Federal Reserve Bank of Minneapolis
Research Department Staff Report 328

Revised April 2005

Business Cycle Accounting

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ABSTRACT

We propose a simple method to help researchers develop quantitative models of economic fluctuations. The method rests on the insight that many models are equivalent to a prototype growth model with time-varying wedges which resemble productivity, labor and capital income taxes, and government consumption. Wedges corresponding to these variables—*efficiency*, *labor*, *investment,* and *government consumption wedges*—are measured with data and then fed back into the model in order to assess the fraction of various fluctuations accounted for by the wedges. For the United States, applications to the Great Depression and the 1982 recession reveal that models with frictions which work as investment wedges are not a promising way to study business cycles. For the Depression, the efficiency and labor wedges together account for essentially all of the fluctuations; for the 1982 recession, only the efficiency wedge matters. In neither period do the other wedges play a significant role.

*We thank the co-editor and three referees for useful comments. We also thank Kathy Rolfe for excellent editorial assistance and the National Science Foundation for support. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

In building detailed, quantitative models of economic fluctuations, researchers face hard choices about where to introduce frictions into their models in order to allow the models to accurately reproduce business cycles. Here we propose a simple method to guide these choices, and we demonstrate how to use it.

Our method has two components: an equivalence result and an accounting procedure. The *equivalence result* is a demonstration that a large class of models, including models with various types of frictions, are equivalent to a prototype model with various types of time-varying wedges that distort the equilibrium decisions of agents operating otherwise in competitive markets. At face value, these wedges look like time-varying productivity, labor income taxes, capital income taxes, and government consumption. We thus label the wedges *efficiency wedges*, *labor wedges*, *investment wedges*, and *government consumption wedges.*

The *accounting procedure* also has two components. It begins by measuring the wedges, using data together with the equilibrium conditions of a prototype model. The measured wedge values are then fed back into the prototype model, one at a time and in combinations, in order to assess how much of the movements in output, labor, and investment in the data can be attributed to each wedge, separately and in combinations. By construction, all four wedges account for all of these observed movements. This accounting procedure is what gives our method its name, *business cycle accounting.*

To demonstrate how the method works, we here apply it to two actual U.S. business cycle episodes: the most extreme in U.S. history, the Great Depression (1929–39), and a downturn less severe and more like those seen often since World War II, the 1982 recession. For the Great Depression period, we find that, in combination, the efficiency and labor wedges account for virtually all of the fall in output, labor, and investment from 1929 to 1933 and the behavior of those variables in the recovery. Over the entire Depression period, the investment wedge not only does not play a role in the downturn; it actually drives output the wrong way, leading to an increase in output during much of the 1930s. Thus, the investment wedge cannot account for either the long, deep downturn or the following slow recovery. Our analysis of the more typical 1982 U.S. recession produces similar, though not identical, results. We find for this period that the efficiency wedge also plays a central role, but here the labor wedge does not; it accounts for little of the fluctuations in output. But in the 1982 period, the investment wedge again is counterproductive, driving output the wrong way. In both episodes, the government consumption wedge plays virtually no role.

We extend our results to the entire postwar period by developing some summary statistics for 1959–2004. The statistics we focus on are the amount of output fluctuations induced by each wedge alone and the correlations between those fluctuations and those actually in the data. Our findings from these statistics are consistent with those from the analyses of the two separate downturns.

We also investigate whether our results are sensitive to alternative assumptions in the prototype model about features like capital utilization rates, labor supply elasticities, and investment adjustment costs. We find that while each of the alternative assumptions we investigate leads to substantial changes in the size of the measured wedges, overall the investment wedge continues to play a quantitatively insignificant role. This exercise suggests that alternative models should be judged not by the size of their wedges, but rather by their equilibrium responses to the wedges.

Here we establish equivalence results that link wedges to detailed models. We show that an economy in which the technology is constant but input-financing frictions vary over time is equivalent to a growth model with efficiency wedges. We show that an open economy with fluctuating borrowing and lending is equivalent to a prototype (closed-economy) model with government consumption wedges. In the appendix, we show that economies with sticky wages and monetary shocks, like that of Bordo, Erceg, and Evans (2000), or unions and antitrust policy shocks, like that of Cole and Ohanian (2004), are equivalent to a growth model with labor wedges. Also in the appendix, we show that an economy with credit market frictions operating through an investment channel, like those of Bernanke and Gertler (1989) and Carlson and Fuerst (1997), is equivalent to a growth model with investment wedges.

Our findings suggest that models with credit market frictions operating through investment channels are not promising avenues for studying the Great Depression or postwar downturns. More promising are sticky wage mechanisms with monetary shocks, such as those of Bordo, Erceg, and Evans (2000), and models with monopoly power, such as that of Cole and Ohanian (2004). In general, this application of our method suggests that successful future work will likely include mechanisms which emphasize the role of efficiency and labor wedges and deemphasize the role of investment wedges. We view these findings as our key substantive contribution.

In terms of method, the equivalence result provides the logical foundation for the way our accounting procedure uses the measured wedges. At a mechanical level, the wedges represent deviations in the prototype model's first-order conditions and in its relationship between inputs and outputs. One interpretation of these deviations, of course, is that they are simply errors, so that their size indicates the goodness-of-fit of the model. Under that interpretation, however, feeding the measured wedges back into the model makes no sense. Our equivalence result leads to a more economically useful interpretation of the deviations by linking them directly to classes of models; that link provides the rationale for feeding the measured wedges back into the model.

Also in terms of method, the accounting procedure goes beyond simply plotting the wedges. Such plots, by themselves, are not useful in evaluating the quantitative importance of competing mechanisms of business cycles because they tell us little about the equilibrium responses to the wedges. Feeding the measured wedges back into the prototype model and measuring the model's resulting equilibrium responses is what allows us to discriminate between competing mechanisms of business cycles.

Our accounting procedure is intended to be a useful first step in guiding the construction of detailed models with various frictions, to help researchers decide which frictions are quantitatively important to business cycle fluctuations. The procedure is not a way to test particular detailed models. If a detailed model is at hand, then it makes sense to confront that model directly with the data. Nevertheless, our procedure is useful in analyzing models with many frictions. For example, some researchers, such as Bernanke, Gertler, and Gilchrist (1999) and Christiano, Gust, and Roldos (2004), have argued that the data are well accounted for by models which include a host of frictions (such as credit market frictions, sticky wages, and sticky prices). Our analysis suggests that the features of these models which lead to investment wedges can be dropped without substantially affecting the models' ability to account for the data.

Our method is not intended to identify the primitive sources of shocks. It is intended simply to help understand the mechanisms through which such shocks lead to economic fluctuations. Many economists think, for example, that monetary shocks drove the U.S. Great Depression, but these economists disagree about the details of the driving mechanism. Our analysis suggests that models in which financial frictions show up as investment wedges are not promising. In our work here, we develop a model, consistent with the views in Bernanke (1983), in which financial frictions show up instead as efficiency wedges. An extension of this model could be fruitful. Other economists, including Cole and Ohanian (1999) and Prescott (1999), emphasize nonmonetary factors behind the Great Depression. For such economists, our findings suggest that the model of Cole and Ohanian (2004), in which fluctuations in the power of unions and cartels lead to labor wedges, and other models in which poor government policies lead to efficiency wedges are also promising.

Our work here is related to a vast business cycle literature that we discuss in detail after we describe and apply our new method.

1. Demonstrating the Equivalence Result

Here we show how various detailed models with underlying distortions can be viewed as equivalent to a prototype growth model with one or more wedges. We choose simple models in order to illustrate how the detailed models map into the prototypes. Since many models map into the same configuration of wedges, identifying one particular configuration does not uniquely identify a model; rather, it identifies a whole class of models consistent with that configuration. In this sense, our method does not uniquely determine the model most promising to analyze business cycle fluctuations. It does, however, guide researchers to focus on the key margins that need to be distorted in order to capture the nature of the fluctuations.

1.1. *The Benchmark Prototype Economy*

The *benchmark* prototype economy that we use later in our accounting procedure is a growth model with four stochastic variables: the *efficiency wedge* A_t, the *labor wedge* $1-\tau_{lt}$, the *investment wedge* $1/(1+\tau_{xt})$, and the *government consumption wedge* g_t.

In the model, consumers maximize expected utility over per capita consumption c_t and per capita labor l_t,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) N_t,$$

subject to the budget constraint

$$c_t + (1+\tau_{xt})x_t = (1-\tau_{lt})w_t l_t + r_t k_t + T_t$$

and the capital accumulation law

$$(1+\lambda)k_{t+1} = (1-\delta)k_t + x_t, \tag{1}$$

where k_t denotes the per capita capital stock, x_t investment, w_t the wage rate, r_t the rental rate on capital, τ_{lt} and τ_{xt} the tax rates on labor and investment, β the discount factor, δ the depreciation rate of capital, N_t the period t population with growth rate equal to $1+\lambda$, and T_t lump-sum taxes.

The firms' production function is $F\big(k_t, (1+\gamma)^t l_t\big)$, where $(1+\gamma)^t$ is the rate of labor-augmenting technical progress, which is assumed to be a constant. Firms maximize $A_t F\big(k_t, (1+\gamma)^t l_t\big) - r_t k_t - w_t l_t$.

The equilibrium of this benchmark prototype economy is summarized by the resource constraint,

$$(2) \qquad c_t + x_t + g_t = y_t,$$

where y_t and g_t denote per capita output and per capita government consumption, together with

$$(3) \qquad y_t = A_t F\big(k_t, (1+\gamma)^t l_t\big),$$

$$(4) \qquad -\frac{U_{lt}}{U_{ct}} = (1-\tau_{lt})A_t(1+\gamma)^t F_{lt}, \text{ and}$$

$$(5) \qquad U_{ct}(1+\tau_{xt}) = \beta E_t U_{ct+1}[A_{t+1}F_{kt+1} + (1-\delta)(1+\tau_{xt+1})],$$

where, here and throughout, notations like U_{ct}, U_{lt}, F_{lt}, and F_{kt} denote the derivatives of the utility function and the production function with respect to their arguments. We assume that g_t fluctuates around a trend of $(1+\gamma)^t$.

Notice that in this benchmark prototype economy, the efficiency wedge resembles the productivity parameter, and the labor wedge and the investment wedge resemble tax rates on labor income and investment income. Other more elaborate models could be considered, models with other kinds of frictions that look like taxes on consumption or on capital income. Consumption taxes induce a wedge between the consumption-leisure marginal rate of substitution and the marginal product of labor in the same way as do labor taxes. Such taxes, if time-varying, also distort the intertemporal margins in (5). Capital income taxes induce a wedge between the intertemporal marginal rate of substitution and the marginal product of capital which is only slightly different from the distortion induced by a tax on investment.

We emphasize that each of the wedges represents the overall distortion to the relevant equilibrium condition of the model. For example, distortions both to labor supply affecting consumers and to labor demand affecting firms distort the static first-order condition (4). Our labor wedge represents the sum of these distortions. Our method identifies the overall wedge induced by both distortions and does not identify each separately. We focus on the overall wedges because what matters in determining business cycle fluctuations is the overall wedges, not each distortion separately.

1.2. *The Mapping—From Frictions to Wedges*

Now we illustrate the mapping between detailed economies and prototype economies for two types of wedges. We show that input-financing frictions in a detailed economy map into efficiency wedges in our prototype economy. Fluctuations in net exports in an open economy map into government consumption wedges in our prototype (closed) economy. In an appendix, we show as well that sticky wages and monetary shocks map into labor wedges and investment-financing frictions map into investment wedges. In earlier work (Chari, Kehoe, and McGrattan (2002)), we have shown that fluctuations in government policy toward monopolies in a model with unions map into labor wedges.

a. *Efficiency Wedges*

In many economies, underlying frictions either within or across firms cause factor inputs to be used inefficiently. These frictions in an underlying economy often show up as aggregate productivity shocks in a prototype economy similar to our benchmark economy. Schmitz (forthcoming) presents an interesting example of within-firm frictions resulting from work rules that lower measured productivity at the firm level. Lagos (2004) also studies how labor market policies can lead to misallocations of labor across firms and, thus, to efficiency wedges.

Here we develop a detailed economy with input-financing frictions and use it to make two points. This economy illustrates the general idea that frictions which lead to inefficient factor utilization map into efficiency wedges in a prototype economy. Beyond that, however, the economy also demonstrates that financial frictions can show up as efficiency wedges rather than as their usual manifestation, investment wedges. In our detailed economy, financing frictions lead to some firms having to finance working capital requirements at higher interest rates than other firms. These frictions lead to a misallocation of inputs across firms, a clear inefficiency.

□ *A Detailed Economy With Input-Financing Frictions*

Consider a simple detailed economy with distortions in the allocation of intermediate inputs across two types of firms, distortions arising from financing frictions. Both types of firms must borrow to pay for an intermediate input in advance of production. One type of firm is financially constrained, in the sense that it pays a higher price for borrowing than the other type. We think of these frictions as capturing the idea that some firms, namely, small firms, often have difficulty borrowing. One motivation for the higher price paid by the financially constrained firms is that moral hazard problems are more severe for small firms.

Specifically, consider the following economy. Aggregate gross output q_t in this economy is a combination of the gross output q_{it} from the economy's two sectors, indexed $i = 1, 2$, according to

$$(6) \qquad q_t = q_{1t}^{\phi} q_{2t}^{1-\phi},$$

where $0 < \phi < 1$. The representative producer of the gross output q_t chooses q_{1t} and q_{2t} to solve this problem:

$$\max \; q_t - p_{1t}q_{1t} - p_{2t}q_{2t}$$

subject to (6), where p_{it} is the price of the output of sector i.

The resource constraint for gross output in this economy is

$$(7) \qquad c_t + k_{t+1} + m_{1t} + m_{2t} = q_t + (1-\delta)k_t,$$

where c_t is consumption, k_t is the capital stock, and m_{1t} and m_{2t} are intermediate goods used in sectors 1 and 2, respectively. Final output, given by $y_t = q_t - m_{1t} - m_{2t}$, is gross output less the intermediate goods used.

The gross output of each sector i, q_{it}, is made from intermediate goods m_{it} and a composite value-added good z_{it} according to

$$(8) \qquad q_{it} = m_{it}^{\theta} z_{it}^{1-\theta},$$

where $0 < \theta < 1$. When composite value-added good is produced from capital k_t and labor l_t according to

(9) $$z_{1t} + z_{2t} = z_t = F(k_t, l_t).$$

The producer of gross output of sector i chooses the composite good z_{it} and the intermediate good m_{it} to solve this problem:

$$\max\ p_{it}q_{it} - v_t z_{it} - R_{it}m_{it}$$

subject to (8). Here v_t is the price of the composite good and R_{it} is the gross within-period interest rate paid on borrowing by firms in sector i. Assume that firms in sector 1 are more financially constrained than those in sector 2, so that $R_{1t} > R_{2t}$. Let $R_{it} = R_t(1+\tau_{it})$, where R_t is the rate consumers earn within period t and τ_{it} measures the within-period spread, induced by financing constraints, between the rate paid to savers and the rate paid by firms in sector i. Since consumers do not discount utility within the period, $R_t = 1$.

In this economy, the representative producer of the composite good z_t chooses k_t and l_t to solve this problem:

$$\max\ v_t z_t - w_t l_t - r_t k_t$$

subject to (9), where w_t is the wage rate and r_t is the rental rate on capital.

Consumers solve this problem:

(10) $$\max \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

$$c_t + k_{t+1} = r_t k_t + w_t l_t + (1-\delta)k_t + T_t,$$

where $l_t = l_{1t} + l_{2t}$ is the economy's total labor supply and $T_t = R_t \sum_i \tau_{it} m_{it}$ is lump-sum transfers. Here we assume that the financing frictions act like distorting taxes, and the proceeds are rebated to consumers. If, instead, we assumed that the financing frictions represent, say, lost gross output, then we would adjust the economy's resource constraint (7) to introduce a government consumption wedge.

□ *The Associated Prototype Economy With Efficiency Wedges*

Now consider a version of the benchmark prototype economy that will have the same aggregate allocations as the input-financing frictions economy just detailed. This prototype economy is identical to our benchmark prototype except that the new prototype economy has taxes on capital income rather than taxes on investment, and here government consumption is set equal to zero.

Now the consumer's budget constraint is

$$c_t + k_{t+1} = (1 - \tau_{kt})r_t k_t + (1 - \tau_{lt})w_t l_t + (1 - \delta)k_t + T_t, \tag{11}$$

and the efficiency wedge is

$$A_t = \kappa(a_{1t}^{1-\phi} a_{2t}^{\phi})^{\frac{\theta}{1-\theta}}[1 - \theta(a_{1t} + a_{2t})], \tag{12}$$

where $a_{1t} = \phi/(1 + \tau_{1t})$, $a_{2t} = (1 - \phi)/(1 + \tau_{2t})$, $\kappa = [\phi^{\phi}(1 - \phi)^{1-\phi}\theta^{\theta}]^{\frac{1}{1-\theta}}$, and τ_{1t} and τ_{2t} are the interest rate spreads in the detailed economy.

Comparing the first-order conditions in the detailed economy with input-financing frictions to those of the associated prototype economy with efficiency wedges leads immediately to this proposition:

PROPOSITION 1: *Consider the prototype economy with resource constraint* (2) *and consumer budget constraint* (11) *with exogenous processes for the efficiency wedge A_t given in* (12), *the labor wedge given by*

$$\frac{1}{1 - \tau_{lt}} = \frac{1}{1 - \theta}\left[1 - \theta\left(\frac{\phi}{1 + \tau_{1t}^*} + \frac{1 - \phi}{1 + \tau_{2t}^*}\right)\right], \tag{13}$$

and the investment wedge given by $\tau_{kt} = \tau_{lt}$, where τ_{1t}^* and τ_{2t}^* are the interest rate spreads from the detailed economy with input-financing frictions. Then the equilibrium allocations in the detailed economy are equilibrium allocations in this prototype economy.

Imagine that in the economy with input-financing frictions, the interest rate spreads τ_{1t} and τ_{2t} fluctuate over time, but in such a way that the weighted average of these spreads

$$(14) \quad a_{1t} + a_{2t} = \frac{\phi}{1+\tau_{1t}} + \frac{1-\phi}{1+\tau_{2t}}$$

is constant while $a_{1t}^{1-\phi} a_{2t}^{\phi}$ fluctuates. Then from (13) we see that the labor and investment wedges are constant, and from (12) we see that the efficiency wedge fluctuates. Thus, on average, financing frictions are unchanged, but relative frictions fluctuate. An outside observer who attempted to fit the data generated by the detailed economy with input-financing frictions to the prototype economy would identify the fluctuations in relative distortions with fluctuations in technology and would see no fluctuations in either the labor wedge $1 - \tau_{lt}$ or the investment wedge τ_{kt}, here represented by a tax on capital income. In particular, periods in which the relative distortions increase would be misinterpreted as periods of technological regress. This observation leads us to label A_t the *efficiency wedge* in the prototype economy.

More generally, fluctuations in the interest rate spreads τ_{1t} and τ_{2t}, which lead to fluctuations in τ_{lt} and τ_{kt}, show up in the prototype economy as fluctuations in all of the wedges.

b. *Government Consumption Wedges*

Now we develop a detailed economy with international borrowing and lending and show that net exports in that economy are equivalent to the government consumption wedge in an associated prototype economy.

□ *A Detailed Economy With International Borrowing and Lending*

Consider a model of a world economy with N countries and a single homogenous good in each period. In each period t, the economy experiences one of finitely many events s_t, which index the shocks. We denote by $s^t = (s_0, \ldots, s_t)$ the history of events up through and including period t. The probability, as of period 0, of any particular history s^t is $\pi(s^t)$. The initial realization s_0 is given.

The representative consumer in country i has preferences

$$\sum \beta^t \pi(s^t) U\big(c_i(s^t), l_i(s^t)\big), \tag{15}$$

where $c_i(s^t)$ and $l_i(s^t)$ denote consumption and labor. The consumer's budget constraint is

$$c_i(s^t) + b_i(s^t) + k_i(s^t) \leq F\big(k_i(s^{t-1}), l_i(s^t)\big) + (1-\delta)k_i(s^{t-1}) + \sum_{s^{t+1}|s^t} q(s^{t+1}) b_i(s^{t+1}), \tag{16}$$

where $b_i(s^{t+1})$ denotes the amount of state-contingent borrowing by the consumer in country i in period t, $q(s^{t+1})$ denotes the corresponding state-contingent price, and $k_i(s^t)$ denotes the capital stock.

An *equilibrium for the detailed economy* is a set of allocations $\big(c_i(s^t), k_i(s^t), l_i(s^t),$ $b_i(s^{t+1})\big)$ and prices $\big(q(s^t)\big)$ such that these allocations both solve the consumer's problem in each country i and satisfy the world resource constraint:

$$\sum_{i=1}^{N} [c_i(s^t) + k_i(s^t)] \leq \sum_{i=1}^{N} \left[F\big(k_i(s^{t-1}), l_i(s^t)\big) + (1-\delta)k_i(s^{t-1})\right]. \tag{17}$$

Note that in this economy the net exports of country i are given by $F\big(k_i(s^{t-1}), l_i(s^t)\big) - [k_i(s^t) - (1-\delta)k_i(s^{t-1})] - c_i(s^t)$.

□ *The Associated Prototype Economy With Government Consumption Wedges*

Consider a prototype economy of a single closed economy i with an exogenous stochastic variable, government consumption $g_i(s^t)$, which we call the *government consumption wedge*. In this economy, consumers maximize (15) subject to their budget constraint

$$c_i(s^t) + k_i(s^t) = w_i(s^t) l_i(s^t) + \left[r_i(s^t) + 1 - \delta\right] k_i(s^{t-1}) + T_i(s^t), \tag{18}$$

where $w_i(s^t)$, $r_i(s^t)$, and $T_i(s^t)$ are the wage rate, the capital rental rate, and lump-sum transfers. In each state s^t, firms choose k and l to maximize $F(k,l) - r_i(s^t)k - w_i(s^t)l$. The government's budget constraint is

$$g_i(s^t) + T_i(s^t) = 0. \tag{19}$$

The resource constraint for this economy is

$$c_i(s^t) + g_i(s^t) + k_i(s^t) = F\big(k_i(s^{t-1}), l_i(s^t)\big) + (1-\delta)k_i(s^{t-1}). \tag{20}$$

An *equilibrium of the prototype economy* is a set of allocations $\big(c_i(s^t), k_i(s^t), l_i(s^t), g_i(s^t),$ $T_i(s^t)\big)$ and prices $\big(w_i(s^t), r_i(s^t)\big)$ such that these allocations are optimal for consumers and firms and the resource constraint is satisfied.

The following proposition shows that the government consumption wedge in the prototype economy consists of net exports in the original economy.

PROPOSITION 2: *Consider the equilibrium allocations $\big(c_i^*(s^t), k_i^*(s^t), l_i^*(s^t), b_i^*(s^{t+1})\big)$ for country i in the detailed economy. Let the government consumption wedge be*

$$g_i(s^t) = F\big(k_i^*(s^{t-1}), l_i^*(s^t)\big) - \big[k_i^*(s^t) - (1-\delta)k_i^*(s^{t-1})\big] - c_i^*(s^t), \tag{21}$$

let the wage and capital rental rates be $w_i(s^t) = F_{li}^*(s^t)$ and $r_i(s^t) = F_{ki}^*(s^t)$, and let $T(s^t)$ be defined by (19). *Then the allocations $\big(c_i^*(s^t), k_i^*(s^t), l_i^*(s^t), g_i^*(s^t), T_i(s^t)\big)$ and the prices $(w_i(s^t), r_i(s^t))$ are an equilibrium for the prototype economy.*

The proof follows by noting that the first-order conditions are the same in the two economies and that, given the government consumption wedge (21), the consumer's budget constraint (16) in the detailed economy is equivalent to the resource constraint (20) in the prototype economy.

Note that for simplicity we have abstracted from government consumption in the detailed economy. If we let that economy have government consumption, then the government consumption wedge in the prototype economy would be the sum of net exports and government consumption in the detailed economy.

2. Applying the Accounting Procedure

Having established our equivalence result, we now describe our accounting procedure and demonstrate how to apply it to the Great Depression and the postwar recession of 1982.

2.1. *The Procedure*

Our accounting procedure works as follows. We choose our benchmark prototype model's parameters of preferences and technology in standard ways, as in the quantitative business cycle literature, and then use the equilibrium conditions of our prototype economy to estimate the parameters of a stochastic process for the wedges. This collection of parameters implies decision rules for output, labor, and investment which can be used with the data to uncover both a stochastic process for the wedges and the realized values of the wedges in the data.

We then ask, How much of the output fluctuations can be accounted for by each of the wedges, separately and in various combinations? To answer this question, we simulate our prototype model using the realized sequence of wedges in the data to assess, separately and in combinations, the contribution of the wedges to fluctuations in output, labor, and investment. The contribution of these wedges is measured by comparing the realizations of variables like output, labor, and investment from the model to the data on these variables. Our approach is an accounting procedure since, by construction, all the wedges together account for all of the movements in the variables.

a. *Measuring the Wedges*

Our process for measuring the wedges has two steps. We use both the data and the models first to estimate the stochastic process for the wedges and then to measure the realized wedges.

□ *Estimating the Stochastic Process for the Wedges*

To estimate the stochastic process for the wedges, we use functional forms and parameter values familiar from the business cycle literature. We assume that the production function has the form $F(k, l) = k^{\alpha} l^{1-\alpha}$ and the utility function the form $U(c, l) = \log c + \psi \log(1 - l)$. We choose the capital share $\alpha = .35$ and the time allocation parameter $\psi = 2.24$. We choose the depreciation rate δ, the discount factor β, and growth rates γ and λ so that, on an annualized basis, depreciation is 4.64%, the rate of time preference is 3%, the population growth rate is 1.5%, and the growth of technology is 1.6%.

Equations (2)–(5) summarize the equilibrium of the benchmark prototype economy. We substitute for consumption c_t in (4) and (5) using the resource constraint (2), then log-linearize (3)–(5) to get three linear equations. We specify a vector autoregressive AR(1) process for the four wedges $s_t = (\log A_t, \tau_{lt}, \tau_{xt}, \log g_t)$ of the form

$$(22) \qquad s_{t+1} = P_0 + P s_t + Q\varepsilon_{t+1},$$

where the shock ε_t is i.i.d. and has a standard normal distribution, and Q is a lower-triangular matrix. This gives us seven equations, three from the equilibrium and four from (22). We then use the maximum likelihood procedure described by Anderson et al. (1996) and data on output, labor, investment, government consumption, and net exports to estimate the parameters P_0, P, and Q of the vector AR(1) process for the wedges.

□ *Measuring the Realized Wedges*

The second step in our measurement procedure is to measure the realized wedges. We measure the government consumption wedge directly from the data as the sum of government spending and net exports. To obtain the values of the other three wedges, we use the data and the model's decision rules. (We construct a series for the capital stock using the capital accumulation law (1), data on investment γ_t, and an initial choice of capital stock k_0.) With y_t^d, l_t^d, x_t^d, and k_0^d denoting the data and $y(s_t, k_t)$, $l(s_t, k_t)$, and $x(s_t, k_t)$ denoting the nonlinear decision rules of the model (solved as McGrattan (1996) does), the realized wedge series s_t^d solves

$$(23) \qquad y_t^d = y(s_t^d, k_t), l_t^d = l(s_t^d, k_t), \text{ and } x_t^d = x(s_t^d, k_t),$$

with $k_{t+1} = (1-\delta)k_t + x_t^d$ and $k_0 = k_0^d$. In effect, we solve for the three unknown elements of the vector s_t using the three equations (3)–(5). We use these values for the wedges in our experiments.

Note that, in order to measure the efficiency and labor wedges, we do not need to compute the decision rules. These wedges can be directly calculated from (3) and (4). The investment wedge cannot be directly calculated from (5) because that requires specifying expectations over future values of consumption, the capital stock, the wedges, and so on. The decision rules from our model implicitly depend on these expectations and, therefore, on the stochastic process driving the wedges. Thus, in terms of measuring the realized wedges, the estimated stochastic process plays a role for only the investment wedge.

b. *The Decomposition*

Now we use the model's measured realizations to decompose movements in variables from an initial date (either 1929 or 1979), with an initial capital stock, into four components—movements in variables driven by each of the four wedges away from their values at the initial date.

We define the *efficiency component* of the wedges by letting $s_{1t} = (\log A_t, \tau_{l0}, \tau_{x0}, \log g_0)$ be the vector of wedges in which, in period t, the efficiency wedge takes on its period t value while the other wedges take on their initial values. Define the other components of the wedges—the *labor component* s_{2t}, the *investment component* s_{3t}, and the *government consumption component* s_{4t}—analogously.

We define the capital stock due to component i, for $i = 1, \ldots, 4$, by $k_{it+1} = k(k_{it}, s_{it})$. Given the capital stock components, define output due to component i by $y_{it} = y(k_{it}, s_{it})$ for $i = 1, \ldots, 4$, and construct labor and investment due to the various components similarly.

We also construct joint components. Define the *efficiency plus labor component* by letting $s_{5t} = (\log A_t, \tau_{lt}, \tau_{x0}, \log g_0)$, and define the other joint components similarly.

2.2. *Accounting Details and Findings*

Now we describe both some details of implementing our procedure and the results of applying the procedure to two historical U.S. business cycles. The efficiency wedge turns out to play a central role in both historical episodes. The labor wedge plays a major role in the slow recovery from the Great Depression, but a modest role in the 1982 recession period. The government consumption wedge plays no role in either period. The most striking result is that the investment wedge plays no role in the Great Depression and only a modest role in the 1982 period.

a. *Details of the Procedure*

In practice, we adjust the U.S. data on output and its components to remove sales taxes and to add the service flow for consumer durables. For the pre–World War II period, we remove military compensation as well. We estimate separate sets of parameters for each of the two historical episodes we analyze. The parameters for the Great Depression analysis are estimated using annual data for 1901–40 (inclusive); those for the postwar analysis, using quarterly data for 1959:1–2004:3 (inclusive). In the Great Depression analysis, we impose the additional restriction that the covariance between the shocks to government consumption and those to the other wedges is zero. This restriction avoids having the large movements in government consumption associated with World War I dominate the estimation of the stochastic process. (In a technical appendix, Chari, Kehoe, and McGrattan (2005), we describe in detail our data sources, computational methods, and estimation procedures.)

Table I displays the resulting estimated values for the parameters P and Q and the associated confidence bands for our two periods. The stochastic process (22) with these values will be used by agents in our economy to form their expectations about future wedges. In the data, we remove a trend of 1.6% from output, investment, and government consumption. Both output and labor are normalized to equal 100 in the base periods: 1929 for the Great Depression and 1979:1 for the 1982 recession. In both episodes, investment (detrended) is divided by the base period level of output. Since the government consumption component accounts for virtually none of the fluctuations in output, labor, and investment, we discuss the government consumption wedge and its components in detail elsewhere (in Chari, Kehoe, and McGrattan (2005)). Here we focus primarily on the fluctuations due to the efficiency, labor, and investment wedges.

b. *Findings: The Great Depression . . .*

Our findings for the period 1929–39, which includes the Great Depression, are displayed in Figures 1–4. We find that the efficiency and labor wedges account for essentially all of the movements of output, labor, and investment in the Depression period and that the investment wedge accounts for almost none.

In Figure 1, we display actual U.S. output along with the three measured wedges for that period: the efficiency wedge A, the labor wedge $(1 - \tau_l)$, and the investment wedge $1/(1+\tau_x)$. We see that the underlying distortions that the three wedges reveal have different patterns. The distortions that manifest themselves as efficiency and labor wedges become substantially worse between 1929 and 1933. By 1939, the efficiency wedge has returned to its trend level, but the labor wedge has not returned to its 1929 level. Over the period, the investment wedge fluctuates, but investment decisions are generally less distorted, in the sense that τ_x is smaller, between 1933 and 1939 than in 1929.

In Figure 2, we plot the 1929–39 data for U.S. output, labor, and investment along with the models' predictions for those variables. Note that labor declines 27% from 1929 to 1933 and stays relatively low for the rest of the decade. Investment also declines sharply from 1929 to 1933, but partially recovers by the end of the decade. Interestingly, in an algebraic sense, about half of output's 36% fall from 1929 to 1933 is due to the decline in investment.

In terms of the models, we start by assessing the separate contributions of the three wedges. In Figure 2, in addition to the data, we plot the values of output, labor, and investment that the model predicts is due to the efficiency wedge and the labor wedge. That is, we plot these variables using the efficiency component s_{1t} and the labor component s_{2t} for the wedges.

Consider the contribution of the efficiency wedge. In Figure 2, we see that the model with this wedge predicts that output declines less than it actually does in the data and that it recovers more rapidly. For example, by 1933, predicted output falls about 25% while output itself falls about 36%. Thus, the efficiency wedge accounts for about two-thirds of the decline of output in the data. By 1939, predicted output is only 3% below trend rather than the observed 22%. As can also be seen in Figure 2, the reason for this predicted rapid recovery is that predicted labor with the efficiency wedge completely misses labor's continued sluggishness in the data from 1933 on. Predicted investment shows a fall similar to that in the data, but a faster recovery.

Consider next the contributions of the labor wedge. In Figure 2, we see that by 1933, the predicted output due to the labor wedge falls only about half as much as output falls in the data: 18% vs. 36%. By 1939, the labor wedge model's predicted output completely captures the slow recovery: it predicts output falling 22%, exactly as output does that year in the data. The reason this model captures the slow output recovery is that predicted labor due to the labor wedge also captures the sluggishness in labor after 1933 remarkably well. The associated prediction for investment is a decline, but not the actual sharp decline from 1929 to 1933.

Summarizing Figure 2, we can say that the efficiency wedge accounts for about two-thirds of output's downturn during the Great Depression, but misses its slow recovery, while the labor wedge accounts for about one-half of this downturn and essentially all of the slow recovery.

Now consider the investment wedge. In Figure 3, we plot along with the data the contributions for output, labor, and investment predicted by the model with the investment wedge. This figure demonstrates that the contributions from the investment wedge completely miss the observed movements in all three variables.

Together Figures 2 and 3 suggest that the efficiency and labor wedges account for essentially all of the movements of output, labor, and investment in the Depression period and that the investment wedge accounts for almost none. This suggestion is confirmed by Figure 4. There we plot the sum of the contributions from the efficiency, labor, and (insignificant) government consumption wedges (labeled *Model With No Investment Wedge*). As can be seen from the figure, essentially all of the fluctuations in output, labor, and investment can be accounted for by movements in the efficiency and labor wedges. For comparison, we also plot the sum of the contributions due to the labor, investment, and government consumption wedges (labeled *Model With No Efficiency Wedge*). This sum does not do well. In fact, comparing Figures 2 and 4, we see that the model with this sum is further from the data than the model with the labor wedge component alone. These findings lead us to conclude that distortions manifested as investment wedges played essentially no role in the U.S. Great Depression.

c. . . . *And the 1982 Recession*

Now we apply our accounting procedure to a more typical U.S. business cycle: the recession of 1982. We find that here the efficiency wedge plays a central role, the labor wedge does not, and the investment wedge actually moderates what would otherwise have been a more severe recession.

We start as we did the Great Depression analysis, by displaying the actual U.S. output over the business cycle period—here, 1979–85—along with the three measured wedges for that period. In Figure 5, we see that output falls nearly 10% relative to trend between 1979 and 1982 and by 1985 is about 1% below trend. We also see that the efficiency wedge falls between 1979 and 1982 and by 1985 is still a little more than 2% below trend. The labor wedge also worsens from 1979 to 1982, but it improves substantially by 1985. The investment wedge, meanwhile, improves fairly steadily over the whole period. Note that this investment wedge pattern does not square with models of business cycles in which financial frictions worsen in downturns and improve in recoveries.

An analysis of the effects of the wedges separately for the 1979–85 period is in Figures 6 and 7. In Figure 6, we see that the model with the efficiency wedge produces a decline in output from 1979 to 1982 of 13%, which is more than the actual decline in that period. With this wedge, output recovers but not as rapidly as in the data. In contrast, the model with the labor wedge accounts for little of the fluctuations. In Figure 7, we see that the model with just the investment wedge actually produces an increase in output of roughly 10% from

1979 to 1982.

Now we examine how well a combination of wedges reproduces the data for the 1982 recession period. In Figure 8, we plot the movements in output, labor, and investment during 1979–85 due to the sum of the efficiency, labor, and (insignificant) government consumption components (labeled *Model With No Investment Wedge*). In terms of output, this sum declines about 18% by 1982, about twice as much as the data, and by 1985 it is still well below the data. The sum of the labor, investment, and government components (labeled *Model With No Efficiency Wedge*) produces a rise in output rather than a recession. These findings suggest that distortions corresponding to investment wedges played, at best, a modest role in the 1982 recession, primarily by preventing the downturn from being even deeper than it was.

3. Extending the Results to the Entire Postwar Period

So far we have analyzed the wedges and their contributions for specific episodes. Now we attempt to extend our method's findings for the entire postwar period by developing some summary statistics for the period from 1959:1 to 2004:3. We first consider the standard deviations of the wedges relative to output as well as correlations of the wedges with each other and with output at various leads and lags. We then consider the standard deviations and the cross correlations of output due to each wedge. These statistics summarize salient features of the wedges and their role in output fluctuations for the entire postwar sample. We think of the wedge statistics as analogs of our plots of the wedges and the output statistics as analogs of our plots of output due to just one wedge.[1] The results for this long period turn out to be consistent with those we have seen for the two specific episodes.

In Tables II and III, we focus on standard deviations and cross correlations using HP-filtered data. Panel A of Table II shows that the efficiency wedge and the labor wedge are positively correlated with output, both contemporaneously and for several leads and lags. The investment wedge and government consumption wedge, meanwhile, are negatively correlated with output, both contemporaneously and for several leads and lags. (Note that the government consumption wedge is the sum of government consumption and net exports and that net exports are negatively correlated with output.) Panel B of Table II shows that the efficiency and labor wedges are positively correlated while the cross correlations of the other combinations of wedges are nearly all negative.

Table III summarizes various statistics of output due to each wedge. Consider first the output fluctuations due to the efficiency wedge. Table III shows that output due to this wedge has a standard deviation relative to output in the data of 1.49 and that output due to the wedge is highly positively correlated with output in the data, both contemporaneously and for several leads and lags. These statistics are consistent with the episodic analysis of the 1982 recession which showed that output due to this wedge both fluctuates substantially more than output and comoves highly with it.

Consider next the role of the other wedges. Output due to the labor wedge fluctuates about half as much as output in the data and is positively correlated with it. Output due to the investment wedge fluctuates three-quarters as much as output in the data but is highly negatively correlated with it. Finally, output due to the government consumption wedge fluctuates about half as much as output in the data and is modestly negatively correlated with it. In panel B of Table III we see that output due to the efficiency and labor wedges are positively correlated and the cross correlations of output due to the other wedges are nearly all negative.

The main point we draw from these summary statistics is the same point we drew from our earlier analyses: models in which the driving forces of business cycles are frictions that manifest themselves as investment wedges are not promising. In fact, the statistics show that recessions in such models will be associated with booms in the data.

Another point we get from the summary statistics is about efficiency wedges. Our decomposition of business cycle fluctuations implies that efficiency wedges play a much larger role in driving fluctuations than they do in much of the literature. Our decomposition differs from that in the early literature on business cycles in two ways, one quantitatively important and one not. The quantitatively important difference is that we allow for multiple correlated shocks while most of the early literature focuses on models with a single shock. The other difference is that we use a realization-based decomposition rather than a population-based decomposition.

To demonstrate the quantitative importance of our shock structure, we analyze a version of our model with only an efficiency wedge. Specifically, we consider the properties of the output component when the efficiency wedge follows a first-order autoregressive process of the form

$$\log A_{t+1} = (1-\rho)\log \bar{A} + \rho \log A_t + \varepsilon_{At+1}.$$

Note that our realization-based decomposition of the log-linearized model is independent of the variance of the innovation ε_{At}. In line with the univariate specifications in the early literature, we vary the autoregressive parameter ρ between .95 and .99. As we do, the standard deviation of output in the model relative to that in the data varies from .90 to .72. These statistics are similar to those in the early literature, which use population-based statistics. This finding leads us to conclude that the main source of the difference is that we have multiple correlated shocks instead of just one.

4. Exploring Sensitivity to Alternative Specifications

Here we investigate whether our results are substantially changed when we use some alternative specifications of the prototype model. We find that they are not.

4.1. *Motivation and Summary*

One reasonable question about our results is the extent to which they rely on the specific stochastic process driving the wedges. To answer that question, we have attempted several alternative specifications of that process, including perfect foresight. Our substantive findings are essentially unaffected by these changes.

In our accounting exercise, we have made three assumptions that could reasonably be conjectured as being important for our results. We have assumed that the capital utilization rate is fixed; that preferences have a particular functional form, that is, logarithmic, in both consumption and leisure; and that the economy has no adjustment costs in investment. Some researchers have argued that capital utilization rates fluctuate systematically over the business cycle; others, that labor supply is less elastic than in our specification; and yet others, that adjustment costs are essential in understanding investment behavior. If those arguments are correct, then our procedure mismeasures the wedges. If capital utilization rates fluctuate systematically, then our procedure mismeasures the efficiency wedge; if labor supply is less elastic than we have assumed, then our procedure mismeasures the labor wedge; and if adjustment costs are important, then our procedure mismeasures the investment wedge. Here we demonstrate that changing these assumptions—allowing for variable capital utilization, less elastic labor supply, or adjustment costs—has little effect on our findings.

We establish these results quantitatively and prove some propositions that provide intuition for them. The changes turn out to produce offsetting effects, leaving our results unchanged. Allowing for variable capital utilization decreases the variability of the efficiency wedge and increases that of the labor wedge. This change in the relative variability of these two wedges does change the relative amounts of the business cycle movements separately accounted for by these wedges. However, the relative variability change has almost no effect on the sum of the contributions due to these two wedges and, thus, also essentially none on the amount of fluctuations accounted for by the investment wedge. As such, allowing for variable capital utilization does not alter our conclusion that investment wedges played almost no role in the Great Depression or the 1982 recession. Similarly, reducing the elasticity of the labor supply increases the variability of the labor wedge. But that increased variability of labor is offset by the reduced responsiveness to it, and the overall effect is minimal. Finally, allowing for adjustment costs effectively rescales the investment wedge but does not otherwise play a quantitatively large role.

These findings suggest that alone the sizes of the measured wedges are not informative for assessing competing business cycle models. The three examples in this section show that the equilibrium responses can be quite similar even though the sizes of the wedges are quite different. Constructing examples in which two models have similar-sized wedges but very different equilibrium responses should be easy. The lesson we draw from these findings is that competing business cycle models should be assessed by the equilibrium responses to the wedges, not by the wedges alone.

4.2. *Details of Alternative Specifications*

a. *Variable Capital Utilization*

Here we consider an extreme view about the amount of variability in capital utilization and show that this change does not alter our main conclusion about the lack of importance of the investment wedge.

Our alternative specification of the technology which allows for variable capital utilization follows the work of Kydland and Prescott (1988) and Hornstein and Prescott (1993). We assume that the production function is now

$$y = A(kh)^{\alpha}(nh)^{1-\alpha}, \tag{24}$$

where n is the number of workers employed and h is the length (or *hours*) of the workweek. The labor input is, then, $l = nh$.

In the data, we measure only the labor input l and the capital stock k. We do not directly measure h or n. The benchmark specification for the production function can be interpreted as assuming that all of the observed variation in measured labor input l is in the number of workers and that the workweek h is constant. Under this interpretation, our benchmark specification with fixed capital utilization correctly measures the efficiency wedge (up to the constant h).

Here we investigate the opposite extreme: assume now that the number of workers n is constant and that all the variation in labor is from the workweek h. Under this *variable capital utilization* specification, the services of capital kh are proportional to the product of the stock k and the labor input l, so that variations in the labor input induce variations in the flow of capital services. Thus, the capital utilization rate is proportional to the labor input l, and the efficiency wedge is proportional to y/k^{α}.

In Figure 9, we plot the measured efficiency wedges for the two specifications during the Great Depression period. Clearly, the efficiency wedge falls less and recovers more by 1939 when capital utilization is variable than when it is fixed. We do not plot either the labor wedge or the investment wedge because they are identical, up to a scale factor, in the two specifications.

In Figure 10, we plot the data and the predicted output due to the efficiency and labor wedges for the 1930s. Comparing Figures 10 and 2, we see that with the remeasured efficiency wedge, the labor wedge plays a much larger role in accounting for the downturn and the slow recovery and the efficiency wedge plays a much smaller role.

In Figure 11, we plot the three data series and the predictions of the model with just the investment wedge. We see that with variable capital utilization, the investment wedge still accounts for none of the movements in the data.

In Figure 12, we compare the contributions of the sum of the efficiency and labor wedges for the two specifications of capital utilization (fixed and variable). The figure shows that these contributions are quite similar. While remeasuring the efficiency wedge as we have changes the relative contributions of the two wedges, it clearly has little effect on their combined contribution. Taking account of variable capital utilization thus does not change the basic result that in the Great Depression period, the efficiency and labor wedges played a central role and the investment wedge a minor role, at best.

Our findings suggest a more general result with regard to capital utilization: if investment wedges account for only a small fraction of fluctuations when capital utilization is fixed, then this fraction will also be small with variable capital utilization. Here we prove a proposition that provides a theoretical rationale for such a result in the extreme case in which the contribution of the investment wedge to fluctuations is zero.

Consider an economy identical to a deterministic version of our benchmark model except that the production function is given by $y = Ak^{\alpha}l^{\gamma}$. Note that setting $\gamma = 1 - \alpha$ yields our benchmark model, while setting $\gamma = 1$ yields the variable capital utilization model. Let there be two such economies indexed by $i = 1, 2$ with γ equal to γ_1 and γ_2, respectively, and the same initial capital stocks. For some given sequence of wedges $(A_{1t}, \tau_{l1t}, \tau_{x1t})$, let y_{1t}, c_{1t}, l_{1t}, and x_{1t} denote the resulting equilibrium outcomes in the economy with $\gamma = \gamma_1$. We then have this proposition:

PROPOSITION 3: *If the sequence of wedges for economy 2 is given by* $A_{2t} = A_{1t}l_{1t}^{(\gamma_1 - \gamma_2)}$, $1 - \tau_{l2t} = \gamma_1(1 - \tau_{l1t})/\gamma_2$, *and* $\tau_{x2t} = \tau_{x1t}$, *then the equilibrium outcomes* y_{2t}, c_{2t}, l_{2t}, *and* x_{2t} *for this economy coincide with the equilibrium outcomes* y_{1t}, c_{1t}, l_{1t}, *and* x_{1t} *for economy 1.*

PROOF: We prove this proposition by showing that the equilibrium conditions of economy 2 are satisfied at the equilibrium outcomes of economy 1. Since $y_{1t} = A_{1t}k_{1t}^{\alpha}l_{1t}^{\gamma_1}$, using the definition of A_{2t}, we have that $y_{1t} = A_{2t}k_{1t}^{\alpha}l_{1t}^{\gamma_2}$. The first-order condition for labor in economy 1 is

$$-\frac{U_{lt}(c_{1t}, l_{1t})}{U_{ct}(c_{1t}, l_{1t})} = (1 - \tau_{l1t})\frac{\gamma_1 y_{1t}}{l_{1t}}.$$

Using the definition of τ_{l2t}, we have that

$$-\frac{U_{lt}(c_{1t}, l_{1t})}{U_{ct}(c_{1t}, l_{1t})} = (1 - \tau_{l2t})\frac{\gamma_2 y_{1t}}{l_{1t}}.$$

The rest of the equations governing the equilibrium are unaffected. *Q.E.D.*

This proposition implies that if τ_{x1t} is a constant, so that the contribution of the investment wedge to fluctuations in economy 1 is zero, then τ_{x2t} is also a constant and, hence, the contribution of the investment wedge to fluctuations in economy 2 is also zero. Extending this proposition to a stochastic environment is just a matter of changing notation.

Notice from Proposition 3 that the size of the measured wedges will be different when the labor exponents, γ_1 and γ_2, are different, but the outcomes will be the same. To understand why, consider the following thought experiment. Generate data from economy 1 and measure the wedges using the parameter values from economy 2. If these measured wedges are fed back into economy 2, then the data generated from economy 1 will be recovered.

b. *Different Labor Supply Elasticities*

Now we consider the effects on our results if another specification is changed: the elasticity of labor supply. Assume now that this elasticity is less than it was in our earlier analysis. We show that for two economies with differing labor supply elasticities, a result analogous to that in Proposition 3 holds: allowing for different labor supply elasticities changes the size of the measured labor wedge, but does not change the measured investment wedge. Therefore, if the contribution of the investment wedge is zero in an economy with a high labor supply elasticity, it is also zero in an economy with a low labor supply elasticity. In other words, our results are not affected by changing the labor supply elasticity.

To see that, consider two economies identical to a deterministic version of our benchmark model except that the utility function is given by

$$U(c) + V_i(1 - l)$$

for $i = 1, 2$. In our benchmark model, both U and V_i are logarithmic. Clearly, by varying the function V_i, we can generate a wide range of alternative labor supply elasticities.

For some given sequence of wedges $(A_{1t}, \tau_{l1t}, \tau_{x1t})$, let y_{1t}, c_{1t}, l_{1t}, and x_{1t} denote the resulting equilibrium outcomes in economy 1. Let the initial capital stocks be the same in economies 1 and 2. We then have this proposition:

PROPOSITION 4: *If the sequence of wedges for economy 2 is given by*

$$1-\tau_{l2t}=(1-\tau_{l1t})\frac{V_2'(1-l_{1t})}{V_1'(1-l_{1t})},$$

and if $A_{2t}=A_{1t}$ *and* $\tau_{x2t}=\tau_{x1t}$, *then the equilibrium outcomes for economy 2 coincide with those of economy 1.*

PROOF: We prove this proposition by showing that the equilibrium conditions of economy 2 are satisfied at the equilibrium outcomes of economy 1. The first-order condition for labor input in economy 1 is

$$-\frac{V_1'(1-l_{1t})}{U'(c_{1t})}=(1-\tau_{l1t})\frac{(1-\alpha)y_{1t}}{l_{1t}}.$$

Using the definition of τ_{l2t}, we have that

$$-\frac{V_2'(1-l_{1t})}{U'(c_{1t})}=(1-\tau_{l2t})\frac{(1-\alpha)y_{1t}}{l_{1t}},$$

so that the first-order condition for labor in economy 2 is satisfied. The rest of the equations governing the equilibrium are unaffected. *Q.E.D.*

Since the investment wedges are the same in both economies, it follows that if the investment wedge is constant in one economy, it is constant in the other, and the contribution of the investment wedge to fluctuations is zero in both economies. Extending this proposition to a stochastic environment is simply a matter of notation.

c. *Adjustment Costs*

Now we consider one more specification change: extending the prototype growth model to allow for investment adjustment costs. This extension does not substantially change our results either.

In this extension, the capital accumulation law is no longer (1), but rather

$$(1+\lambda)k_{t+1}=(1-\delta)k_t+x_t-\phi\left(\frac{x_t}{k_t}\right)k_t, \tag{25}$$

where ϕ represents the per unit cost of adjusting the capital stock. We assume that

$$\phi\left(\frac{x}{k}\right)=\frac{a}{2}\left(\frac{x}{k}-\delta-\gamma-\lambda\right)^2. \tag{26}$$

In our technical appendix, Chari, Kehoe, and McGrattan (2005), we compare the economies with and without adjustment costs. For the Great Depression period, we find that allowing for adjustment costs leads our measured investment wedge to fluctuate much less than it did without these costs. We also find that the investment wedge alone still accounts for essentially none of the movements in output, labor, and investment in the Great Depression period.

To get some intuition for these findings, note that the equilibrium conditions are the same with adjustment costs as in the prototype growth model, except for the capital accumulation law (25) and the intertemporal Euler equation (5), which is now

$$\frac{1+\tau_{xt}}{1-\phi'(x_t/k_t)}U_{ct}=$$

$$\beta E_t U_{ct+1}\left\{A_{t+1}F_{kt+1}+\frac{1+\tau_{xt+1}}{1-\phi'(x_{t+1}/k_{t+1})}\left[1-\delta-\phi\left(\frac{x_{t+1}}{k_{t+1}}\right)+\phi'\left(\frac{x_{t+1}}{k_{t+1}}\right)\left(\frac{x_{t+1}}{k_{t+1}}\right)\right]\right\}$$

The last two terms in the brackets, $-\phi(\frac{x_{t+1}}{k_{t+1}})+\phi'(\frac{x_{t+1}}{k_{t+1}})(\frac{x_{t+1}}{k_{t+1}})$, are easily shown to be small compared to δ. To a first approximation, then, the effect on the intertemporal Euler equation of introducing adjustment costs is to rescale the investment tax. (This rescaling occurs in the sense that in terms of the intertemporal Euler equation, a tax of $1+\tau_{xt}$ in the adjustment cost economy acts like a tax of $(1+\tau_{xt})/[1-\phi'(x_t/k_t)]$ in the economy without adjustment costs.) Adjustment costs, of course, have other effects as well, by using up real resources. If these other effects were zero, then adjustment costs would simply lead the investment wedge to be rescaled, but would have no effect on output, labor, or investment. Our quantitative exercises indicate that these other effects are small.

5. Reviewing The Related Literature

Our work here is related to the existing literature in terms of methodology and the interpretation of the wedges.

5.1. *Methodology*

Our basic method is to use restrictions from economic theory to back out wedges from the data, formulate stochastic processes for these wedges, and then put them back into a quantitative general equilibrium model for an accounting exercise. This basic idea is at the heart of an enormous amount of work in the real business cycle theory literature. Prescott (1986), for example, explicitly asks what fraction of the variance of output can plausibly be attributed to productivity shocks, which we have referred to as the *efficiency wedge*. Subsequent studies have expanded this general equilibrium accounting exercise to include a wide variety of other shocks. (See, for example, the studies in Cooley's 1995 volume.)

An important difference between our method and others is that we back out the labor wedge and the investment wedge from the combined consumer and firm first-order conditions, while most of the recent business cycle literature uses direct measures of labor and investment shocks. Perhaps the most closely related precursor of our method is McGrattan's (1991); she uses the equilibrium of her model to infer the implicit wedges. Ingram, Kocherlakota, and Savin (1994) advocate a similar approach.

5.2. *Wedge Interpretations and Assessments*

The idea that taxes of various kinds distort the relation between various marginal rates is the cornerstone of public finance. Taxes are not the only well-known distortions; monopoly power by unions or firms is also commonly thought to produce a labor wedge. And the idea that a labor wedge is produced by sticky wages or sticky prices is the cornerstone of the new Keynesian approach to business cycles; see, for example, the recent survey by Rotemberg and

Woodford (1999). One contribution of our work here is to show the precise mapping between the wedges and general equilibrium models with frictions.

Many studies have plotted one or more of the four wedges. The efficiency wedge has been extensively studied. (See, for example, the studies in the 1995 Cooley volume and those introduced by Kehoe and Prescott (2002).) The labor wedge has also been studied. For example, Parkin (1988), Hall (1997), and Gali, Gertler, and López-Salido (2002) all graph and interpret the labor wedge for the postwar data. Parkin (1988) discusses how monetary shocks might drive the wedge. Hall (1997) mostly interprets the wedge as a preference shock, but also discusses a search interpretation. Gali, Gertler, and López-Salido (2002) discuss a variety of interpretations, as do Rotemberg and Woodford (1991, 1999). Mulligan (2002a, b) plots the labor wedge for the United States for much of the 20th century, including the Great Depression period. He interprets movements in this wedge as arising from changes in labor market institutions and regulation, including features we discuss here. Cole and Ohanian (2002) plot the labor wedge for the Great Depression and offer interpretations similar to ours. The investment wedge has been investigated by McGrattan (1991), Braun (1994), Carlstrom and Fuerst (1997), and Cooper and Ejarque (2000).

6. Conclusion

This study is aimed at applied theorists who are interested in building detailed, quantitative models of economic fluctuations. Once such theorists have chosen the primitive sources of shocks to economic activity, they need to choose the mechanisms through which the shocks lead to business cycle fluctuations. We have shown that these mechanisms can be summarized by their effects on four wedges in the standard growth model. Our business cycle accounting method can be used to judge which mechanisms are promising and which are not, thus helping theorists narrow down their options.

Here we have demonstrated how our method works by applying it to the Great Depression and to a typical U.S. recession. We have found that efficiency and labor wedges, in combination, account for essentially all of the decline and recovery in these business cycles; investment wedges play, at best, a minor role. These results hold in summary statistics of the entire postwar period and in alternative specifications of the growth model. The findings imply that existing models of credit market frictions, such as those of Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997), can account for only a small fraction of the fluctuations in the Great Depression or more typical U.S. downturns. These findings are our primary substantive contribution.

These findings do not imply, of course, that frictions in financial markets are irrelevant for business cycle fluctuations. Indeed, we have shown that a detailed economy with input-financing frictions is equivalent to a prototype economy with efficiency wedges. In this sense, while existing models of credit market frictions are not promising, new models in which financial frictions show up as efficiency and labor wedges are.

More generally, our results suggest that future theoretical work should focus on developing models which lead to fluctuations in efficiency and labor wedges. Many existing models produce fluctuations in labor wedges. The challenging task is to develop detailed models in which primitive shocks lead to fluctuations in efficiency wedges as well.

Notes

[1]In Chari, Kehoe, and McGrattan (2005), we apply a spectral method to determine the contributions of the wedges based on the population properties of the stochastic process generated by the model. We do this in both periods and find that the investment wedge plays only a modest role.

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APPENDIX: Mapping of Labor and Investment Wedges

Here we demonstrate the mapping from two more detailed economies with frictions to two prototype growth economies with wedges. In the preceding text, we have described the mapping for efficiency and government consumption wedges. In this appendix, we describe it for labor and investment wedges as well.

A. Labor Wedges Due to Sticky Wages

Here we show that a sticky wage monetary economy maps into a (real) prototype growth economy with labor wedges. In the detailed economy, the primitive shocks are to monetary policy, while in the prototype economy, they are to the labor wedge.

a. *A Detailed Economy With Sticky Wages*

Consider a monetary economy populated by a large number of identical, infinitely lived consumers, with notation for uncertainty similar to that above. The economy consists of a competitive final goods producer and a continuum of monopolistically competitive unions that set their nominal wages in advance of the realization of the shocks. Each union represents all consumers who supply a specific type of labor.

In each period t, the commodities in this economy are a consumption-capital good, money, and a continuum of differentiated types of labor indexed by $j \in [0,1]$. The technology for producing final goods from capital and a labor aggregate at history s^t has constant returns to scale and is given by

$$(27) \quad y(s^t) = F\big(k(s^{t-1}), l(s^t)\big),$$

where $y(s^t)$ is output of the final good, $k(s^{t-1})$ is capital, and

$$(28) \quad l(s^t) = \left[\int l(j,s^t)^v \, dj\right]^{\frac{1}{v}}$$

is an aggregate of the differentiated types of labor $l(j,s^t)$.

The final goods producer behaves competitively. This producer has some initial capital stock $k(s^{-1})$ and accumulates capital according to

$$(29) \quad k(s^t) = (1-\delta)k(s^{t-1}) + x(s^t)$$

where $x(s^t)$ is investment. The present discounted value of profits for this producer is

$$(30) \quad \sum_{t=0}^{\infty}\sum_{s^t} Q(s^t)\left[P(s^t)y(s^t) - P(s^t)x(s^t) - W(s^{t-1})l(s^t)\right],$$

where $Q(s^t)$ is the price of a dollar at s^t in an abstract unit of account, $P(s^t)$ is the dollar price of final goods at s^t, and $W(s^{t-1})$ is the aggregate nominal wage at s^t which depends on only s^{t-1} because of wage stickiness.

The producer's problem can be stated in two parts. First, the producer chooses sequences for capital $k(s^{t-1})$, investment $x(s^t)$, and aggregate labor $l(s^t)$ in order to maximize (30) subject to (27) and (29). The first-order conditions can be summarized by

$$(31) \quad P(s^t)F_l(s^t) = W(s^{t-1}) \text{ and}$$

$$(32) \quad Q(s^t)P(s^t) = \sum_{s_{t+1}} Q(s^{t+1})P(s^{t+1})[F_k(s^{t+1}) + 1 - \delta].$$

Second, for any given amount of aggregate labor $l(s^t)$, the producer's demand for each type of differentiated labor is given by the solution to

$$(33) \quad \min_{\{l(j,s^t)\}, j\in[0,1]} \int W(j, s^{t-1})l(j, s^t)\, dj$$

subject to (28), where $W(j, s^{t-1})$ is the nominal wage for differentiated labor of type j. Nominal wages are set by unions before the realization of the event in period t; thus, wages can depend on, at most, s^{t-1}. The demand for labor of type j by the final goods producer is

$$(34) \quad l^d(j, s^t) = \left[\frac{W(s^{t-1})}{W(j, s^{t-1})}\right]^{\frac{1}{1-v}} l(s^t),$$

where $W(s^{t-1}) \equiv \left[\int W(j, s^{t-1})^{\frac{v}{v-1}}\, dj\right]^{\frac{v-1}{v}}$ is the aggregate nominal wage. The minimized value in (33) is thus $W(s^{t-1})l(s^t)$.

Consumers can be thought of as being organized into a continuum of unions indexed by j. Each union consists of all the consumers in the economy with labor of type j. Each union realizes that it faces a downward-sloping demand for its type of labor, given by (34). In each period, these new wages are set before the realization of the current shocks.

The preferences of a representative consumer in the jth union is

$$(35) \quad \sum_{t=0}^{\infty}\sum_{s^t} \beta^t \pi(s^t)\, U\big(c(j, s^t), l(j, s^t), M(j, s^t)/P(s^t)\big),$$

where $c(j, s^t), l(j, s^t), M(j, s^t)$ are the consumption, labor supply, and money holdings of this consumer and $P(s^t)$ is the economy's overall price level. This economy has complete markets for state-contingent nominal claims. We represent the asset structure by having complete, contingent, one-period nominal bonds. We let $B(j, s^{t+1})$ denote the consumers' holdings of such a bond purchased in period t and with history s^t with payoffs contingent on some particular event s_{t+1} in $t+1$, where $s^{t+1} = (s^t, s_{t+1})$. One unit of this bond pays one dollar in period $t+1$ if the particular event s_{t+1} occurs and 0 otherwise. Let $Q(s^{t+1}|s^t)$ denote the dollar price of this bond in period t and at history s^t. Clearly, $Q(s^{t+1}|s^t) = Q(s^{t+1})/Q(s^t)$.

The problem of the jth union is to maximize (35) subject to the budget constraint

$$P(s^t)c(j, s^t) + M(j, s^t) + \sum_{s_{t+1}} Q(s^{t+1}|s^t)B(j, s^{t+1})$$
$$\leq W(j, s^{t-1})l^d(j, s^t) + M(j, s^{t-1}) + B(j, s^t) + T(s^t) + D(s^t)$$

and the borrowing constraint $B(s^{t+1}) \geq -P(s^t)\bar{b}$, where $l^d(j, s^t)$ is given by (34). Here $T(s^t)$ is transfers and the positive constant $\bar{b}$ constrains the amount of real borrowing by the consumer. Also, $D(s^t) = P(s^t)y(s^t) - P(s^t)x(s^t) - W(s^{t-1})l(s^t)$ are the dividends paid by the firms. The initial conditions $M(j, s^{-1})$ and $B(j, s^0)$ are given and assumed to be the same for all j. Notice that in this problem, the union chooses the wage and agrees to supply whatever labor is demanded at that wage.

The first-order conditions for this problem can be summarized by

$$(36) \quad \frac{U_m(j, s^t)}{P(s^t)} - \frac{U_c(j, s^t)}{P(s^t)} + \beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t) \frac{U_c(j, s^{t+1})}{P(s^{t+1})} = 0,$$

$$(37) \quad Q(s^t|s^{t-1}) = \beta\pi(s^t|s^{t-1}) \frac{U_c(j, s^t)}{U_c(j, s^{t-1})} \frac{P(s^{t-1})}{P(s^t)}, \text{ and}$$

$$(38) \quad W(j, s^{t-1}) = -\frac{\sum_{s^t} Q(s^t)P(s^t)U_l(j, s^t)/U_c(j, s^t)l^d(j, s^t)}{v\sum_{s^t} Q(s^t)l^d(j, s^t)}.$$

Here $\pi(s^{t+1}|s^t) = \pi(s^{t+1})/\pi(s^t)$ is the given conditional probability of s^{t+1}. Notice that in a steady state, this condition reduces to $W/P = (1/v)(-U_l/U_c)$, so that real wages are set as a markup over the marginal rate of substitution between labor and consumption. Clearly, given the symmetry among the unions, we know that all of them choose the same consumption, labor, money balances, bond holdings, and wages, which we denote simply by $c(s^t)$, $l(s^t)$, $M(s^t)$, $B(s^{t+1})$, and $W(s^{t-1})$.

Consider next the specification of the money supply process and the market-clearing conditions. The nominal money supply process is given by $M(s^t) = \mu(s^t)M(s^{t-1})$, where $\mu(s^t)$ is a stochastic process. New money balances are distributed to consumers in a lump-sum fashion by having nominal transfers satisfy $T(s^t) = M(s^t) - M(s^{t-1})$. The resource constraint for this economy is $c(s^t) + k(s^t) = y(s^t) + (1 - \delta)k(s^{t-1})$. Bond market–clearing requires that $B(s^{t+1}) = 0$.

b. *The Associated Prototype Economy With Labor Wedges*

Consider now a prototype economy with money and labor wedges and a technology given by (27). The representative firm maximizes (30) subject to (29). The first-order conditions can be summarized by (31) and (32). The representative consumer maximizes

$$(39) \quad \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t)\, U\big(c(s^t), l(s^t), M(s^t)/P(s^t)\big)$$

subject to the budget constraint

$$P(s^t)c(s^t) + M(s^t) + \sum_{s_{t+1}} Q(s^{t+1}|s^t)B(s^{t+1})$$
$$\leq W(s^t)[1 - \tau_l(s^t)]l(s^t) + M(s^{t-1}) + B(s^t) + T(s^t) + D(s^t)$$

and a bound on bond holdings, where the lump-sum transfer $T(s^t) = M(s^t) - M(s^{t-1}) + \tau_l(s^t)l(s^t)$ and the dividends $D(s^t) = P(s^t)y(s^t) - P(s^t)x(s^t) - W(s^{t-1})l(s^t)$. Here the first-order conditions for money and bonds are identical to those in (36) and (37) once symmetry has been imposed on them. The first-order condition for labor is given by

$$-\frac{U_l(s^t)}{U_c(s^t)} = [1 - \tau_l(s^t)]\frac{W(s^t)}{P(s^t)}.$$

Consider an equilibrium of the sticky wage economy for some given stochastic process $M^*(s^t)$ on money growth. Denote all of the allocations and prices in this equilibrium with asterisks. Then we can easily establish this proposition:

PROPOSITION 5: *Consider the prototype economy just described with a given stochastic process for money growth $M(s^t) = M^*(s^t)$ and labor wedges given by*

$$(40) \quad 1 - \tau_l(s^t) = -\frac{U_l^*(s^t)}{U_c^*(s^t)}\frac{1}{F_l^*(s^t)},$$

where $U_l^*(s^t), U_c^*(s^t)$, and $F_l^*(s^t)$ are evaluated at the equilibrium of the sticky wage economy. Then the equilibrium allocations and prices in the sticky wage economy are in the prototype economy.

The proof of this proposition is immediate from comparing the first-order conditions, the budget constraints, and the resource constraints for the prototype economy with money and labor wedges to those of the sticky wage economy. The key idea is that distortions between the marginal rate of substitution between leisure and consumption and the marginal product of labor implicit in (38) for the sticky wage economy are perfectly captured by the labor wedge (40) in the prototype economy.

Suppose next that the utility function of consumers in the sticky wage economy is additively separable in money, so that $U(c, l, m) = u(c, l) + v(m)$. Consider a real version of the prototype economy with labor wedges. Let the utility function be

$$(41) \quad \sum_{t=0}^{\infty}\sum_{s^t} \beta^t \pi(s^t)\, u\big(c(s^t), l(s^t)\big)$$

and the technology be the same as in the monetary prototype economy. Define the rest of the economy in the obvious way. The following is immediate:

COROLLARY 1: *Consider the real prototype economy just described with a given stochastic process for labor wedges*

$$1 - \tau_l(s^t) = -\frac{u_l^*(s^t)}{u_c^*(s^t)}F_l^*(s^t),$$

where $u_l^*(s^t), u_c^*(s^t)$, and $F_l^*(s^t)$ are evaluated at the equilibrium of the sticky wage economy with preferences of the form (41). Then the equilibrium allocations in the sticky wage economy coincide with those in the real prototype economy.

B. Investment Wedges Due to Investment Frictions

A variety of investment frictions affect the economy by raising the cost of investment. These frictions show up in prototype economies as taxes on investment. Some investment frictions also show up as wasted resources in both the resource constraint and the capital accumulation equation. One example of that sort of friction is due to Carlstrom and Fuerst (1997), who exposit a quantitative version of Bernanke and Gertler's (1989) model. Here we show the equivalence between the Carlstrom and Fuerst model and a prototype growth model with adjustment costs.

a. *A Detailed Economy With Investment Frictions*

The Carlstrom and Fuerst model has a continuum of risk-neutral entrepreneurs of mass η and a continuum of consumers of mass 1. The timing is as follows. At the beginning of each period, each consumer supplies l_t units of labor, each entrepreneur supplies l_{et} units of labor, and each consumer and each entrepreneur rent capital denoted k_{ct} and k_{et} to firms that produce output according to the technology $F(k_{ct}+\eta k_{et}, l_t, \eta l_{et})$. These firms solve

$$\max \; F(k_{ct}+\eta k_{et}, l_t, \eta l_{et}) - r_t(k_{ct}+\eta k_{et}) - w_t l_t - w_{et} l_{et}$$

where r_t is the rental rate on capital and w_t and w_{et} are the wage rates of consumers and entrepreneurs.

Consumers solve the problem of maximizing utility

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

$$c_t + q_t[k_{ct+1} - (1-\delta)k_{ct}] = w_t l_t + r_t k_{ct} + T_t,$$

where c_t is their consumption, q_t is the price of the investment good in units of the consumption good, and T_t is a lump-sum transfer. Combining the first-order conditions for the firms and consumers gives

$$(42) \qquad -\frac{U_{lt}}{U_{ct}} = F_{lt}, \text{ and}$$

$$(43) \qquad q_t U_{ct} = \beta U_{ct+1}[q_{t+1}(1-\delta) + F_{kt+1}].$$

Consumption goods can be transformed into capital goods only by entrepreneurs. Each entrepreneur owns a technology that transforms i_t units of consumption goods at the beginning of any period t into $\omega_t i_t$ units of capital goods at the end of the period, where the shock ω_t is i.i.d. across entrepreneurs and time and has density ϕ and c.d.f. Φ. The realization of ω_t is private information to the entrepreneur. At the beginning of each period, each entrepreneur supplies one unit of labor inelastically, receives labor income w_{et}, receives rental income $r_t k_{et}$, and pays taxes T_{et}. The value of the entrepreneur's capital is $q_t k_{et}(1-\delta)$. Thus, the entrepreneur's net worth in period t is

$$(44) \qquad a_t = w_{et} + k_{et}[r_t + q_t(1-\delta)] - T_{et}.$$

Entrepreneurs can use their net worth together with funds borrowed from financial intermediaries to purchase consumption goods and transform them into capital goods. The financial intermediaries can monitor the realized output $\omega_t i_t$ by paying a proportional cost μi_t units of the capital good. This cost is the key investment friction in the model.

The key restriction on trade in this economy is that entrepreneurs are allowed only to enter into within-period deterministic contracts that are made before the realization of ω_t and pay off after that. (In particular, the risk-neutral entrepreneurs are prohibited from entering into contracts that share aggregate risk with the consumers.) With such a restriction, we know from Townsend (1979), the optimal contract is a type of risky debt in which the entrepreneur pays a fixed amount $R_t(i_t - a_t)$ if ω_t is greater than some cutoff level $\bar{\omega}_t$ and $\omega_t i_t$ otherwise, where $R_t(i_t - a_t) = \bar{\omega}_t i_t.$ The intermediaries monitor the entrepreneur if and only if $\omega_t < \bar{\omega}_t$.

Under such a contract, the expected income of the entrepreneur is

$$q_t i_t \left[\int_{\bar{\omega}_t}^{\infty} (\omega_t - \bar{\omega}_t) \phi(\omega)\ d\omega \right] \equiv q_t i_t f(\bar{\omega}_t),$$

and the expected income of the financial intermediary is

$$q_t i_t \left[\int_0^{\bar{\omega}_t} (\omega_t - \mu) \phi(\omega)\ d\omega + [1 - \Phi(\bar{\omega}_t)] \bar{\omega}_t \right] \equiv q_t i_t g(\bar{\omega}_t).$$

The funds the intermediary lends are from the consumers. The consumers can either store their consumption goods from the beginning until the end of the period at a zero rate of return or lend their goods to the entrepreneur by saving them at the financial intermediaries. The mass of entrepreneurs is sufficiently small that the optimal contract maximizes their expected income subject to the constraint that an intermediary's gross return on the investment of $i_t - a_t$ is at least one.

The contract then solves

$$\max_{i_t, \bar{\omega}_t}\ q_t i_t f(\bar{\omega}_t)$$

subject to

$$q_t i_t g(\bar{\omega}_t) \geq i_t - a_t. \tag{45}$$

The first-order conditions imply that

$$\frac{f'(\bar{\omega}_t)}{f(\bar{\omega}_t)} + \frac{q_t g'(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} = 0, \tag{46}$$

and since (45) holds with equality, the optimal investment level is given by

$$i_t = \frac{a_t}{1 - q_t g(\bar{\omega}_t)}. \tag{47}$$

The expected income of each entrepreneur is, thus,

$$q_t i_t f(\bar{\omega}_t) = \frac{a_t q_t f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)}, \tag{48}$$

which, by the law of large numbers, is the aggregate income of entrepreneurs.

From (47), we know that each entrepreneur's investment is linear in that entrepreneur's net worth, so that aggregate investment is linear in aggregate net worth. Together the aggregation result and the law of large numbers imply that the aggregate capital held by entrepreneurs has the following law of motion:

$$(49) \quad c_{et} + q_t k_{et+1} = \big[w_{et} + k_{et}\big(r_t + q_t(1-\delta)\big) - T_{et}\big]\frac{q_t f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)}.$$

After substitution from (44) and (48), the right side of (49) is simply $q_t i_t f(\bar{\omega}_t)$.

The entrepreneur's utility function is

$$(50) \quad \sum_{t=0}^{\infty}(\beta\gamma)^t c_{et},$$

where $\gamma < 1$. We assume that entrepreneurs discount the future at a higher rate than consumers. This assumption is needed because the within-period rate of return earned by entrepreneurs is (weakly) greater than the rate of return earned by consumers. If entrepreneurs discounted the future at the same rate as consumers, then the entrepreneurs would postpone consumption indefinitely, and no equilibrium would exist.

Given the risk-neutrality of the entrepreneurs and the aggregation result, it should be clear that the optimal decisions of the entrepreneurs can be obtained by maximizing (50) subject to (49). The lump-sum tax levied on entrepreneurs is redistributed to the consumers, and hence, $T_t = \eta T_{et}$.

b. *The Associated Prototype Economy With Investment Wedges*

In the prototype economy associated with the Carlstrom and Fuerst model with investment frictions just detailed, the resource constraint is given by $c_t + x_t + g_t = F(k_t, l_t, \eta)$. The firm maximizes $F(k_t, l_t, \eta) - w_t l_t - r_t k_t$ with first-order conditions $F_{kt} = r_t$ and $F_{lt} = w_t$. Consumers maximize $\sum_{t=0}^{\infty}\beta^t U(c_t, l_t)$ subject to

$$c_t + (1+\tau_{xt})x_t = w_t l_t + r_t k_t + T_t + \pi_t \text{ and}$$

$$k_{t+1} = (1-\delta)k_t + x_t(1-\theta_t),$$

where π_t denotes profits. In equilibrium, the lump-sum transfer T_t is given by $\tau_{xt}x_t$. The first-order conditions are summarized by

$$(51) \quad -\frac{U_{lt}}{U_{ct}} = w_t \text{ and}$$

$$(52) \quad \frac{1+\tau_{xt}}{1-\theta_t}U_{ct} = \beta U_{ct+1}\left[r_{t+1} + \frac{1+\tau_{xt+1}}{1-\theta_{t+1}}(1-\delta)\right].$$

Denoting the equilibrium allocations in the Carlstrom and Fuerst economy with asterisks gives the following proposition:

PROPOSITION 6: *Consider the prototype economy just described, with given stochastic processes for adjustment costs $\theta_t = \Phi(\bar{\omega}_t^*)\mu$, capital income taxes $1+\tau_{xt} = q_t^*(1-\theta_t)$, and government* *consumption* $g_t = \eta c_{et}$. *The aggregate equilibrium allocations for this prototype economy coincide with those of the detailed economy with investment frictions.*

In this proposition, we are measuring aggregate consumption by $c_t + \eta c_{et}$ in the Carlstrom and Fuerst economy and by $c_t + g_t$ in the associated prototype economy. Proposition 6 is similar to one established by Carlstrom and Fuerst.

TABLE I

Parameters of Vector AR(1) Stochastic Process in Two Historical Episodes[a]

Estimated Using Maximum Likelihood with U.S. Data on
Output, Labor, Investment, and Government Consumption

Annual Data, 1901–40

Coefficient matrix on lagged states (P) | Coefficient matrix on shocks (Q)

$$
\begin{bmatrix}
\substack{.804 \\ (.544,1.05)} & \substack{.0510 \\ (-.0304,.145)} & \substack{-.150 \\ (-.561,.502)} & 0 \\
\substack{-.0924 \\ (-.362,.221)} & \substack{1.05 \\ (.870,1.10)} & \substack{.538 \\ (-.0466,.960)} & 0 \\
\substack{-.0262 \\ (-.468,.205)} & \substack{-.0304 \\ (-.226,.152)} & \substack{.170 \\ (-.310,.390)} & 0 \\
0 & 0 & 0 & \substack{.747 \\ (.413,.805)}
\end{bmatrix}
\begin{bmatrix}
\substack{.0516 \\ (.0381,.0622)} & 0 & 0 & 0 \\
\substack{-.0154 \\ (-.0309,.00544)} & \substack{.0459 \\ (.0231,.0548)} & 0 & 0 \\
\substack{-.00725 \\ (-.0394,.0167)} & \substack{-.00471 \\ (-.0193,.0123)} & \substack{.0327 \\ (.0093,.0429)} & 0 \\
0 & 0 & 0 & \substack{.221 \\ (.145,.277)}
\end{bmatrix}
$$

$$\text{Means of states } = [.544\,(.506, .595),\ -.186\,(-.262, -.0800),\ .278\,(.216, .355),\ -2.78\,(-2.94, -2.53)]$$

Quarterly Data, 1959:1–2004:3[b]

Coefficient matrix on lagged states (P) | Coefficient matrix on shocks (Q)

$$
\begin{bmatrix}
.764 & \substack{.0455 \\ (.0315,.0451)} & \substack{.434 \\ (.397,411)} & \substack{-.0486 \\ (-.0882,-.0433)} \\
\substack{-.0236 \\ (-.0392,-.0141)} & \substack{.995 \\ (.949,.995)} & \substack{.0397 \\ (.0298,.0475)} & \substack{-.00442 \\ (-.00758,.0121)} \\
\substack{-.0958 \\ (-.0899,-.0670)} & \substack{.0240 \\ (.0212,.0366)} & \substack{1.17 \\ (1.12,1.15)} & \substack{-.0201 \\ (-.0315,-.0170)} \\
\substack{-.0254 \\ (-.0384,-.00233)} & \substack{.0398 \\ (.0439,.0699)} & \substack{.000948 \\ (-.00333,.0185)} & \substack{.992 \\ (.973,.991)}
\end{bmatrix}
\begin{bmatrix}
\substack{.0125 \\ (.00874,.0128)} & 0 & 0 & 0 \\
\substack{-.00245 \\ (-.00487,-.00245)} & \substack{.00554 \\ (.00108,.00443)} & 0 & 0 \\
\substack{.00457 \\ (.00219,.00419)} & \substack{.000607 \\ (-.000598,.00161)} & \substack{.00185 \\ (.000746,.00239)} & 0 \\
\substack{-.00418 \\ (.00870,-.00301)} & \substack{.00436 \\ (-.00288,.0108)} & \substack{.0136 \\ (.00472,.0132)} & \substack{6.97e-7 \\ (7.39e-9,1.87e-5)}
\end{bmatrix}
$$

$$\text{Means of states } = [-.0375\,(-.0472, -.0223),\ .304\,(.294, .316),\ .356\,(.326, .359),\ -1.570\,(-1.60, -1.56)]$$

[a] To ensure stationarity, we added a penalty term to the likelihood function proportional to $\max(|\lambda_{max}| - .995, 0)^2$, where λ_{max} is the maximal eigenvalue of P. Numbers in parentheses are 90 percent confidence intervals for a bootstrapped distribution with 500 replications.

[b] The (1,1) element of P is set residually after imposing the condition that one eigenvalue is equal to 0.995. This was done to achieve better performance in hill climbing when computing confidence intervals.

Sources of basic data: See Chari, Kehoe, and McGrattan (2005).

TABLE II

Properties of the Wedges, 1959:1–2004:3[a]

A. Summary Statistics

Wedges	Standard Deviation Relative to Output	Cross Correlation of Wedge with Output at Lag $k =$ −2	−1	0	1	2
Efficiency	.62	.65	.76	.85	.61	.35
Labor	.92	.52	.65	.71	.73	.68
Investment	.25	−.49	−.60	−.70	−.50	−.31
Government Consumption	1.51	−.42	−.42	−.33	−.24	−.11

B. Cross Correlations

Wedges (X, Y)	Cross Correlation of X with Y at Lag $k =$ −2	−1	0	1	2
Efficiency, Labor	.57	.48	.30	.28	.17
Efficiency, Investment	−.34	−.59	−.85	−.61	−.37
Efficiency, Government Consumption	−.27	−.33	−.34	−.35	−.31
Labor, Investment	−.18	−.18	−.15	−.28	−.35
Labor, Government Consumption	−.02	−.22	−.38	−.47	−.50
Investment, Government Consumption	−.02	−.05	−.16	−.04	.01

[a] Series are first logged and detrended using the HP filter.

TABLE III

PROPERTIES OF THE OUTPUT COMPONENTS, 1959:1–2004:3[a]

A. Summary Statistics

Output Components	Standard Deviation Relative to Output	Cross Correlation of Component with Output at Lag $k =$ −2	−1	0	1	2
Efficiency	1.49	.64	.77	.87	.63	.39
Labor	.60	.52	.67	.73	.76	.71
Investment	.74	−.47	−.59	−.70	−.52	−.34
Government Consumption	.55	−.44	−.45	−.38	−.29	−.15

B. Cross Correlations

Output Components (X, Y)	Cross Correlation of X with Y at Lag $k =$ −2	−1	0	1	2
Efficiency, Labor	.58	.51	.35	.33	.21
Efficiency, Investment	−.37	−.60	−.85	−.61	−.37
Efficiency, Government Consumption	−.28	−.34	−.36	−.37	−.33
Labor, Investment	−.19	−.20	−.17	−.29	−.36
Labor, Government Consumption	−.12	−.32	−.48	−.54	−.55
Investment, Government Consumption	−.02	−.04	−.13	−.02	.03

[a] Series are first logged and detrended using the HP filter.

Figures 1–4

Examining the U.S. Great Depression
Annually, 1929–39; Normalized to Equal 100 in 1929

Figure 1
U.S. Output and Three Measured Wedges

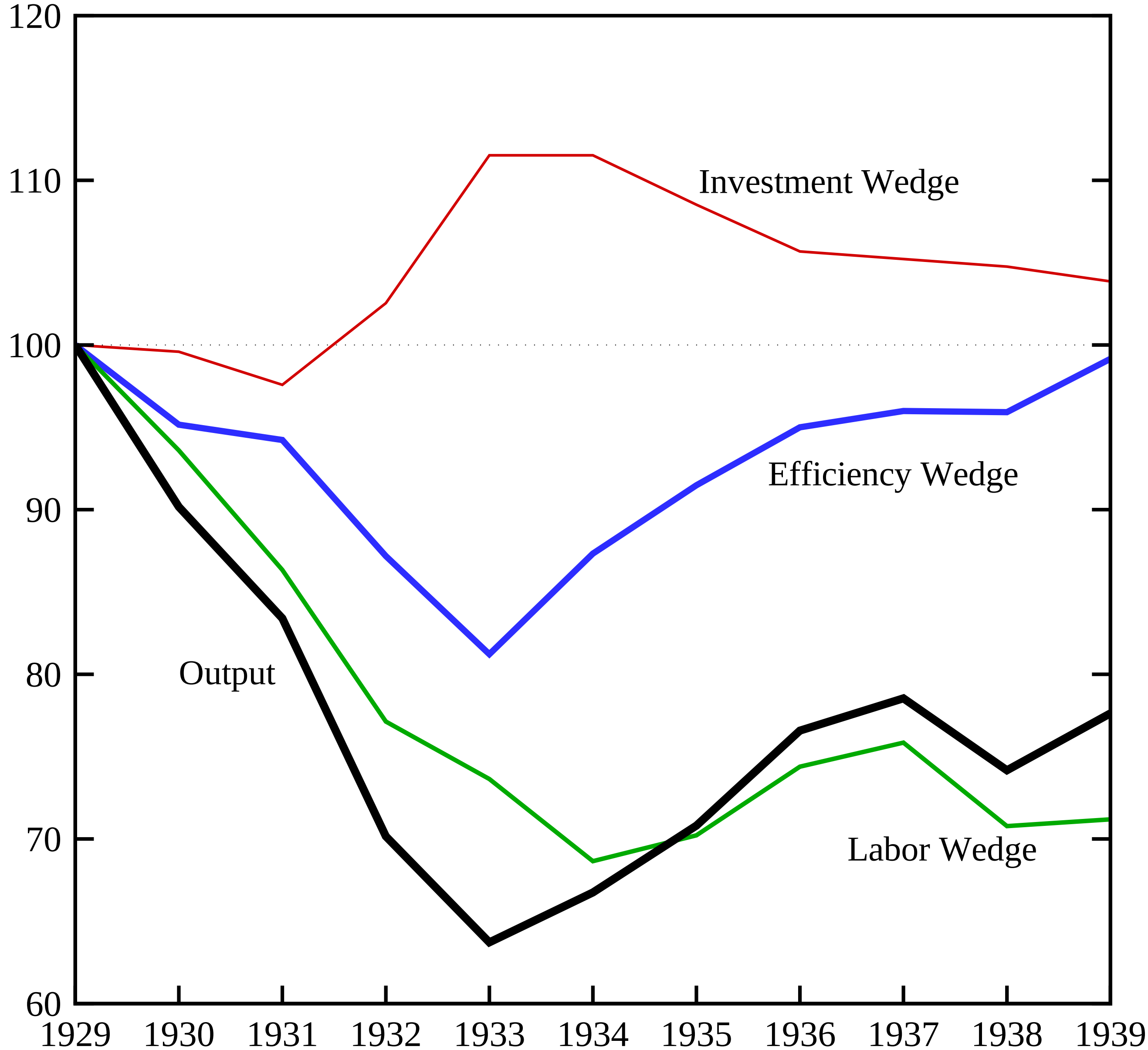


Figure 2
Data and Predictions of Models With Just One Wedge

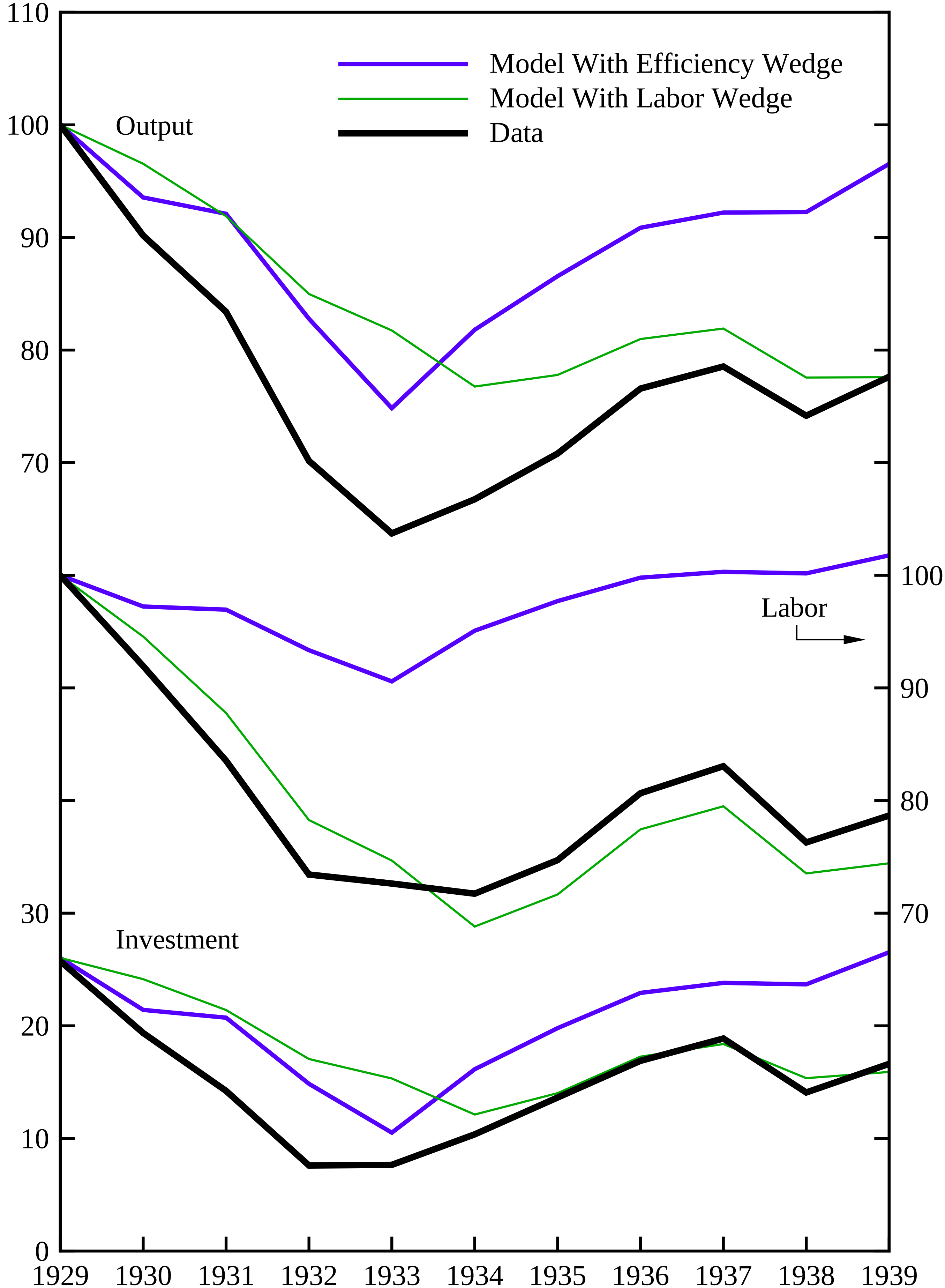


Figure 3
Data and Predictions of a Model With Just the Investment Wedge

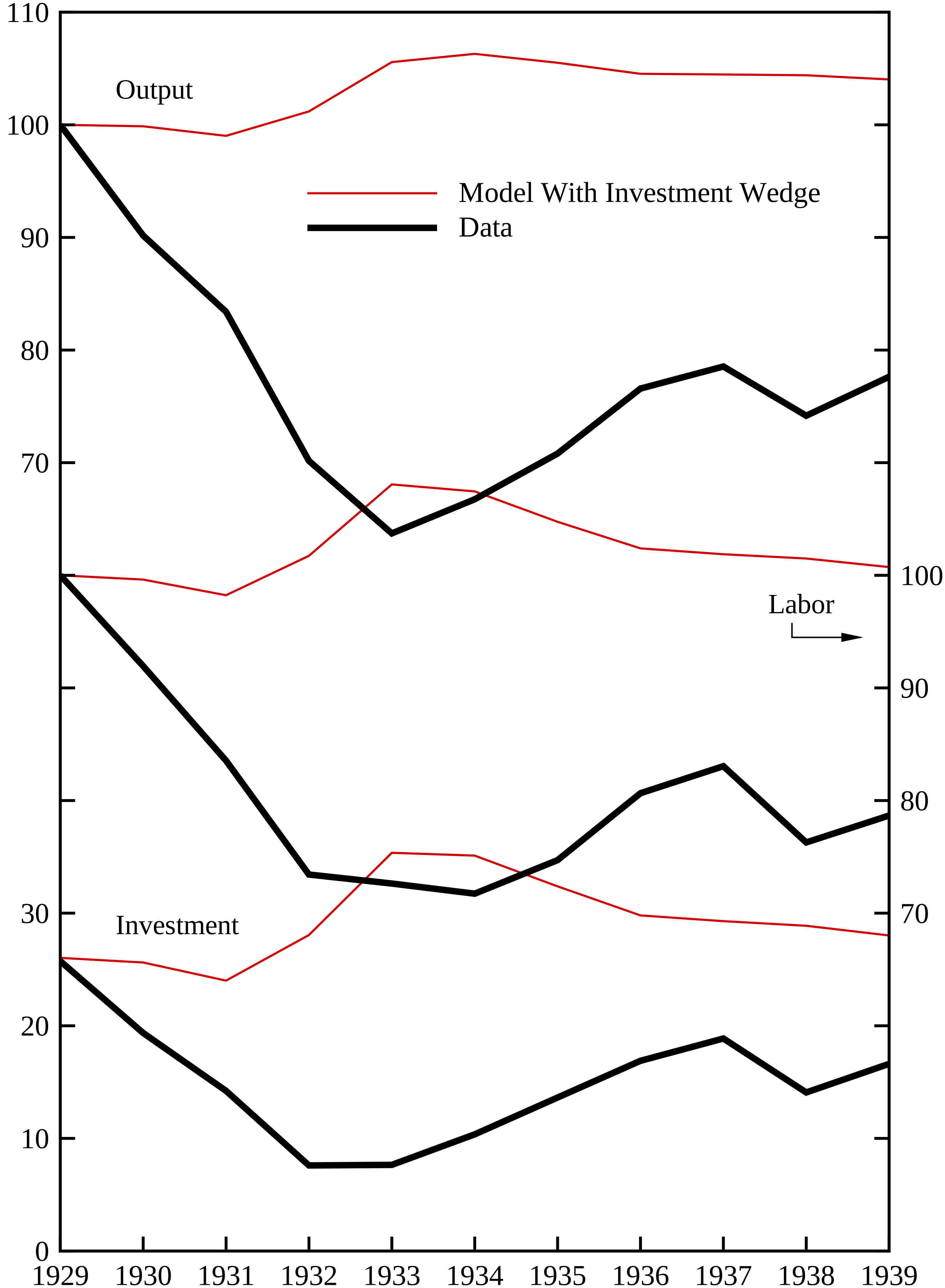
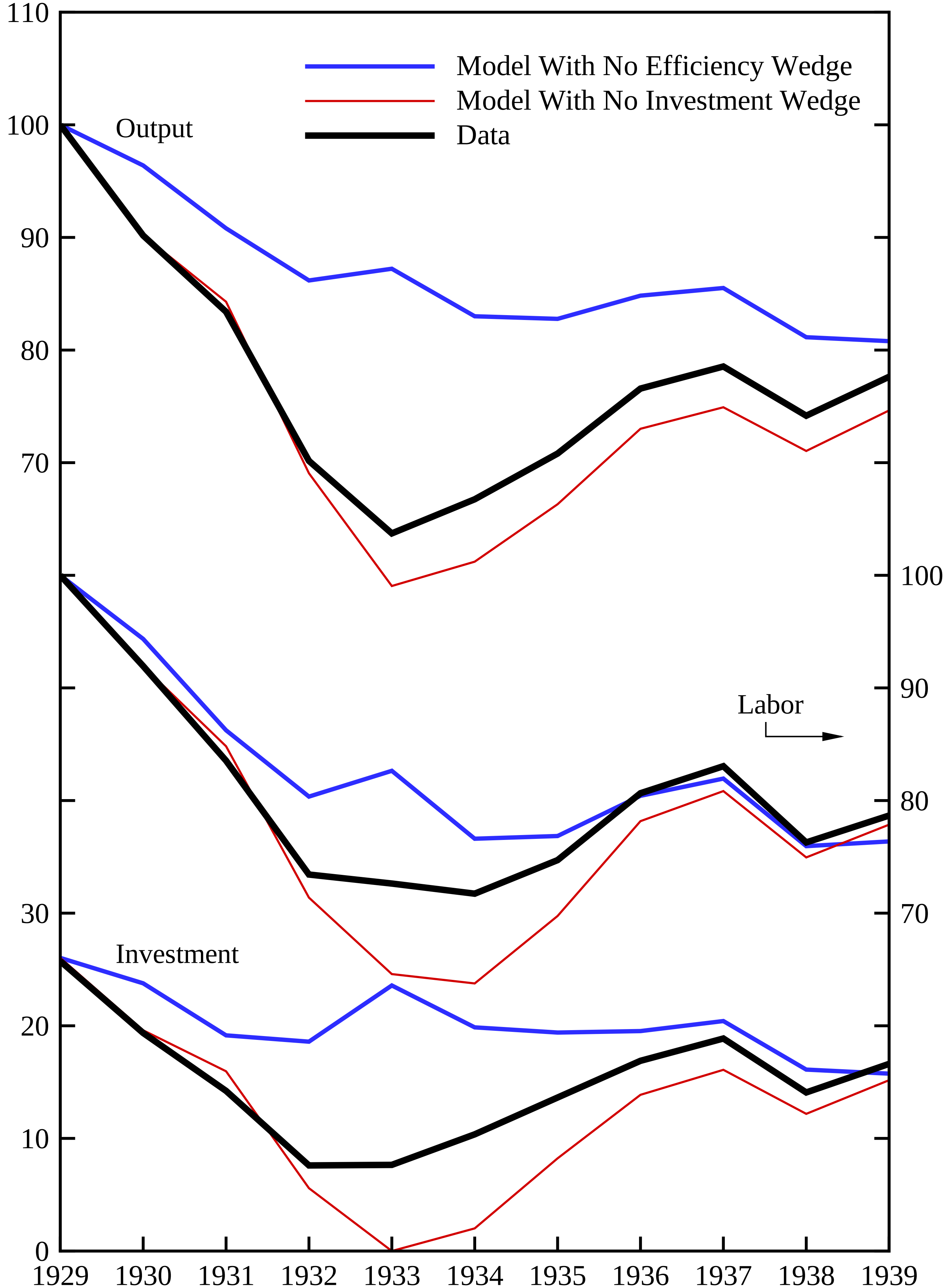


Figure 4
Data and Predictions of Models With All But One Wedge



Figures 5–8

Examining the 1982 U.S. Recession
Quarterly, 1979:1–1985:4; Normalized to Equal 100 in 1979:1

Figure 5
U.S. Output and Three Measured Wedges

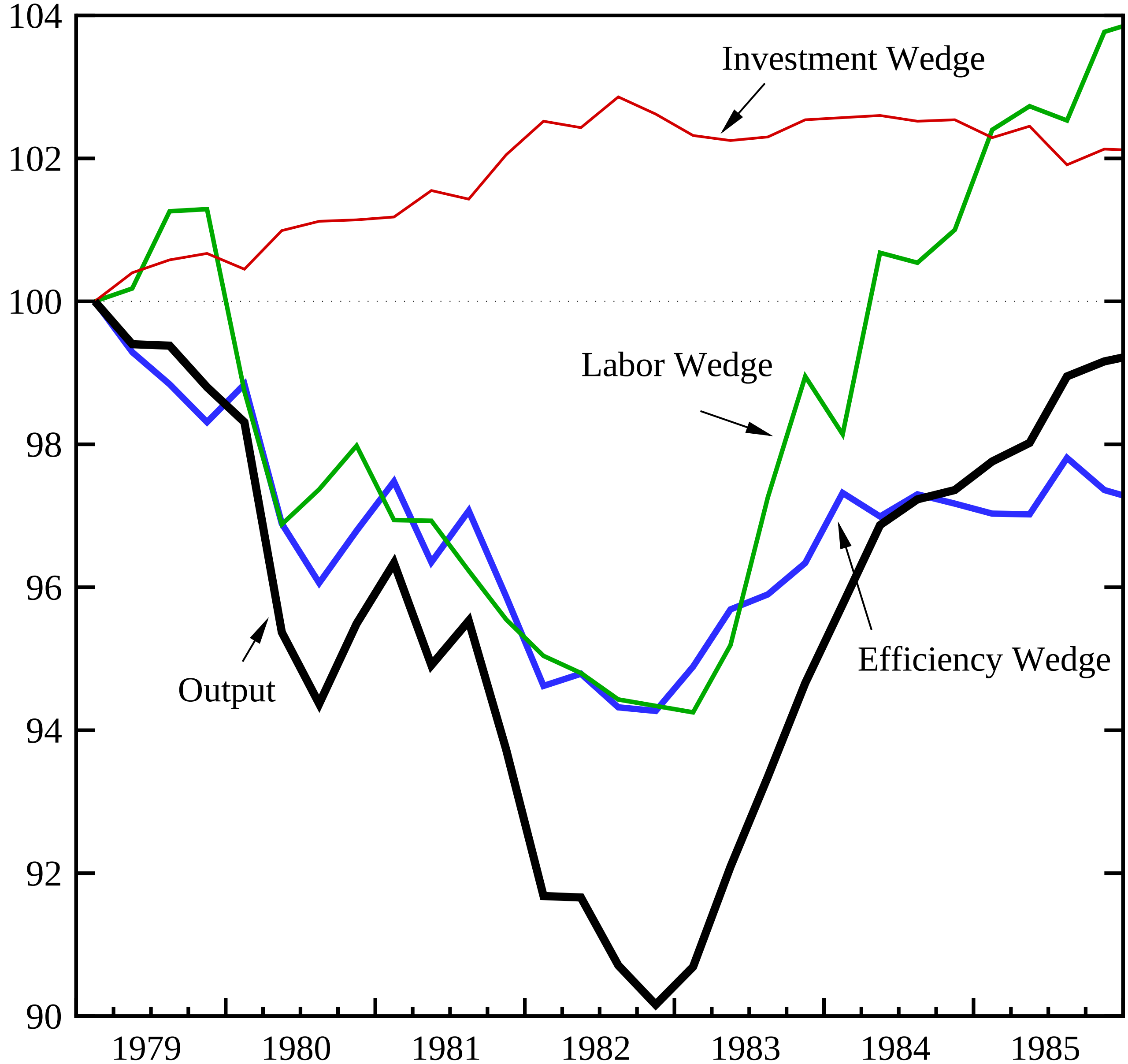


Figure 6
Data and Predictions of Models with Just One Wedge

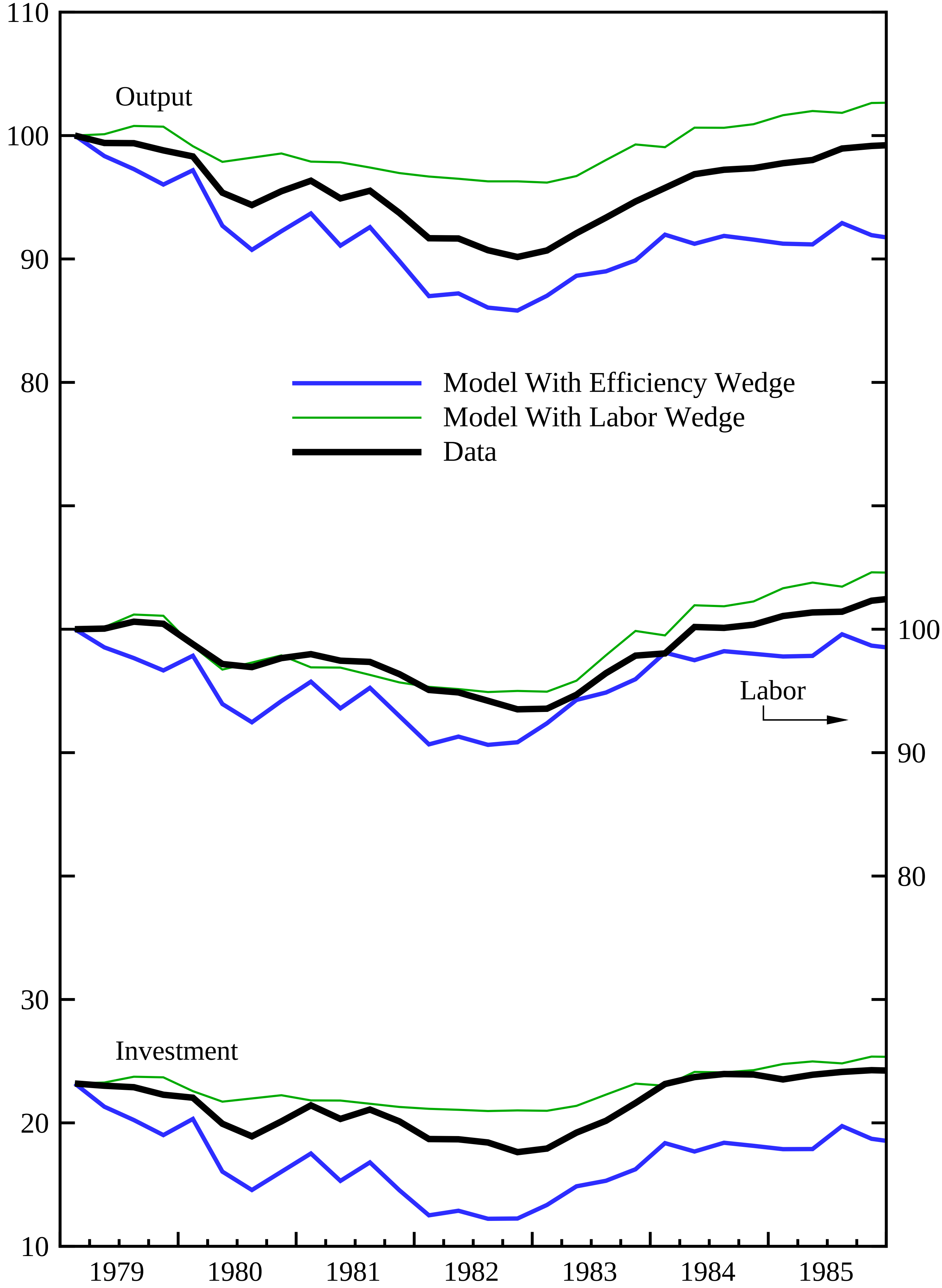


Figure 7

Data and Predictions of a Model With Just the Investment Wedge

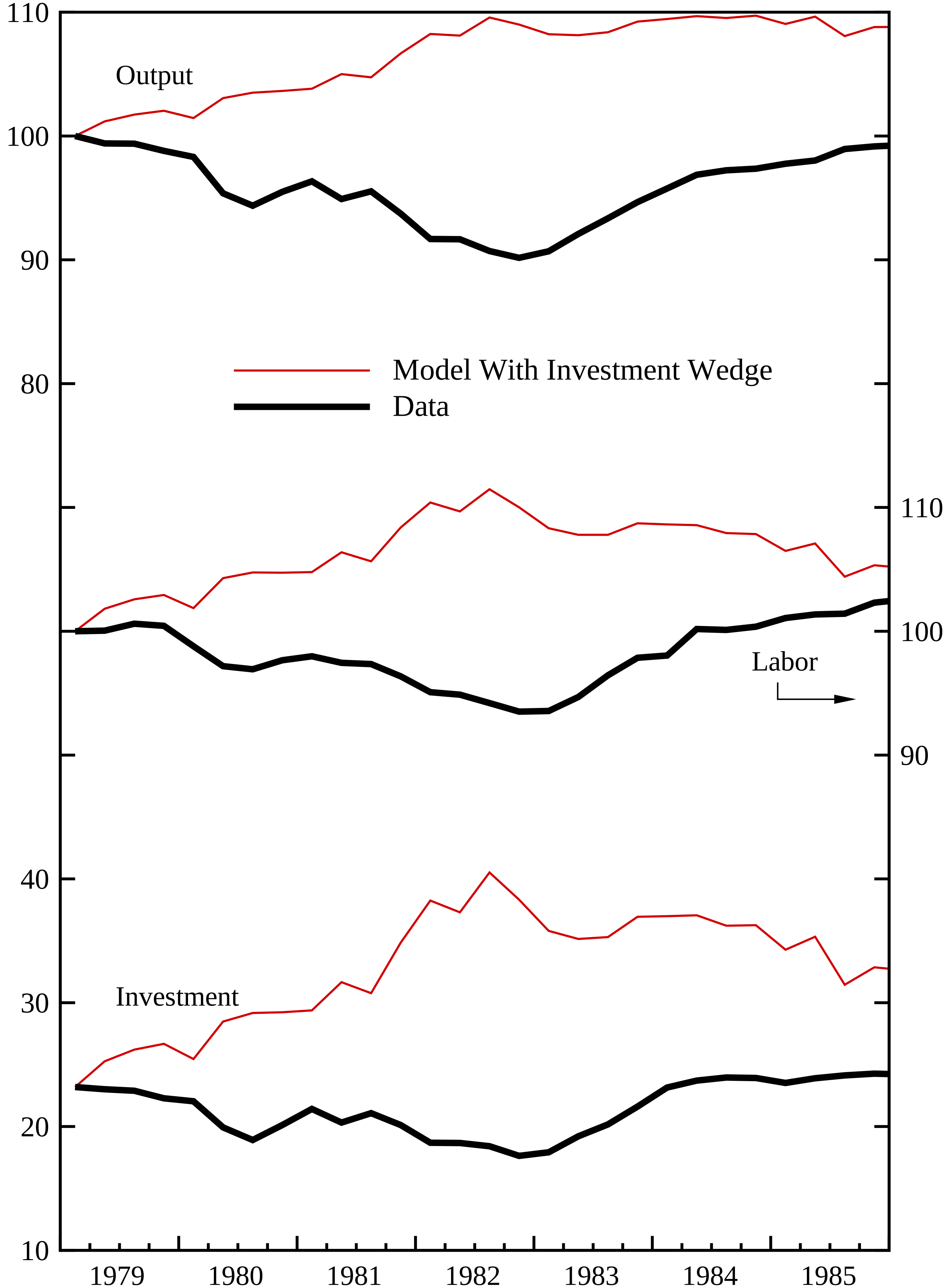
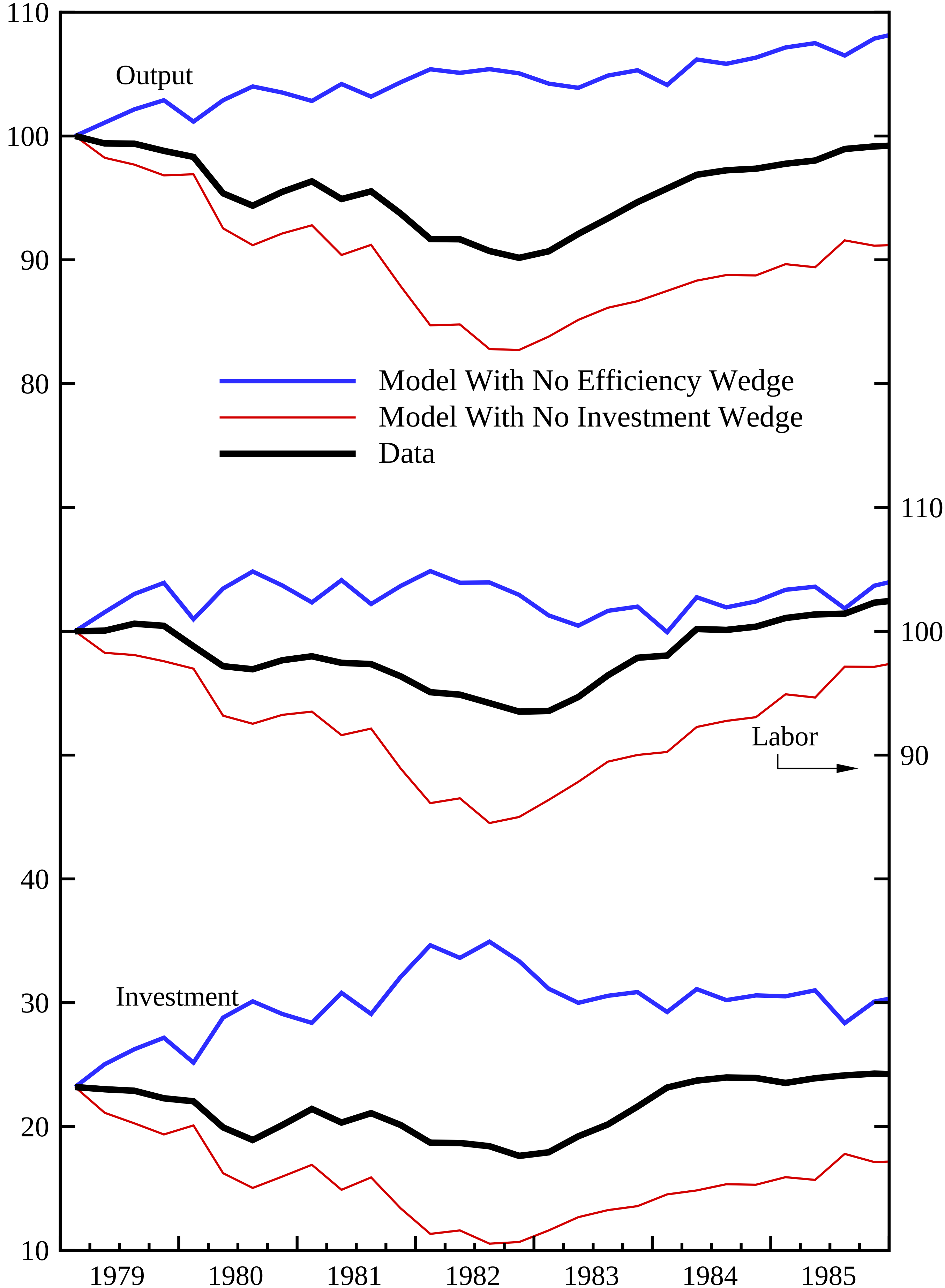


Figure 8
Data and Predictions of Models With All But One Wedge



Figures 9–12

Varying the Capital Utilization Specification
During the Great Depression Period, 1929–39

Figure 9
Measured Efficiency Wedges for Two Capital Utilization Specifications

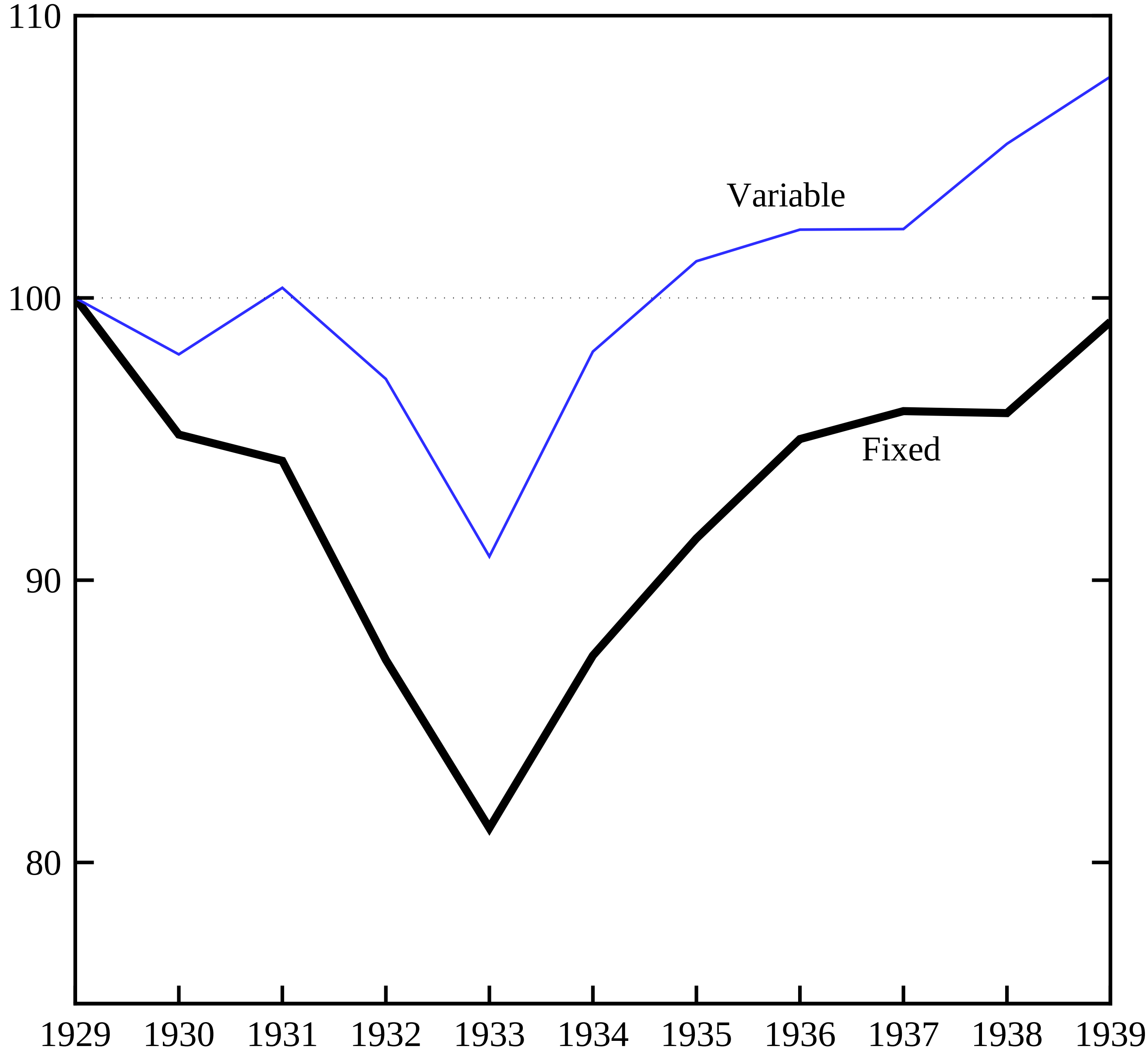


Figure 10
Data and Predictions of Models With
Variable Capital Utilization and Just One Wedge

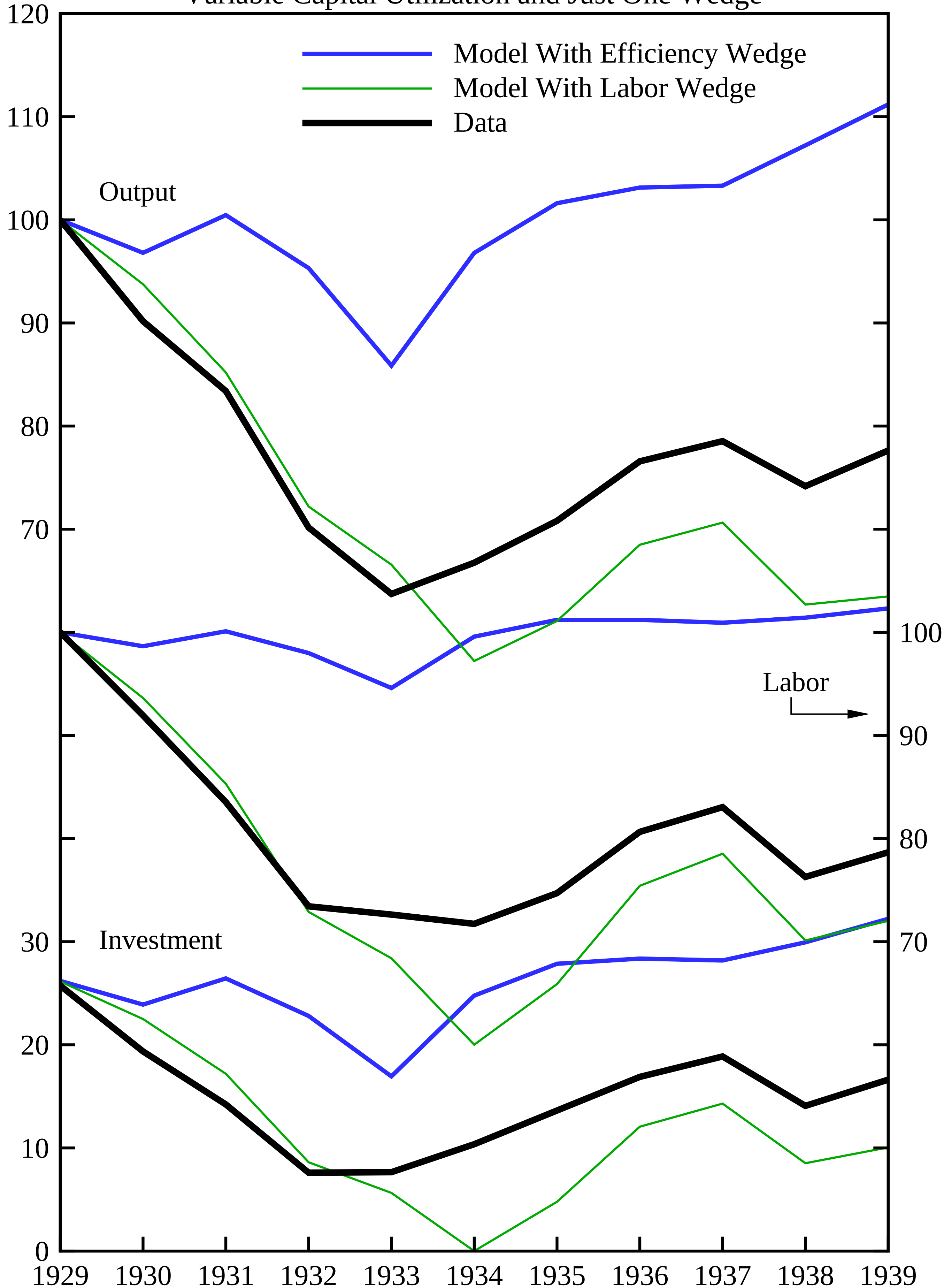


Figure 11
Data and Predictions of a Model With
Variable Capital Utilization and Just the Investment Wedge

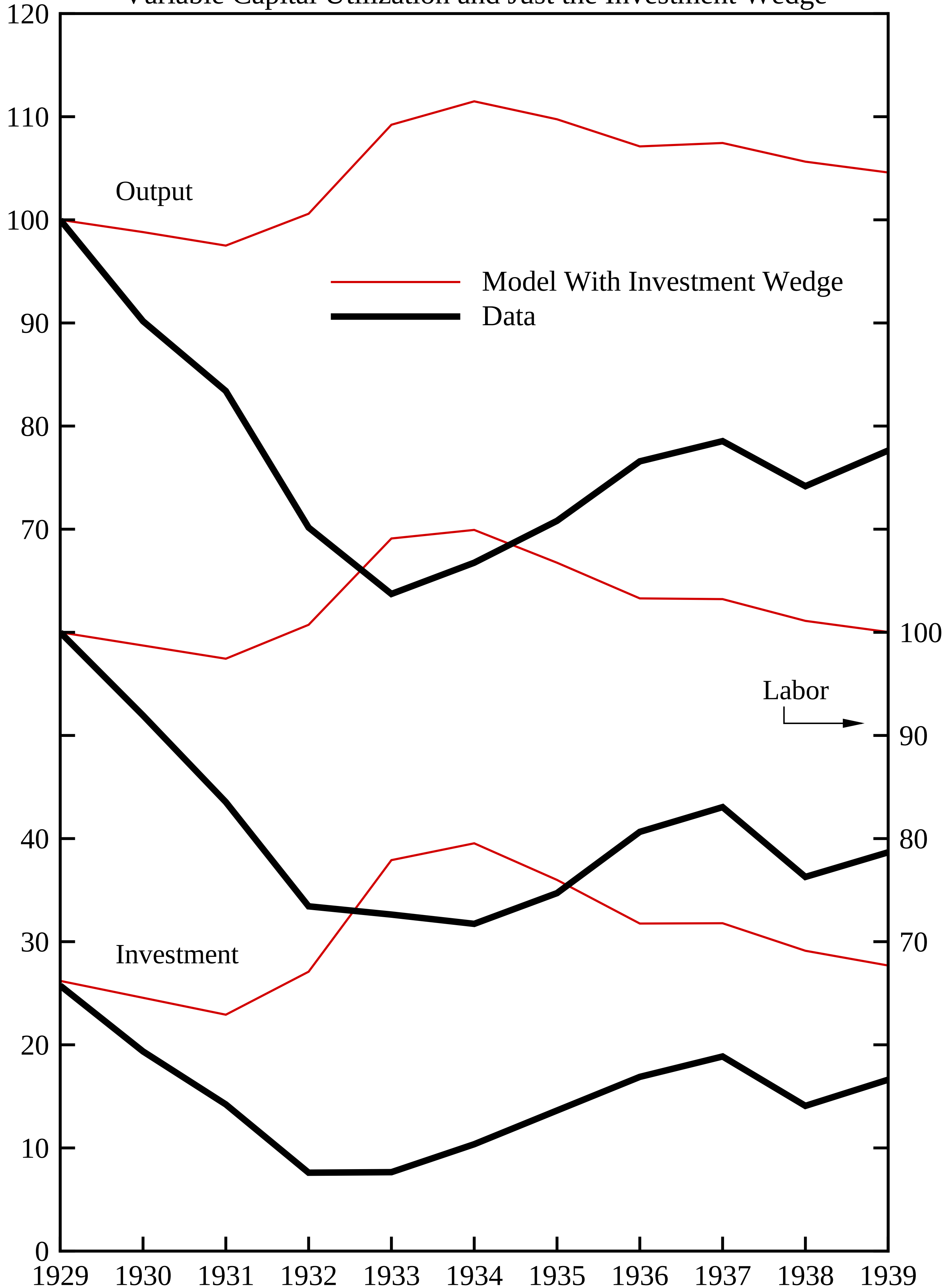


Figure 12
Predictions of Models with Fixed and Variable
Capital Utilization and With All But the Investment Wedge