature
- ξ_w wages reoptimized on average every 3.6 quarters
- Parameters important in subsequent discussion....
  - σ_a costly to vary utilization of capital
  - λ_f close to perfect competition
  - γ amazingly low! (similar to estimates reported in literature)

Estimated Parameters of Exogenous Shocks, ζ_2

ρ_M	σ_M	ρ_{μ_z}	σ_{μ_z}	ρ_{xz}	c_z	c_z^p	ρ_{μ_Υ}	σ_{μ_Υ}	$\rho_{x\Upsilon}$	c_Υ	c_Υ^p
.24	.26	.92	.06	.37	3.36	1.19	.21	.31	.67	.38	.26

Figure 10: Benchmark model – dynamic response to a monetary policy shock

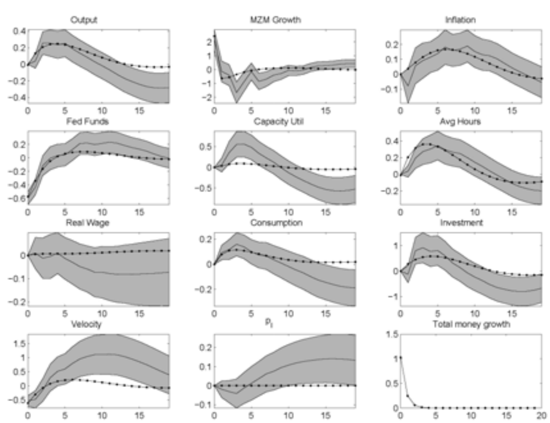
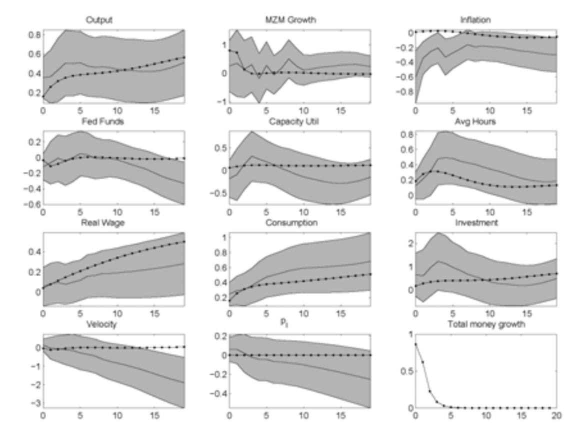
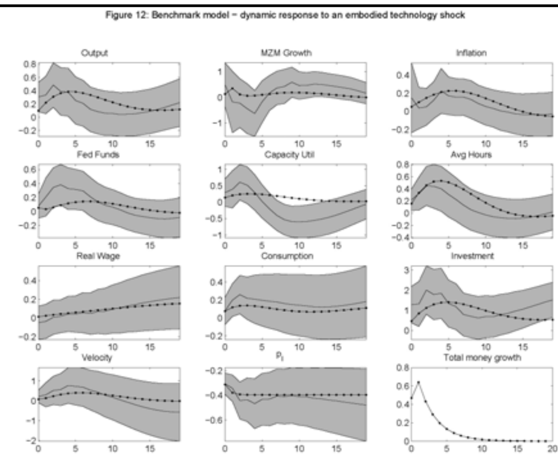


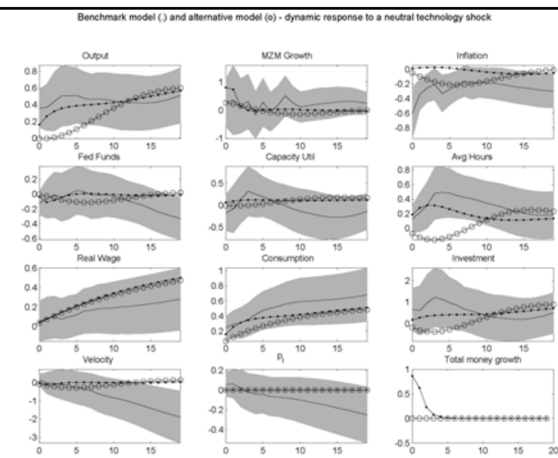
Figure 11: Benchmark model – dynamic response to a neutral technology shock

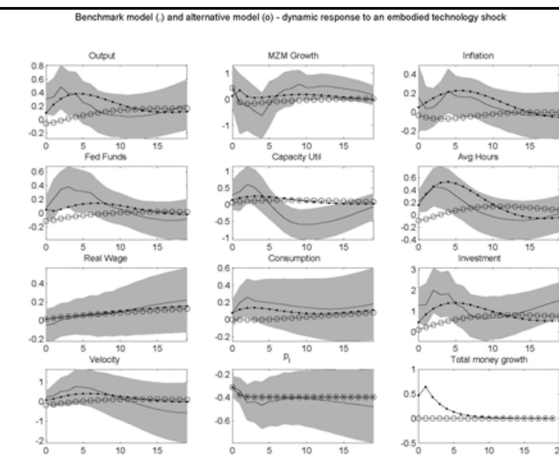




Monetary Policy and Technology Shocks

- How would the economy have responded to technology shocks if monetary policy had not been accommodative?





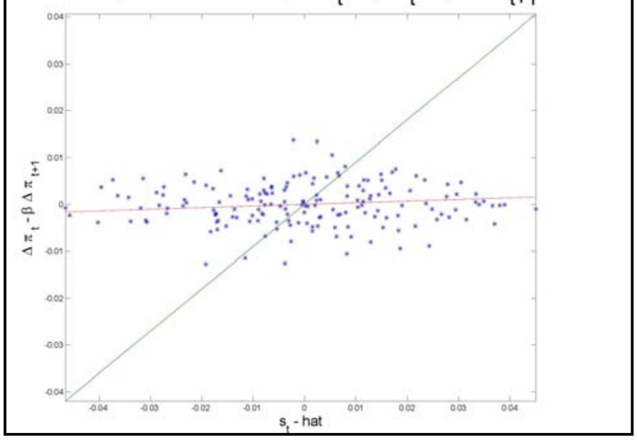
Understanding the Microeconomic Price Implications of the Model

- Reduced Form Parameter, γ:
  - Model: $\gamma = 0.035$ 'a 1 percent temporary rise in marginal cost leads to a 0.03% rise in price level'

$$\hat{\pi}_t \;=\; \beta E_t \hat{\pi}_{t+1} + \gamma \hat{s}_t$$

  - Direct Analysis of Macro Data Supports Low Estimate of γ
- Inflation Inertia:
  - Conventional Def: Sluggish Response of Inflation to (Monetary) Shocks
  - Conventional Solution: Identify Model Features that Slow Rise in Marginal Cost After Shock
  - Does not help with low γ!

Analysis of relation, $\Delta \pi_t = \gamma s_t + \beta \Delta \pi_{t+1}$



Microeconomic Price Implications of the Model....

- Homogeneous Capital Model:

$$\gamma \;=\; \frac{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)}{\xi_p}$$

$$\gamma \;=\; 0.035 \rightarrow \xi_p = 0.83,\; \frac{1}{1-\xi_p} = 5.8$$

- Homogeneous Capital Model:
  - Price Seems Not to Respond Much to Marginal Cost - Prices Must be VERY Sticky!
  - Striking Conflict Between Macro (γ) and Micro Evidence

Microeconomic Price Implications of the Model....

- Firm-specific Capital Model:

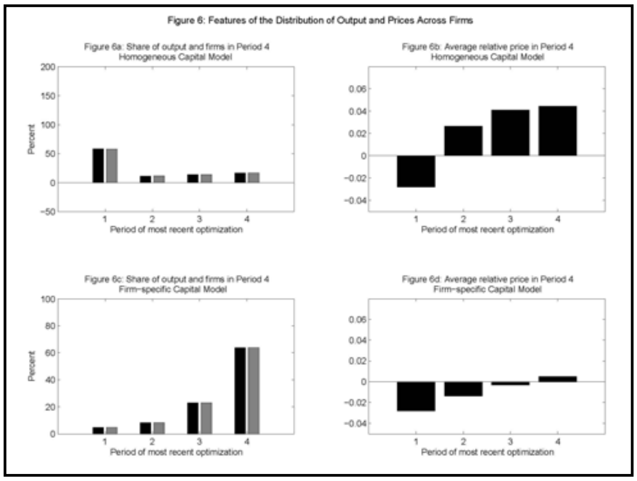
$$\gamma \;=\; \frac{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)}{\xi_p}\chi(\sigma_a, \lambda_f, \xi_p)$$

$$0.031 \;=\; \chi(\sigma_a = 2, \lambda_f = 1.01, \xi_p = 0.36)$$

- Firm-specific capital breaks Micro/Macro by introducing endogenous price stickiness
- How does it do it?

The Experiment

- Begin at steady state and assume there is an expansionary monetary policy shock in period 1.
- Period 1
  - Prices and output is the same for all firms.
- Period 2
  - $(1-\xi_p)$ firms re-optimize and implement new price, ξ_p do not.
- Period 3: there are 4 types of firms.
  - $(1-\xi_p)^2$ re-optimize in period 2 and 3,
  - ξ_p^2 don't re-optimize in either period 2 or period 3.
  - $(1-\xi_p)\xi_p$ re-optimized in period 2 but not in period 3.
  - $\xi_p(1-\xi_p)$ did not re-optimize in period 2 but did re-optimize in period 3
- In period s there are 2^{s-1} different firms.
- For each period s we calculated the distribution of output and prices across firms.



Micro Findings

- Homogeneous and Firm-Specific Capital Models are Indistinguishable from the Point of View of Aggregate Data
- Different Implications for
  - Degree of Price Stickiness in Micro Data
  - Dispersion of Prices and Output Across Firms
    - Homogenous Capital Model – Some Firms *Reduce* Output In Wake of Positive Technology Shock
- Firm-Specific Capital Model Seems to Have Better Micro Implications

A Check on the Econometric Procedure

- CKM Have Used an Example to Question Whether Estimated VARs are a Reliable Estimator of Impulse Response Functions to a Shock
- We Did an Experiment to Investigate Whether We Have the Problems They Describe

Basic Idea

- Generate Artificial Data from Economic Model, then Feed it to 10 Variable VAR Program Which Was Applied to Actual Data
- Wait!
  - Economic Model Only Has Three Shocks
  - Can't Fit 10 Variable VAR to Data From Model
- Solution
  - Empirical Procedure Recognizes We're Short on Shocks
  - Offers a Natural Solution

Background

- Recall, Structural Form of VAR:

$$A_0 Y_t = A(L) Y_{t-1} + e_t$$

- Reduced Form:

$$Y_t = B(L) Y_{t-1} + C e_t$$

where

$$B(L) = A_0^{-1} A(L),\ C = A_0^{-1}.$$

and

$$e_t = \left(\underbrace{e_{\Upsilon,t}}_{1\times 1} \ \underbrace{e_{z,t}}_{1\times 1} \ \underbrace{e_{1t}'}_{1\times 6} \ \underbrace{e_{Rt}}_{1\times 1} \ \underbrace{e_{2t}}_{1\times 1} \right)'$$

Background ...

- Can Write:

$$Ce_t = C_1 \begin{pmatrix} e_{\Upsilon,t} \\ e_{z,t} \\ e_{R_t} \end{pmatrix} + C_2 \begin{pmatrix} e_{1t} \\ e_{2,t} \end{pmatrix}$$

- So

$$Y_t = B(L) Y_{t-1} + C_1 \begin{pmatrix} e_{\Upsilon,t} \\ e_{z,t} \\ e_{R_t} \end{pmatrix} + C_2 \begin{pmatrix} e_{1t} \\ e_{2,t} \end{pmatrix}$$

- Stochastic Process for Y_t Can Be Decomposed Into Two Orthogonal Pieces:

$$Y_t = Y_t^{Model} + Y_t^{Other}$$

$$Y_t^{Model} = B(L) Y_{t-1}^{Model} + C_1 \begin{pmatrix} e_{\Upsilon,t} \\ e_{z,t} \\ e_{R_t} \end{pmatrix}$$

$$Y_t^{Other} = B(L) Y_{t-1}^{Other} + C_2 \begin{pmatrix} e_{1t} \\ e_{2,t} \end{pmatrix}$$

Background ...

$$Y_t = Y_t^{Model} + Y_t^{Other}$$

- Piece, Y_t^{Model}, Is Captured By the Equilibrium Model
- Piece, Y_t^{Other}, Is Left Out.
- Implications of the Analysis:
  - Data Generated From Model Corresponds to Y_t^{Model}
  - Data Generated From Model is Missing Y_t^{Other}.
- To Run VAR in Artificial Data
  - Generate Artificial Data From Economic Model, $Y_t^{Economic\ Model}$
  - Generate Y_t^{Other} From

$$Y_t^{Other} = B(L) Y_{t-1}^{Other} + C_2 \begin{pmatrix} e_{1t} \\ e_{2,t} \end{pmatrix}$$

  - Construct:

$$Y_t = Y_t^{Economic\ Model} + Y_t^{Other}.$$

  - Fit VAR to Y_t

Experiment

- Generate Artificial Data
  - Extremely Long Data Set to Get Plim (20,000 Observations)
  - Many Data Sets of Length 170 Each
- Feed Each Data Set to *Same* VAR Fit to US Data
- Compute Impulse Response Functions
  - Dotted Lines: Small Sample Means
  - Dashed Lines: Plims

Figure 16: Monetary policy shock

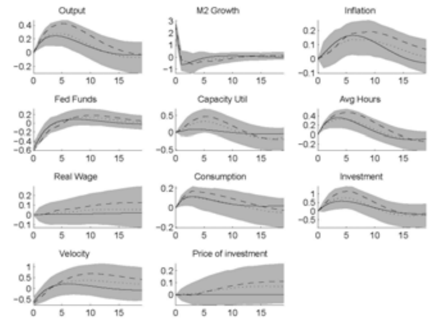


Figure 18: Neutral technology shock

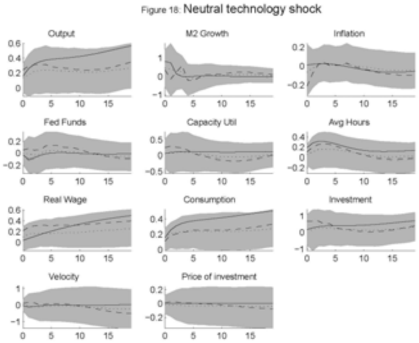
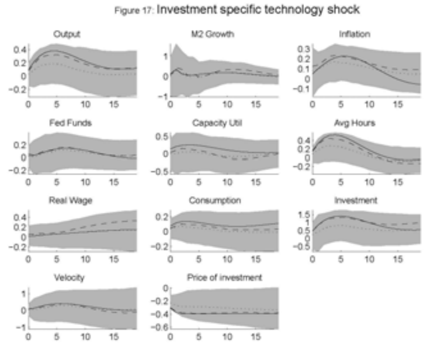


Figure 17: Investment specific technology shock



Summary

- We constructed a dynamic GE model of cyclical fluctuations.
- Given assumptions satisfied by our model, we identified dynamic response of key US economic aggregates to 3 shocks
  - Monetary Policy Shocks
  - Neutral Technology Shocks
  - Capital Embodied Technology Shocks
- These shocks account for substantial cyclical variation in output.
- Estimated GE model does a good job of accounting for response functions (However, Misses on Inflation Response to Neutral Shock)
- Have Made Progress on Micro/Macro Conflict

Summary…

- Calvo Sticky Prices and Wages Seems Like Good Reduced Form
  - What is the Underlying Structure?