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Vector Autoregressions

based on work with

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Background

- Structural Vector Autoregressions Address the Following Type of Question:
  - How Does the Economy Respond to Particular Economic Shocks?
  - The Answer to this Type of Question Can Be Very Useful in the Construction of Dynamic, General Equilibrium Models
- To be useful in practice, estimators of response functions must have good sampling properties.

Objective

- Investigate the Sampling Properties of SVARs, When Data are Generated by Estimated DSGE Models.

  - Bias Properties of Impulse Response Function Estimators

    - Bias: Mean of Estimator Minus True Value of Object Being Estimated

  - Accuracy of Standard Estimators of Sampling Uncertainty

  - Is Inference Sharp?

    - How *Large* is Sampling Uncertainty?

Objective ...

- Throughout, We Assume The Identification Assumptions Motivated by Economic Theory Are Correct
  - Example: ‘Only Shock Driving Labor Productivity in Long Run is Technology Shock’
- In Practice, Implementing VARs Involves Auxiliary Assumptions (Cooley-Dwyer)
  - Example: Lag Length Specification of VARs
  - Failure of Auxilliary Assumptions May Induce Distortions

Objective ...

- We Look at Two Classes of Identifying Restrictions

- Long-run identification

  – Exploit implications that some models have for long-run effects of shocks

- Short-run identification

  – Exploit model assumptions about the timing of decisions relative to the arrival of information.

Key Substantive Findings

- With Short Run Restrictions, SVARs Work *Remarkably* Well
  - Inference Sharp (Sampling Uncertainty Small), Essentially No Bias.
- With Long Run Restrictions,
  - For Model Parameterizations that Fit the Data Well, SVARs Work Well
    - Inference is correct but not necessarily sharp.
    - Sharpness is example specific.
  - Examples Can Be Found In Which There is Noticeable Bias
    - But, Analyst Who Looks at Standard Errors Would Not Be Misled

Technical Objectives

- Describe Estimation of VARs
- Identification
- Discuss Some Relevant Frequency Domain Concepts

Outline

- Analyze Performance of SVARs Identified with Long Run Restrictions
  - Reconcile Our Findings for Long-Run Identification with CKM
- Analyze Performance of SVARs Identified with Short Run Restrictions
- We Focus on the Question:
  - How do hours worked respond to a technology shock?

A Conventional RBC Model

- Preferences:

$$E_0 \sum_{t=0}^{\infty} \left(\beta\left(1+\gamma\right)\right)^t \left[\log c_t + \psi \log\left(1-l_t\right)\right].$$

- Constraints:

$$c_t + \left(1+\tau_x\right)\left[\left(1+\gamma\right)k_{t+1} - \left(1-\delta\right)k_t\right] \ \leq \ \left(1-\tau_{lt}\right)w_t l_t + r_t k_t + T_t.$$

$$c_t + \left(1+\gamma\right)k_{t+1} - \left(1-\delta\right)k_t \ \leq \ k_t^{\theta}\left(z_t l_t\right)^{1-\theta}.$$

- Shocks:

$$\begin{aligned} \Delta \log z_t &= \mu_Z + \sigma_z \varepsilon_t^z \\ \tau_{lt+1} &= \left(1-\rho_l\right)\bar{\tau}_l + \rho_l \tau_{lt} + \sigma_l \varepsilon_{t+1}^l \end{aligned}$$

- Information: Time t Decisions Made After Realization of All Time t Shocks

Long-Run Properties of RBC Model

- ε_t^z is only shock that has a permanent impact on output and labor productivity

$$a_t \equiv y_t/l_t.$$

- *Exclusion property:*

$$\lim_{j\to\infty}\left[E_t a_{t+j} - E_{t-1}a_{t+j}\right] = f\left(\varepsilon_t^z \text{ only}\right),$$

- *Sign property:*

f is an increasing function.

Parameterizing the Model

- Parameters:
  - Exogenous Shock Processes: We Estimate These
  - Other Parameters: Same as CKM

β	θ	δ	ψ	γ	$\bar{\tau}_x$	$\bar{\tau}_l$	μ_z
$0.98^{1/4}$	$\frac{1}{3}$	$1-(1-.06)^{1/4}$	2.5	$1.01^{1/4}-1$	0.3	0.243	$1.02^{1/4}-1$

- Baseline Specifications of Exogenous Shocks Processes:
  - Our Baseline Specification
  - Chari-Kehoe-McGrattan (July, 2005) Baseline Specification

Our Baseline Model (*KP Specification*):

- Technology shock process corresponds to Prescott (1986):

$$\Delta \log z_t \;=\; \mu_Z + 0.011738 \times \varepsilon_t^z.$$

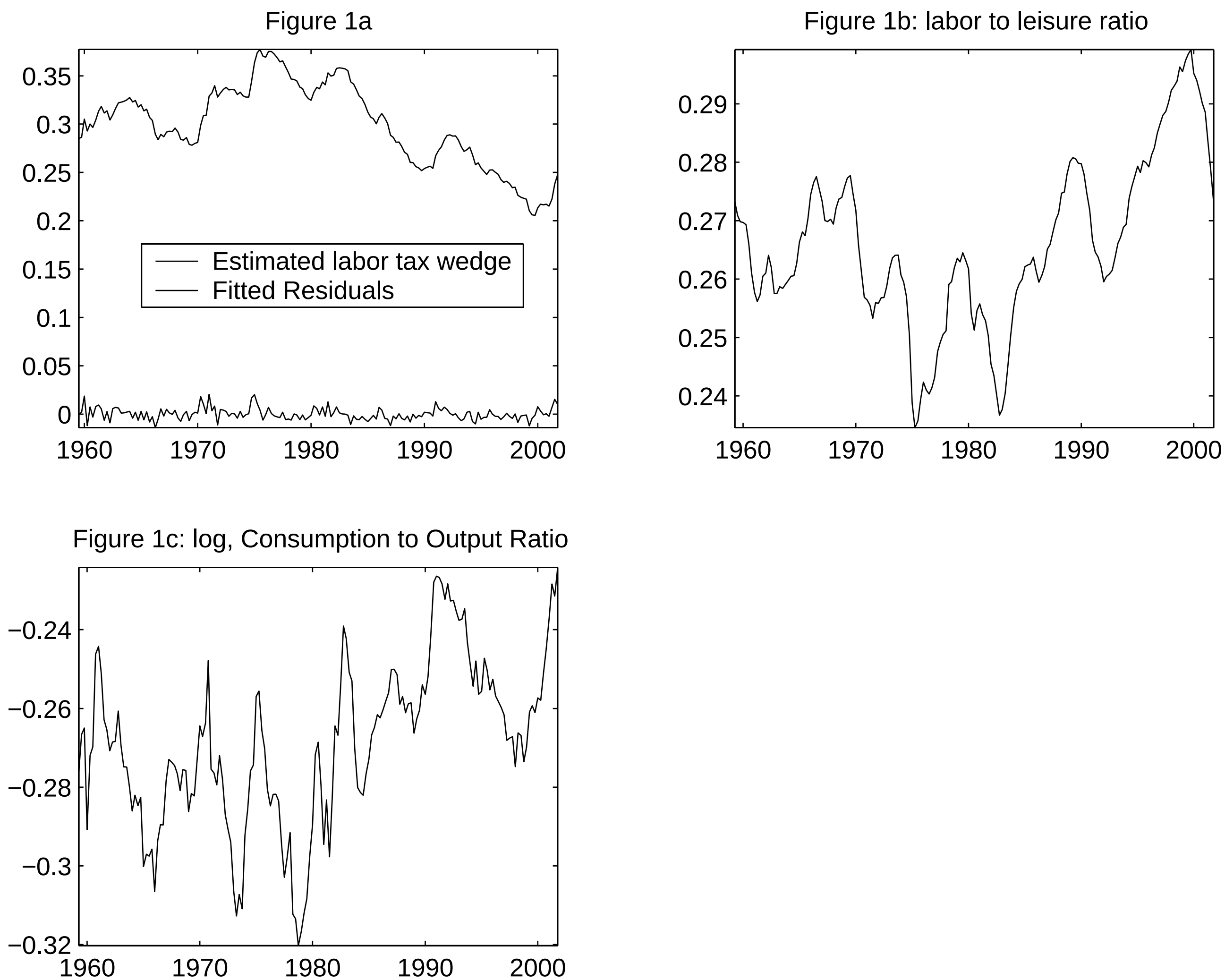
- Law of motion for Preference Shock, $\tau_{l,t}$:

$$\tau_{l,t} \;=\; 1 - \left(\frac{c_t}{y_t}\right)\left(\frac{l_t}{1-l_t}\right)\left(\frac{\psi}{1-\theta}\right) \text{ (Household and Firm Labor Fonc)}$$

$$\tau_{l,t} \;=\; \bar{\tau}_l + 0.9934 \times \tau_{l,t-1} + .0062 \times \varepsilon_t^l.$$

- Estimation Results Robust to Maximum Likelihood Estimation -
  - Output Growth and Hours Data
  - Output Growth, Investment Growth and Hours Data (here, τ_{xt} is stochastic)

Figure 1 - The Labor Tax Wedge and Its Components



CKM Baseline Model

- Exogenous Shocks: also estimated via maximum likelihood

$$\Delta \log z_t = 0.00516 + 0.0131 \times \varepsilon_t^z$$
$$\tau_{lt} = \bar{\tau}_l + 0.952\tau_{l,t-1} + 0.0136 \times \varepsilon_t^l.$$

- Note: the shock variances (particularly τ_{lt}) are very large compared with KP

- We Will Investigate Why this is so, Later

Using VARs to Estimate the Effects of a Positive Technology Shock

- VAR:

$$X_t = B_1 X_{t-1} + B_2 X_{t-2} + ... + B_p X_{t-p} + u_t,$$

$$Eu_t u_t' = V, \; u_t = Ce_t, \, Ee_t e_t' = I, \; CC' = V$$

$$X_t = \begin{pmatrix} \Delta \log a_t \\ \log l_t \\ x_t \end{pmatrix}, \; C = [C_1 \vdots C_2 \vdots C_3], \; e_t = \begin{pmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{pmatrix}, \; a_t = \frac{Y_t}{l_t}$$

- Impulse Response to Positive Technology Shock (e_{1t}):

$$X_t - E_{t-1} X_t = C_1 e_{1t}, \; E_t X_{t+1} - E_{t-1} X_{t+1} = B_1 C_1 e_{1t}$$

$$E_t X_{t+2} - E_{t-1} X_{t+2} = B_1^2 C_1 e_{1t} + B_2 C_1 e_{1t}$$

- Need: $B_1, ..., B_p, C_1$.

Identification Problem

- From Applying OLS To Both Equations in VAR, We 'Know':

$$B_1, ..., B_p,\ V$$

- Problem, Need first Column of C, C_1
- Restrictions (Bivariate Case): three equations in four unknowns

$$CC' = V$$

- Identification Problem:

Not Enough Restrictions to Pin Down C_1

- Need More Restrictions

Identification Problem ...

- Two Key Properties of DGP:
  - Long-Run Exclusion Restriction:

$$\lim_{j\to\infty}\left[E_t a_{t+j} - E_{t-1} a_{t+j}\right] = f(\varepsilon_t^z \text{ only})$$

  - Sign Restriction:

$$f \text{ increasing in } \varepsilon_t^z$$

- These Properties Provide Sufficient Additional Restrictions to Pin Down C_1

Characterizing Restrictions

- Note:

$$E_t[a_{t+1}] - E_{t-1}[a_{t+1}] = [E_t \Delta a_{t+1} - E_{t-1}\Delta a_{t+1}] + [\Delta a_t - E_{t-1}\Delta a_t]$$

- Then ($p = 1$)

$$E_t[a_{t+1}] - E_{t-1}[a_{t+1}] \;=\; (1,0)\,[B+I]\,Ce_t$$

$$E_t[a_{t+2}] - E_{t-1}[a_{t+2}] \;=\; (1,0)\left[B^2+B+I\right]Ce_t$$

$$E_t[a_{t+j}] - E_{t-1}[a_{t+j}] \;=\; (1,0)\left[B^j+B^{j-1}+...+B^2+B+I\right]Ce_t$$

as $j \longrightarrow \infty$:

$$\begin{aligned}
&\lim_{j\to\infty} E_t[a_{t+j}] - E_{t-1}[a_{t+j}] \\
=\;& \lim_{j\to\infty}(1,0)\left[...+B^j+B^{j-1}+...+B^2+B+I\right]Ce_t \\
=\;& (1,0)\,[I-B]^{-1}\,Ce_t
\end{aligned}$$

Characterizing Restrictions ...

- As $j \rightarrow \infty$ (for arbitrary p) :

$$\lim_{j\rightarrow\infty} E_t[a_{t+j}] - E_{t-1}[a_{t+j}] \; = \; (1,0)\,[I - B(1)]^{-1}\, Ce_t$$

Characterizing Restrictions ...

- The VAR:

$$X_t \;=\; B_1 X_{t-1} + B_2 X_{t-2} + ... + B_p X_{t-p} + u_t$$

- Identification: Solve for C Such that -

$$\text{(exclusion restriction)}\;\; (1,2)\;\; \text{element of}\;\; [I - B(1)]^{-1}\, C = \begin{bmatrix} \text{number} & 0, ..., 0 \\ \text{numbers} & \text{numbers} \end{bmatrix}$$

$$\text{(sign restriction)}\;\; (1,1)\;\; \text{element of}\;\; [I - B(1)]^{-1}\, C \text{ is } positive$$

$$CC' = V$$

- There Are Many C That Satisfy These Constraints. All Have the Same C_1.

Proof of Uniqueness of C_1

- Let
$$D \equiv [I - B(1)]^{-1} C$$
$$\text{so, } DD' = [I - B(1)]^{-1} V [I - B(1)']^{-1} \equiv S_0 \text{ (Since } CC' = V\text{)}$$
- Exclusion Restriction Requires:
$$D = \begin{bmatrix} d_{11} & 0 \\ D_{21} & D_{22} \end{bmatrix}$$
- So
$$DD' = \begin{bmatrix} d_{11}^2 & d_{11} D_{21}' \\ D_{21} d_{11} & D_{21} D_{21}' + D_{22} D_{22}' \end{bmatrix} = \begin{bmatrix} S_0^{11} & S_0^{21\prime} \\ S_0^{21} & S_0^{22} \end{bmatrix}.$$
- Sign Restriction:
$$d_{11} > 0.$$
- Then, First Column of D Uniquely Pinned Down:
$$d_{11} = \sqrt{S_0^{11}},\ D_{21} = S_0^{21} / d_{11}$$
- First Column of C Uniquely Pinned Down:
$$C_1 = [I - B(1)]\, D_1.$$

Five Results from the Frequency Domain

- Time Series Representation:

$$Y_t = D(L)e_t, \; Ee_te_t' = V$$

$$= \left[D_0 + D_1 L + D_2 L^2 + ...\right] e_t.$$

- Define:

$$S_Y(z) = D(z)V D(z^{-1}), \text{ where } z \text{ is a variable.}$$

- **Result #1**:

$$S_Y(z) = C(0) + C(1)z + C(2)z^2 + C(3)z^3 + ...$$

$$+C(-1)z^{-1} + C(-2)z^{-2} + C(-3)z^{-3} + ...$$

where

$$C(\tau) \equiv EY_tY_{t-\tau}'$$

- Proof: do the multiplication and collect terms in powers of z.

- Example:

$$Y_t = D_0 e_t + D_1 e_{t-1} = [D_0 + D_1 L]\, e_t$$

$$C(0) \equiv EY_t Y_t' = D_0 V D_0' + D_1 V D_0'$$

$$C(\tau) \equiv EY_t Y_{t-\tau}' = \begin{cases} D_1 V D_0' & \tau = 1 \\ 0 & \tau > 1 \\ D_1 V D_0' & \tau = -1 \\ 0 & \tau < -1 \end{cases},$$

- Note,

$$\begin{aligned} & D(z)\, V D\left(z^{-1}\right) \\ = & \; [D_0 + D_1 z]\, V \left[D_0' + D_1' z^{-1}\right] \\ = & \; D_0 V D_0' + D_1 V D_1' \\ & + D_1 V D_0' z + D_0 V D_1 z^{-1} \\ \\ = & \; C(0) + C(1) z + C(-1) z^{-1}. \end{aligned}$$

- **Result #2**

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-i\omega h} d\omega = \begin{cases} 1 & h = 0 \\ 0 & h \neq 0. \end{cases}$$

- Proof:
  - Result Obvious for $h = 0$. Consider $h \neq 0$
  - Note:

$$\begin{aligned} e^{-i\omega h} &= \cos(-\omega h) + i\sin(-\omega h) \\ &= \cos(\omega h) - i\sin(\omega h) \end{aligned}$$

  - Note:

$$\sin(\pi k) = 0, \text{ for all integer } k.$$

  - Then,

$$\begin{aligned} \int_{-\pi}^{\pi} e^{-i\omega h} d\omega &= \frac{-1}{ih}\left[e^{-i\pi h} - e^{i\pi h}\right] \\ &= \frac{-1}{ih}\left[\cos(\pi h) - i\sin(\pi h) - (\cos(\pi h) + i\sin(\pi h))\right] \\ &= \frac{2}{h}\sin(\pi h) = 0 \end{aligned}$$

- Result #1 and #2 Imply **Result #3**:

$$C(\tau) \;=\; \frac{1}{2\pi}\int_{-\pi}^{\pi} S_Y(e^{-i\omega})e^{i\omega\tau}d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} D(e^{-i\omega})VD(e^{i\omega})'e^{i\omega\tau}d\omega.$$

- The Effects of Filtering
  – Consider the Filtered Data:

$$\tilde{Y}_t \;=\; F(L)Y_t,\; F(L) = \sum_{j=-\infty}^{\infty} F_j L^j$$

  – From Result #3:

$$C_{\tilde{Y}}(\tau) \;=\; \frac{1}{2\pi}\int_{-\pi}^{\pi} F(e^{-i\omega})S_Y(e^{-i\omega})F(e^{i\omega})'e^{i\omega\tau}d\omega$$

– A Filter of Particular Interest (the Band Pass Filter):

$$F_D(L), \text{ such that } F_D\left(e^{-i\omega}\right) = \begin{cases} 1 & \omega \in D \equiv \{\omega : \omega \in [a,b] \cup [-b,-a]\} \\ 0 & \text{otherwise} \end{cases}$$

– Then,

$$C_{\tilde{Y}}(\tau) = \frac{1}{2\pi}\int_{-\pi}^{\pi} F(e^{-i\omega})S_Y(e^{-i\omega})F(e^{i\omega})'e^{i\omega\tau}d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-b}^{-a} S_Y(e^{-i\omega})e^{i\omega\tau}d\omega + \int_{a}^{b} S_Y(e^{-i\omega})e^{i\omega\tau}d\omega\right]$$

– Band Pass Filter ‘Kills’ All Components of Y_t with $\omega \notin [a,b] \cup [-b,-a]$.

– Finding the Time Domain Representation of the Band Pass Filter Using Result #2

$$\begin{aligned} F_D(L) \;=\; & F_{D,0} + F_{D,1}L + F_{D,2}L^2 + \ldots \\ & + F_{D,-1}L^{-1} + F_{D,-2}L^{-2} + \ldots \end{aligned}$$

– By Result #2:

$$F_{D,\tau} = \frac{1}{2\pi}\int_{-\pi}^{\pi} F_D\left(e^{-i\omega}\right) e^{i\omega\tau} d\omega$$

– By Definition of Band-Pass Filter:

$$F_{D,\tau} \;=\; \frac{1}{2\pi}\int_{a}^{b} e^{i\omega\tau} d\omega + \frac{1}{2\pi}\int_{-b}^{-a} e^{i\omega\tau} d\omega$$

$$=\; \frac{1}{2\pi}\frac{1}{i\tau}\left[e^{i\omega\tau}\mid_a^b + e^{i\omega\tau}\mid_{-b}^{-a}\right]$$

Five Results from the Frequency Domain ...

$$
\begin{aligned}
F_{D,\tau} &= \frac{1}{2\pi}\int_a^b e^{i\omega\tau}d\omega + \frac{1}{2\pi}\int_{-b}^{-a} e^{i\omega\tau}d\omega \\
&= \frac{1}{2\pi}\frac{1}{i\tau}\left[e^{i\omega\tau}\mid_a^b + e^{i\omega\tau}\mid_{-b}^{-a}\right] \\
&= \frac{1}{2\pi}\frac{1}{i\tau}\left[e^{ib\tau} - e^{ia\tau} + \left(e^{-ia\tau} - e^{-ib\tau}\right)\right] \\
&= \frac{1}{2\pi}\frac{1}{i\tau}[\cos(b\tau) + i\sin(b\tau) - (\cos(a\tau) + i\sin(a\tau)) \\
&\quad + \cos(a\tau) - i\sin(a\tau) - \cos(b\tau) + i\sin(b\tau)] \\
&= \frac{1}{2\pi}\frac{1}{i\tau}[2i\sin(b\tau) - 2i\sin(a\tau)] \\
&= \frac{1}{\pi}\left[\frac{\sin(b\tau) - \sin(a\tau)}{\tau}\right]
\end{aligned}
$$

– Note: Band-Pass Filter Symmetric, Infinite Ordered

– Time-Domain Application of Band-Pass Filter:

$$F_D(L)\,x_t = F_{D,0}x_t + F_{D,1}x_{t+1} + F_{D,2}x_{t+2} + ...$$

$$+F_{D,-1}x_{t-1} + F_{D,-2}x_{t-2} + ...$$

$$= \sum_{\tau=-\infty}^{\infty} F_{D,\tau}x_{t-\tau}.$$

– In Practice, Have Only Finite Data

– Application of Two Band Pass Filters

∗ Suppose $D \cap D' = \varnothing$, Then **Result #4:**

$$F_D(L)Y_t \;\perp\; F_{D'}(L)Y_t.$$

∗ To See Why, Consider:

$$Z_t \;=\; \begin{pmatrix} F_D(L)Y_t \\ F_{D'}(L)Y_t \end{pmatrix} = \begin{pmatrix} F_D(L) \\ F_{D'}(L) \end{pmatrix} D(L)e_t,$$

∗ So

$$C_Z(\tau) \;=\; \frac{1}{2\pi}\int_{-\pi}^{\pi} \begin{bmatrix} F_D(e^{-i\omega})S_Y(e^{-i\omega})F_D(e^{i\omega})' & F_D(e^{-i\omega})S_Y(e^{-i\omega})F_{D'}(e^{i\omega})' \\ F_{D'}(e^{-i\omega})S_Y(e^{-i\omega})F_D(e^{i\omega})' & F_{D'}(e^{-i\omega})S_Y(e^{-i\omega})F_{D'}(e^{i\omega})' \end{bmatrix} e^{i\omega\tau}d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \begin{bmatrix} F_D(e^{-i\omega})S_Y(e^{-i\omega})F_D(e^{i\omega})' & 0 \\ 0 & F_{D'}(e^{-i\omega})S_Y(e^{-i\omega})F_{D'}(e^{i\omega})' \end{bmatrix} e^{i\omega\tau}d\omega$$

∗ Note that the Upper Right and Lower Left Blocks Are Zero.

$*$ Now Supppose

$$D \cap D' \; = \; \varnothing \text{ and } D \cup D' = [-\pi, \pi].$$

$*$ Then, **result #5**

$$F_D(L)Y_t + F_{D'}(L)Y_t \; = \; Y_t,$$

Represents an Orthogonal Decomposition of Y_t.

- Interpretation of Spectral Density:

  – Provides an Orthogonal Decomposition of Y_t By Frequency.
    - $S_Y(e^{-i\omega})$ is 'Variance of Process at Frequency, ω'

  – Closely Related Spectral Decomposition Theorem: Any Covariance Stationary Process Can Be Written:

$$Y_t = \int_0^{\pi} [a(\omega)\cos(\omega t) + b(\omega)\sin(\omega t)]\, d\omega.$$

  – cos and sin are periodic with period 2π

$$\begin{aligned} \cos(\omega t) &= \cos(\omega t'), \\ \omega t' &= \omega t + 2\pi, \\ t' - t &= \frac{2\pi}{\omega}. \end{aligned}$$

  – So, Frequency ω Corresponds to Period $2\pi/\omega$.

- This Completes 'Tour' of Frequency Domain!

Back to VARs

- The VAR:

$$X_t \;=\; B_1 X_{t-1} + B_2 X_{t-2} + ... + B_p X_{t-p} + u_t,\; u_t = C\varepsilon_t,\; CC' = V$$

$$D \;\equiv\; [I - B(1)]^{-1} C.$$

- Identification:

$$\text{(exclusion restriction) } (1,2) \text{ element of } D = \begin{bmatrix} \text{number} & 0, ..., 0 \\ \text{numbers} & \text{numbers} \end{bmatrix}$$

$$\text{(sign restriction) } (1,1) \text{ element of } D \text{ is } positive$$

- To Find C_1, Solve:

$$DD' = [I - B(1)]^{-1} V \left[I - B(1)'\right]^{-1},\; C_1 = [I - B(1)]\, D_1$$

Frequency Zero Spectral Density

- Note:
$$DD' = [I - (B_1 + ... + B_p)]^{-1} V \left[I - (B_1 + ... + B_p)'\right]^{-1} = S_0$$
- What is S_0?
- Spectral Density of Y_t (result #1):
$$\begin{aligned} S_Y(e^{-i\omega}) &= \left[I - B(e^{-i\omega})e^{-i\omega}\right]^{-1} V \left[I - B(e^{i\omega})'e^{i\omega}\right]^{-1} \\ &= C(0) + C(1)e^{-i\omega} + C(2)e^{-i2\omega} + C(3)e^{-i3\omega} + ... \\ &\quad +C(-1)e^{i\omega} + C(-2)e^{i2\omega} + C(-3)e^{i3\omega} + ... \end{aligned}$$
- So, S_0 is 'Zero Frequency Spectral Density of Y_t':

$$\begin{aligned} S_0 &= S_Y(e^{-i\times 0}) = \left[I - B(e^{-i\times 0})e^{-i\times 0}\right]^{-1} V \left[I - B(e^{i\times 0})'e^{i\times 0}\right]^{-1} = \sum_{k=-\infty}^{\infty} C(k), \\ C(k) &= EY_t Y'_{t-k}. \end{aligned}$$

Frequency Zero Spectral Density

- So, S_0 is 'Zero Frequency Spectral Density of Y_t':

$$S_0 = S_Y(e^{-i\times 0}) = \left[I - B(e^{-i\times 0})e^{-i\times 0}\right]^{-1} V \left[I - B(e^{i\times 0})' e^{i\times 0}\right]^{-1} = \sum_{k=-\infty}^{\infty} C(k),$$

$$C(k) = EY_t Y_{t-k}'.$$

- Note:

$$DD' = [I - (B_1 + ... + B_p)]^{-1} V \left[I - (B_1 + ... + B_p)'\right]^{-1} = S_0$$

- An Alternative Way to Compute D_1 (and, hence, C_1) Is to Use a Different Estimator of S_0

$$S_0 = \sum_{k=-r}^{r} |1 - \frac{k}{r}| \hat{C}(k), \quad \hat{C}(k) = \frac{1}{T} \sum_{t=k}^{T} EY_t Y_{t-k}'$$

Frequency Zero Spectral Density ...

- Modified SVAR Procedure Similar to Extending Lag Length, But Non-Parametric

Experiments with Estimated Models

- Simulate 1000 data sets, each of length 180 observations, using DSGE model as Data Generating Mechanism.

- On Each Data Set: Estimate a four lag VAR.

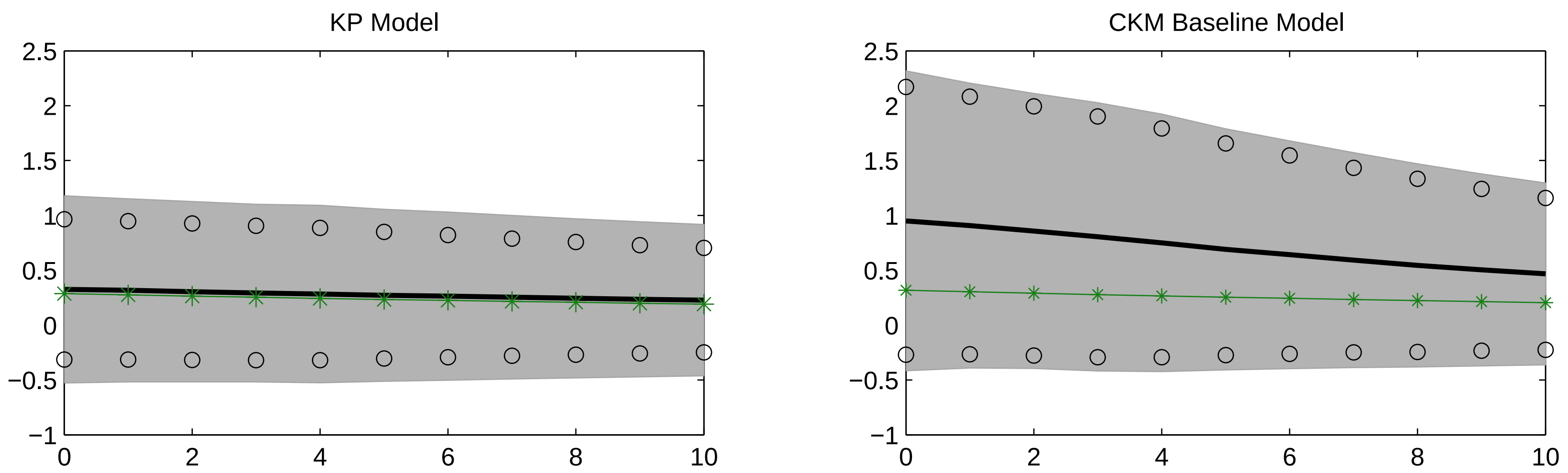
  – Report Mean Impulse Response Function Across 1000 Synthetic data sets (Solid, Black Line)

  – Report Mean, Plus/Minus Two Standard Deviations of Estimator (Grey area)

  – Report Mean of Econometrician's Confidence Interval Across 1000 synthetic data sets (Circles)

Response of Hours to A Technology Shock

Long−Run Identification Assumption



Results: Long Run Restrictions

- KP Model: VAR based analysis is reliable.
  - Bias Properties Excellent
  - Sampling Uncertainty Substantial,
    - But, Econometrician Would Know it
    - Econometrician Does Understate Slightly the Sampling Uncertainty

- CKM Baseline Model
  - Substantial bias
  - But,
    - Substantial Sampling Uncertainty
    - Econometrician Would Know It
      - Would See High Standard Errors on Average
      - Would Not Make Sharp Inferences

Diagnosing the Results

- What is Going on in CKM Example: Why is There Bias?
  - Problem Lies in Difficulty of Estimating the Sum of VAR Coefficients.
    - Recall:

$$C_1 = f_{LR}(V, B_1 + ... + B_p)$$

    - Regressions Only Care About $B_1 + ... + B_q$ If There is Lots of Power at Low Frequencies

- CKM Example Would Have Had Less Bias if:
  - If VAR Was Better at Low-Frequency Part of Estimation
  - If there Were More Low-Frequency Power in CKM Example

Sims' Approximation Theorem

- Suppose that the True VAR Has the Following Representation:

$$Y_t \ = \ B(L)Y_{t-1} + u_t, \ u_t \perp Y_{t-s}, \ s > 0.$$

- Econometrician Estimates Finite-Parameter Approximation to $B(L)$:

$$Y_t \ = \ \hat{B}(L;a)Y_{t-1} + \hat{u}_t$$

  – Concern: $\hat{B}(L;a)$ May Have Too Few Lags

  – How Does Specification Error Affect Inference About Impulse Responses?

- OLS Estimation Focuses on Residual:

$$\begin{aligned}\hat{u}_t &= Y_t - \hat{B}(L;a)Y_{t-1} \\ &= \left[B(L) - \hat{B}(L;a)\right] Y_{t-1} + u_t\end{aligned}$$

- By Orthogonality of u_t and past Y_t :

$$\begin{aligned}Var(\hat{u}_t) &= Var\left(\left[B(L) - \hat{B}(L;a)\right] Y_{t-1}\right) + V \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left[B(e^{-i\omega}) - \hat{B}\left(e^{-i\omega}\right)\right] S_Y(e^{-i\omega})\left[B(e^{i\omega})' - \hat{B}\left(e^{i\omega}\right)'\right] d\omega + V\end{aligned}$$

$$\begin{aligned}B(e^{-i\omega}) &= B_0 + B_1 e^{-i\omega} + B_2 e^{-2i\omega} + ... \\ \hat{B}(e^{-i\omega}) &= \hat{B}_0 + \hat{B}_1 e^{-i\omega} + \hat{B}_2 e^{-2i\omega} + ... + \hat{B}_p e^{-pi\omega}\end{aligned}$$

Sims' Approximation Theorem ...

- In Population, $\hat{B}$, $\hat{V}$ Chosen to Solve (Sims, 1972)

$$\hat{V} = \min_{\hat{B}} \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[B(e^{-i\omega}) - \hat{B}\left(e^{-i\omega}\right)\right] S_Y(e^{-i\omega}) \left[B(e^{i\omega})' - \hat{B}\left(e^{i\omega}\right)'\right] d\omega + V$$

  – With No Specification Error $\hat{B}(L) = B(L)$, $\hat{V} = V$

  – With Short Lag Lengths,

    ∗ $\hat{V}$ Accurate

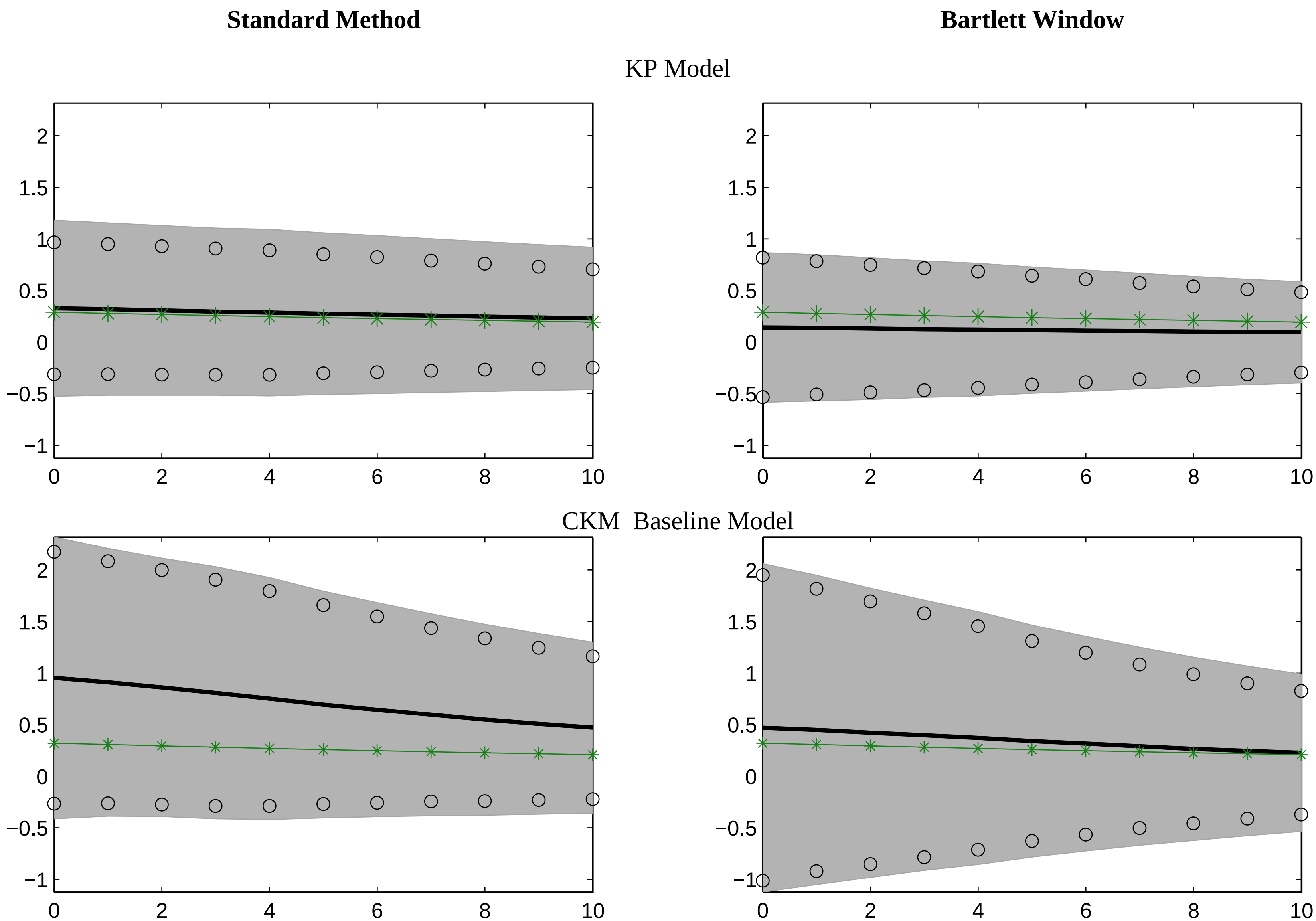
    ∗ $\hat{B}_1 + ... + \hat{B}_p$ Accurate Only By Chance (i.e., if $S_Y(e^{-i0})$ large)

    ∗ No Reason to Expect $\hat{S}_0$ to be Accurate

Modified Long-run SVAR Procedure

- Replace $\hat{S}_0$ Implicit in Standard SVAR Procedure, with Non-parametric Estimator of S_0

The Importance of Frequency Zero



The Importance of Power at Low Frequencies

- Preference Shock in CKM Example:

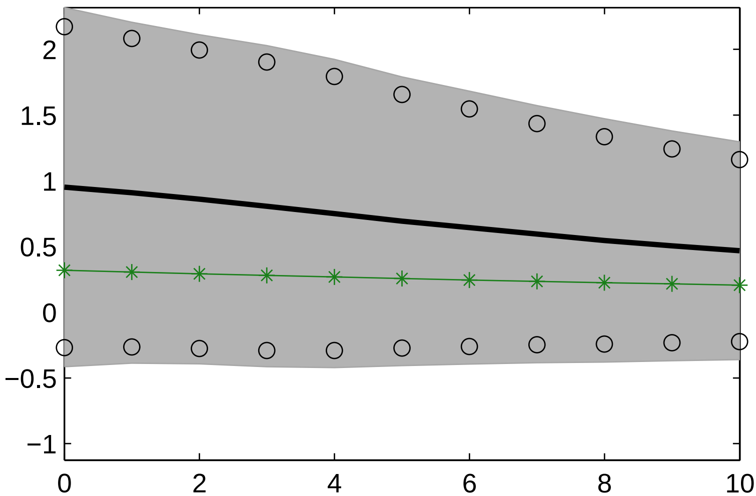
$$\tau_{lt} = \bar{\tau}_l + 0.952\tau_{l,t-1} + 0.0136 \times \varepsilon_t^l.$$

- Replace it with:

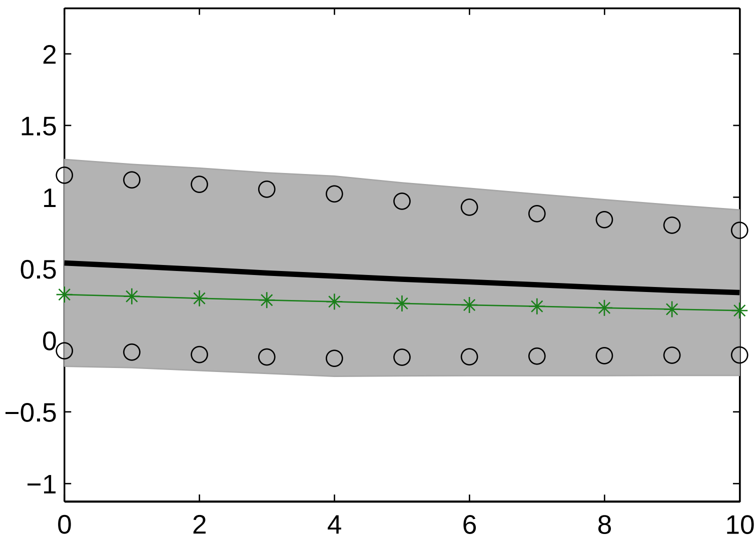
$$\tau_{lt} = \bar{\tau}_l + 0.995\tau_{l,t-1} + \sigma \times \varepsilon_t^l,$$

where

σ adjusted so Variance(τ_{lt}) Unchanged



CKM Baseline Model except $\rho_l = 0.995$ with la



Reconciling with CKM

- CKM Conclude Long-run SVARs Not Fruitful for Building DSGE Models.

- We Disagree: Two Reasons

  - The Data *Overwhelmingly* Reject CKM's Parameterization

  - Even if the World *Did* Correspond to One of CKM's Examples, No Econometrician Would Be Misled

    - A Feature of CKM Examples is That Econometrician's Standard Errors are Huge

CKM Baseline Model is Rejected by the Data

- CKM estimate their Baseline Model using MLE with Measurement Error.
  - Let

$$Y_t = (\Delta \log y_t, \ \log l_t, \ \Delta \log i_t, \ \Delta \log G_t)',$$

  - Observer Equation:

$$Y_t = X_t + u_t, \ E u_t u_t' = R,$$

R is a diagonal matrix,

u_t : 4×1 vector of iid measurement error,

X_t : model implications for Y_t

CKM Baseline Model is Rejected by the Data ...

- CKM Allow for Four Shocks

$$(\tau_{l,t}, z_t, \tau_{xt}, g_t)$$

.

$$G_t \;=\; g_t z_t$$

- CKM fix the elements on the diagonal of R to equal $1/100 \times Var(Y_t)$

- For Purposes of Estimating the Baseline Model, Assume:

$$g_t \;=\; \bar{g},\; \tau_{xt} = \tau_x.$$

- So,

$$\Delta \log G_t \;=\; \Delta \log z_t + \text{small measurement error}_t \;.$$

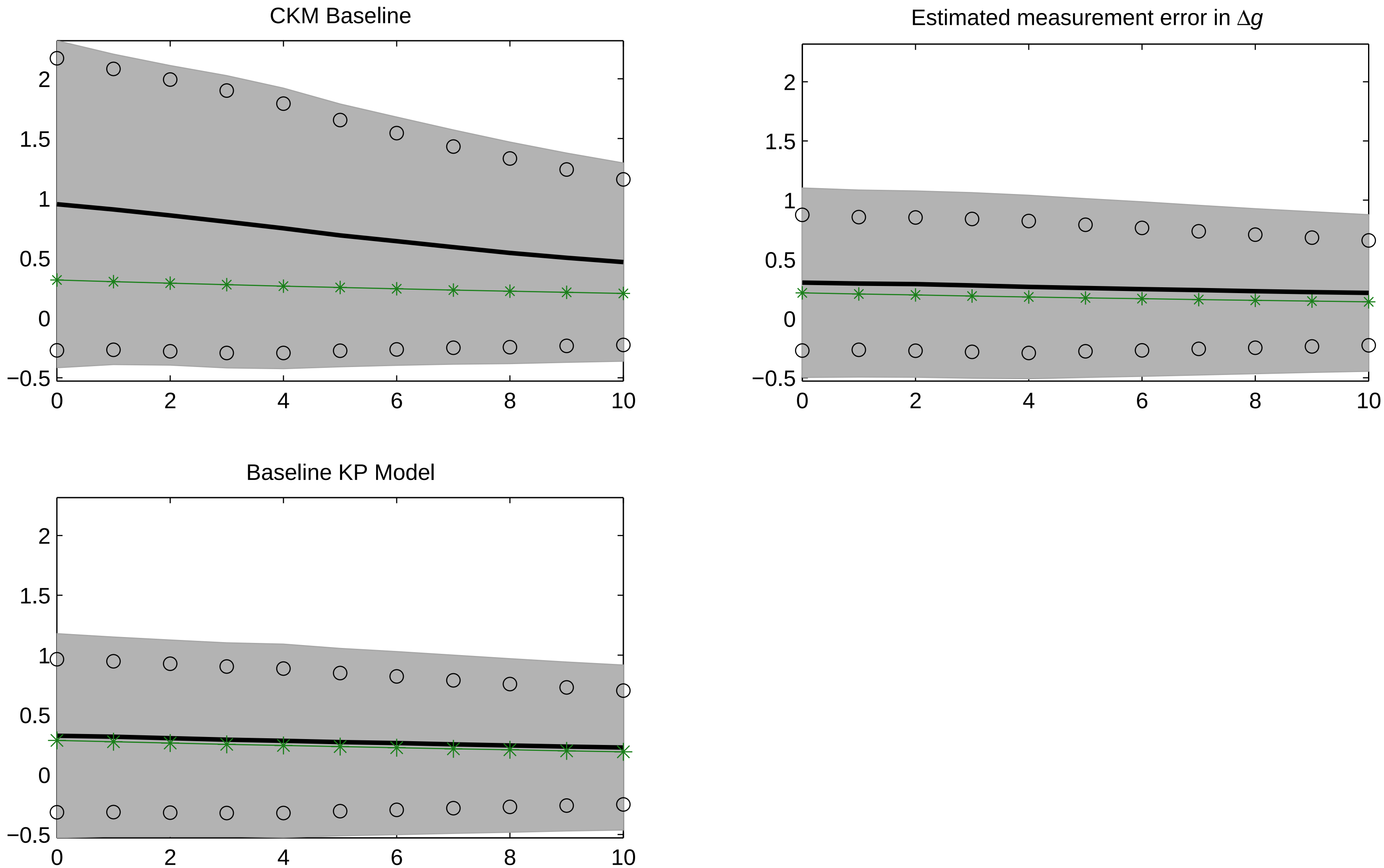
CKM Baseline Model is Rejected by the Data ...

- Overwhelming Evidence Against CKM Baseline Model

		Likelihood Ratio Statistic
	Likelihood Value	(degrees of freedom)
Estimated model	-328	
Freeing Measurement Error on $g = z$	2159	4974 (1)
Freeing All Four Measurement Errors	2804	6264 (4)

- Evidence of Bias in Estimated CKM Model Reflects CKM Choice of Measurement Error
  - Free Up Measurement error on $g = z$
    - Produces Model With Good Bias Properties: Similar to KP Benchmark Model

The Role of Δg



Alternate CKM Model With Government Spending Also Rejected

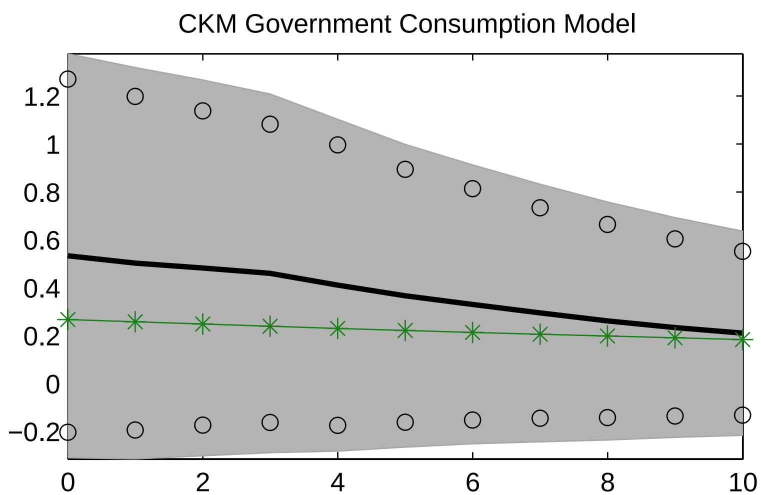
- CKM Model With G_t:

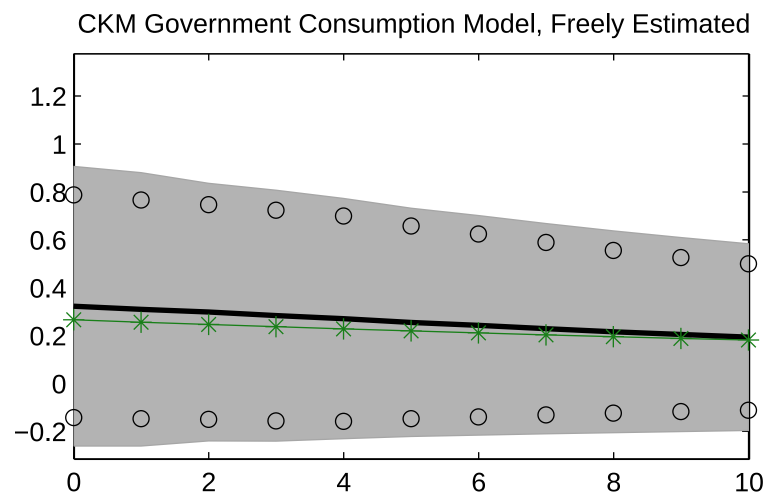
$$G_t = g_t z_t$$

g_t First Order Autoregression

- Model Estimated Holding Measurement Error Fixed As Before.
  - Resulting Model Implies Noticeable Bias in SVARs
  - But, Sampling Uncertainty is Big and Econometrician Would Know it
  - When Restriction on Measurement Error is Dropped Resulting Model Implies Bias in SVARs Small

The Role of Government Spending





LLF = 2695.46

LLF = 2842.96

Likelihood Ratio Statistic: 295 with 4 degrees of freedom

CKM Assert that SVARs Perform Poorly for ‘Large’ Range of Parameter Values

- Problem With CKM Assertion
  - Allegation Applies only to Parameter Values that are Extremely Unlikely
  - Even in the Extremely Unlikely Region,
    - Econometrician Who Looks at Standard Errors is Innoculated from Error

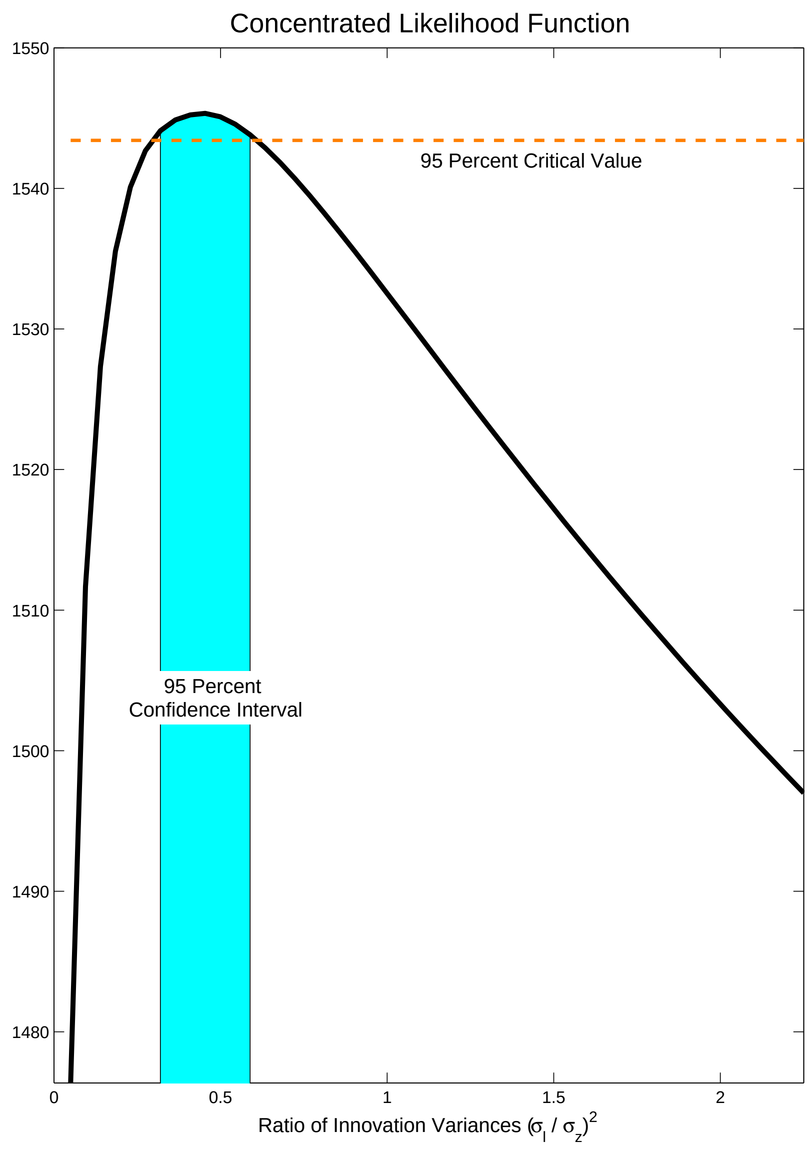
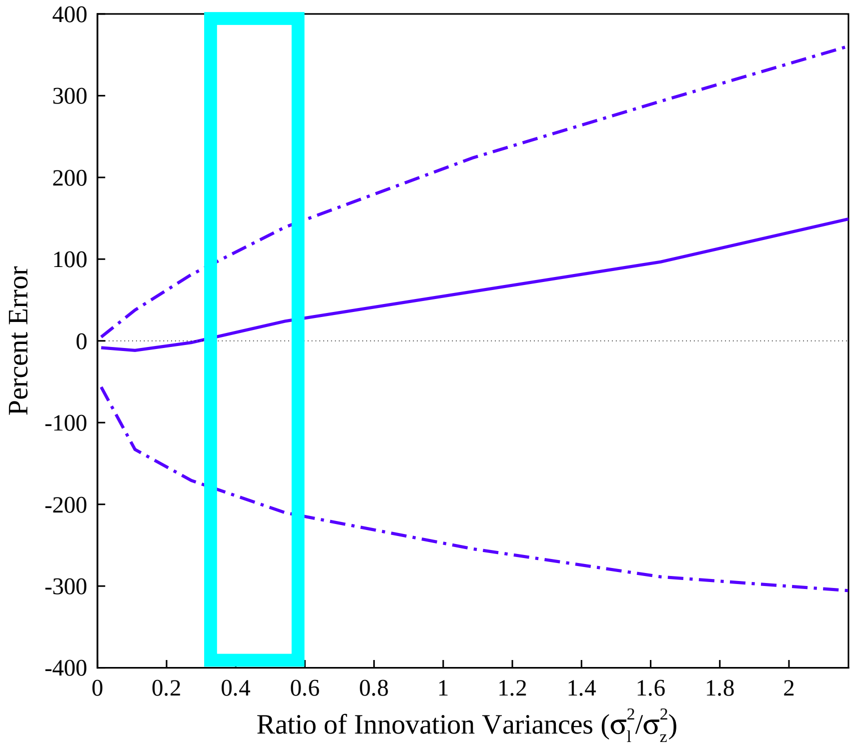


FIGURE A6

Combined Error in the Mean Impact Coefficient (solid line) and the Mean of 95% Bootstrapped Confidence Bands (dashed lines) Averaged Across 1,000 Applications of the Four-Lag LSVAR Procedure with $\rho = .99$ to Model Simulations of Length 180, Varying the Ratio of Innovation Variances



NOTE: The combined error is defined to be the percent error in the small sample SVAR response of hours to technology on impact relative to the model's theoretical response. This error combines the specification error and the small sample bias.

A Summing Up So Far

- With Long Run Restrictions,
  - For RBC Models that Fit the Data Well, Structural VARs Work Well
  - Examples Can Be Found With Some Bias
    * Reflects Difficulty of Estimating Sum of VAR Coefficients
    * Bias is Small Relative to Sampling Uncertainty
    * Econometrician Would Correctly Assess Sampling Uncertainty
- Golden Rule: Pay Attention to Standard Errors!

Turning to SVARS with Short Run Identifying Restrictions

- Bulk of SVAR Literature Concerned with Short-Run Identification

- Substantive Economic Issues Hinge on Accuracy of SVARs with Short-run Identification

- Ed Green's Review of Mike Woodford's Recent Book on Monetary Economics

  – Recent Monetary DSGE Models Deviate from Original Rational Expectations Models (Lucas-Prescott, Lucas, Kydland-Prescott, Long-Plosser, and Lucas-Stokey) By Incorporating Various Frictions.

  – Motivated by Analysis of SVARs with Short-run Identification.

SVARS with Short Run Identifying Restrictions

- Adapt our Conventional RBC Model, to Study VARs Identified with Short-run Restrictions
  - Results Based on Short-run Restrictions Allow Us to Diagnose Results Based on Long-run Restrictions

- *Recursive version of the RBC Model*
  - First, τ_{lt} is observed
  - Second, labor decision is made.
  - Third, other shocks are realized.
  - Then, everything else happens.

The Recursive Version of the RBC Model

- Key Short Run Restrictions:

$$\log l_t = f\left(\varepsilon_{l,t}, \text{lagged shocks}\right)$$
$$\Delta \log \frac{Y_t}{l_t} = g\left(\varepsilon_t^z, \varepsilon_{l,t}, \text{lagged shocks}\right),$$

- Recover ε_t^z:
  - Regress $\Delta \log \frac{Y_t}{l_t}$ on $\log l_t$
  - Residual is measure of ε_t^z.

- This Procedure is Mapped into an SVAR identified with a $Choleski$ decompostion of $\hat{\text{V}}$.

- The Estimated VAR:

$$\begin{aligned} Y_t &= B_1 Y_{t-1} + B_2 Y_{t-2} + ... + B_p Y_{t-p} + u_t,\ E u_t u_t' = V \\ u_t &= C\varepsilon_t,\ CC' = V. \\ C &= \left[C_1 \vdots C_2\right],\ \varepsilon_t = \begin{pmatrix} \varepsilon_t^z \\ \varepsilon_{2t} \end{pmatrix} \end{aligned}$$

- Impulse Response Response Functions Require: $B_1, ..., B_p, C_1$

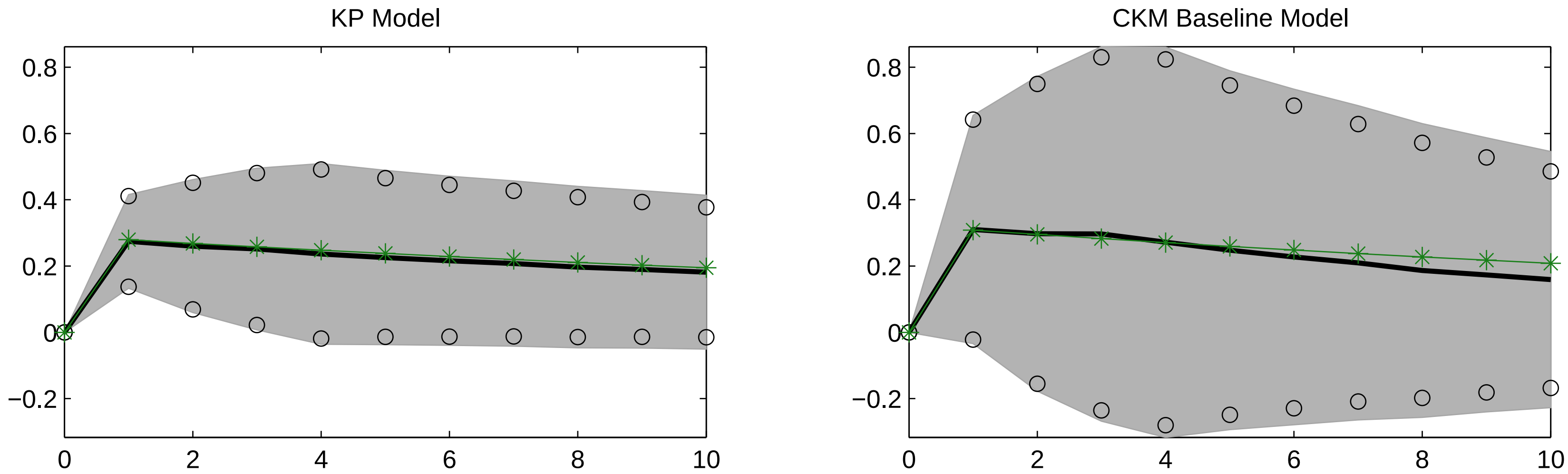
- Short-run Restrictions Uniquely Pin Down C_1 :

$$C_1 = f_{SR}\left(\hat{V}\right)$$

- Note: Sum of VAR Coefficients Not Needed

Response of Hours to A Technology Shock

Short−Run Identification Assumption



SVARs with Short Run Restrictions

- Perform remarkably well
  - Inference is Sharp and Correct

Short Run Versus Long Run Restrictions

- Recursive Results Helpful For Diagnosing Results with Long-run Identification

- Corroborates Theme: When there is Bias with Long-run Identification, It is Because of Difficulties with Estimating Sum of VAR Coefficients

  – Long-run Identification:

$$C_1 = f_{LR}\left(\hat{V}, \hat{B}_1 + ... + \hat{B}_p\right)$$

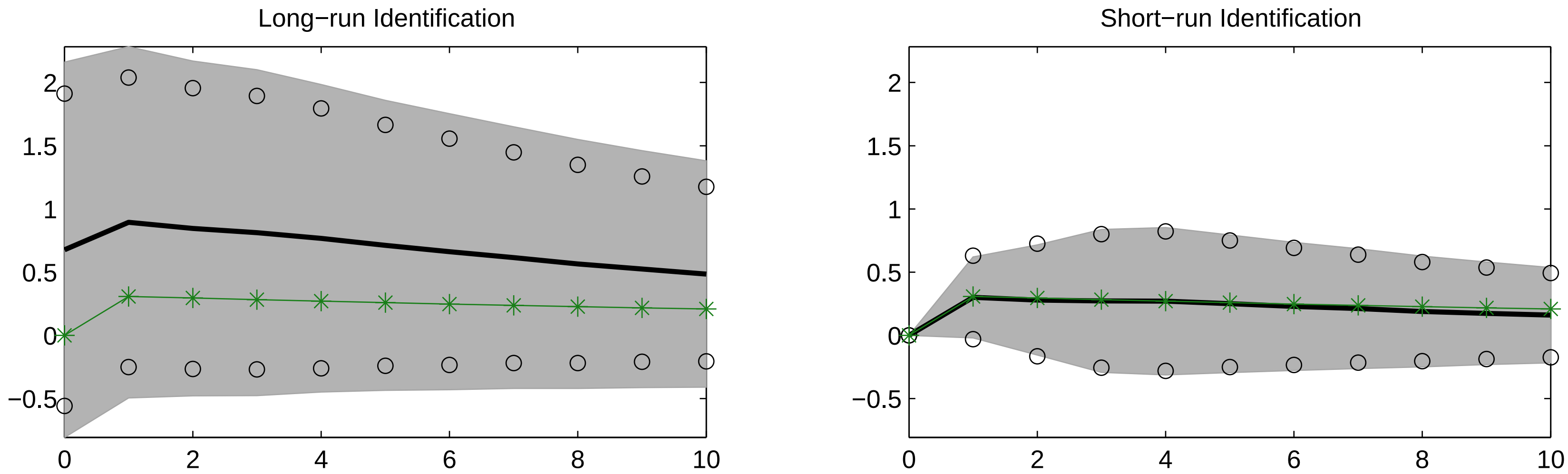
  – Short-run Identification:

$$C_1 = f_{SR}\left(\hat{V}\right)$$

- Recursive Version of CKM Model Rationalizes Both Short and Long-run Identification

The Importance of Frequency Zero: Another View

Analysis of Recursive Version of Baseline CKM Model



VARs and Models with Nominal Frictions

- Data Generating Mechanism: an estimated DSGE model embodying nominal wage and price frictions as well as real and monetary shocks ACEL (2004)

- Three shocks

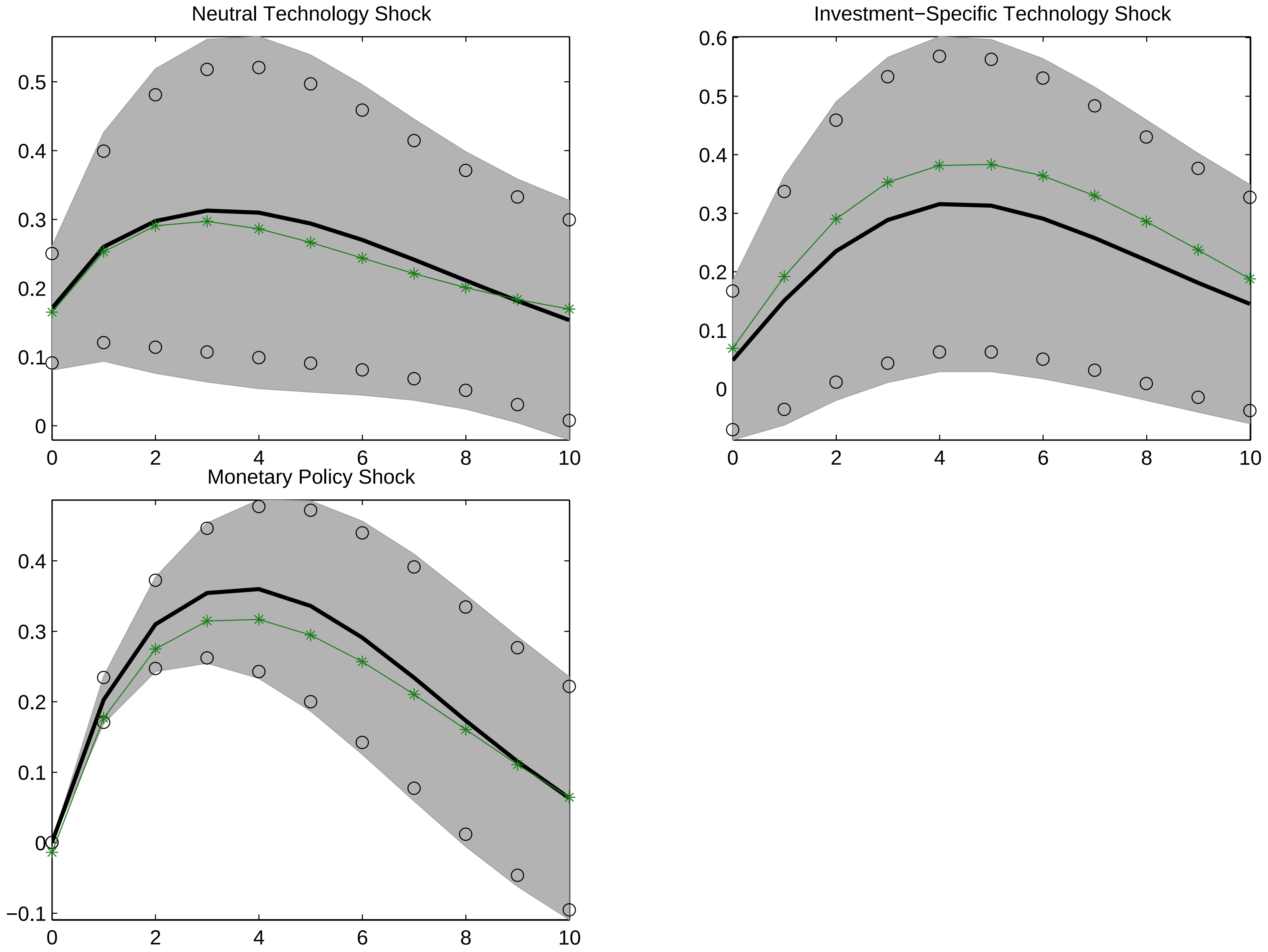
  – Neutral shock to technology,

  – Shock to capital-embodied technology

  – Shock to monetary policy.

- Each shock accounts for about 1/3 of cyclical output variance in the model

Analysis of VARS using the ACEL model as DGP



Conclusion

- We studied the properties of SVARs.
  - With short run restrictions, SVARs perform remarkably well in All Examples Considered
    - ∗ VAR Coefficients Reasonably Accurately Estimated With 4 Lags (Despite Presence of Capital)
  - With long run restrictions, SVARs also perform well for Data Generating Mechanisms that Fit the Data Well
    - ∗ Bias is Small & Sampling Uncertainty Characterized Accurately

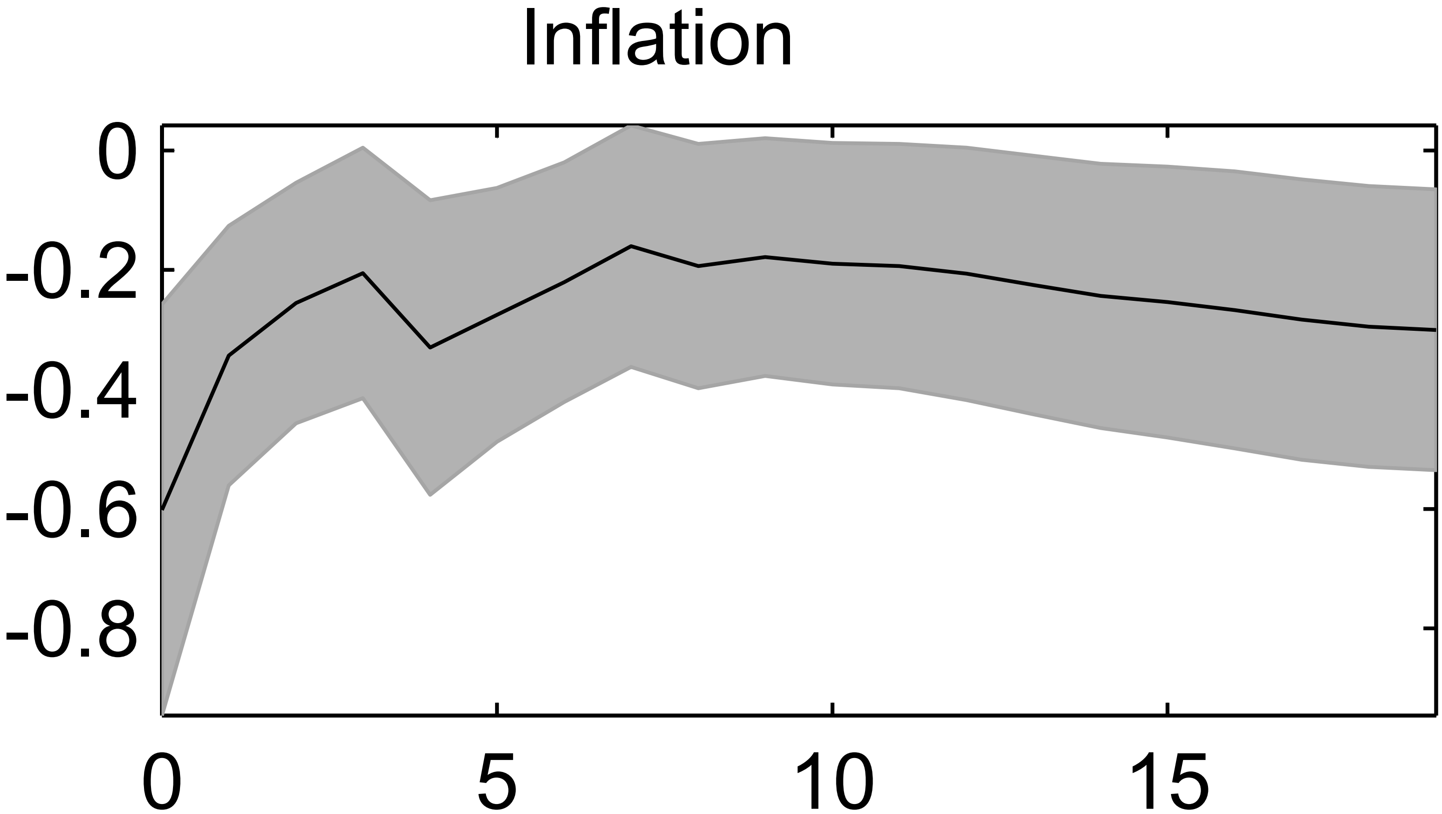
Conclusion ...

- There do exist cases when long run SVARs Exhibit Some Bias,
  - When there is Bias, Reflects Difficulty of Estimating Sum of VAR Coefficients Accurately
  - However,
    - Cases are Based on Models that are Overwhelmingly Rejected by the US Data
    - In Any Event, Econometrician Would See Large Standard Errors and Discount the Evidence
- Rule for Staying Out of Trouble With Long-Run SVARs:

Pay Attention to Standard Errors!!

Conclusion ...

- In The RBC Examples Shown With Long-run Restrictions:
  - Sampling Uncertainty High
- High Sampling Uncertainty Does Not Always Occur
  - Ex #1: ACEL Simulations
  - Ex #2: In ACEL Estimated SVAR, Inflation Responds Strongly to Neutral Technology Shock
    - Simulations (Cautiously) Suggest We Should Trust Standard Errors from SVARs with Long-Run Restrictions
    - Result Casts a Cloud Over Models with Price Frictions


Inflation
0
-0.2
-0.4
-0.6
-0.8
0
5
10
15