1. In a recent paper ("A Question of Timing"), Valerie Ramey of UCSD summarizes the state of the literature on fiscal policy shocks. She describes two branches of that literature. One measures shocks using the ‘narrative method’, which constructs dummy variables that turn on in dates when it first became known in magazines like Business Week that an expansion in US military spending would soon occur. Ramey also summarizes results based on the ‘VAR method’, in which military spending is one variable in a multivariate VAR and the fiscal spending shock is identified by the assumption that military spending shocks impact on other macro variables contemporaneously, while other shocks do not impact on military spending contemporaneously. The VAR method resembles the monetary policy shock approach described in class, except that government spending is ordered first in the VAR. The vector, $C_1$ (in the notation of my lecture notes) is the first column of the lower triangular Choleski factorization of the variance-covariance matrix of the VAR fitted disturbances. In Ramey’s summary, she argues that the narrative method and VAR approach yield different implications for the effects of fiscal shocks. The narrative method implies results that are consistent with the neoclassical model: after a positive government spending shock, output rises while consumption and the real wage fall. Ramey argues that the VAR approach implies that after a government spending shock output, consumption and the real wage all rise. She argues that the VAR results are distorted, because shocks to government spending show up with a delay in the government spending data (a jump in government spending is preceded by a period of planning and debate and there is a delay between orders and actual purchases). One component of Ramey’s argument is based on a simulation of artificial data from a closed economy rbc model. In the data, government spending is driven by signals that arrive two quarters in advance (I discussed these things in the last lecture). She shows that
the VAR method applied to the artificial data produce results that re-
semble the ones found in the literature. This question asks you to see
whether Ramsey’s findings also occur in the open economy, monetary
model that you worked on in your last homework assignment.
Here is a quick review of VAR estimation and impulse responses to
orthogonalized shocks. A $p$–lag VAR is written
\[
y_t = c + \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + \varepsilon_t,
\]
where $y_t$ is an $n$ dimensional column vector. Also, $\varepsilon_t$ is iid over time,
$E\varepsilon_t\varepsilon_t' = V$, and $\varepsilon_t$ is independent of $y_{t-s}$, $s > 0$. Write the VAR in
transposed form:
\[
y_t' = x_t' \Pi + \varepsilon_t',
\]
where
\[
x_t = \begin{pmatrix}
1 \\
y_{t-1} \\
\vdots \\
y_{t-p}
\end{pmatrix}, \quad \Pi' = \begin{bmatrix}
c \\
\phi_1 \\
\vdots \\
\phi_p
\end{bmatrix}.
\]
Suppose we have data over the period $t = 1, \ldots, T$ and write:
\[
Y = \begin{pmatrix}
y_1' \\
y_2' \\
\vdots \\
y_T'
\end{pmatrix}, \quad X = \begin{pmatrix}
x_1' \\
x_2' \\
\vdots \\
x_T'
\end{pmatrix}, \quad \varepsilon = \begin{pmatrix}
\varepsilon_1' \\
\varepsilon_2' \\
\vdots \\
\varepsilon_T'
\end{pmatrix},
\]
so that the VAR can be written as follows:
\[
Y_{T \times n} = X_{T \times (np+1)} \Pi_{(np+1) \times n} + \varepsilon_{T \times n},
\]
where $\Pi$ is the matrix of true VAR coefficients. Premultiply this by $X'$
and then take expectations:
\[
E [X'Y] = E [X'X] \Pi + E [X'\varepsilon].
\]
Note that
\[
EX'\varepsilon = E \sum_{t=1}^T x_t \varepsilon_t' = 0,
\]
because of the assumed independence properties of $\varepsilon_t$. Then,

$$\Pi = \{E [X'X]\}^{-1} E [X'Y].$$

This motivates using the following estimator for $\Pi$:

$$\hat{\Pi} = (X'X)^{-1} X'Y.$$

An estimator of the variance-covariance matrix, $V$, is

$$\hat{V} = \frac{\hat{\varepsilon}' \hat{\varepsilon}}{T}, \quad \hat{\varepsilon} = Y - X\hat{\Pi}.$$ 

It can be shown that $\hat{\Pi}$ and $\hat{V}$ are the maximum likelihood estimators of $\Pi$ and $V$, respectively, conditional on the initial observations. It is convenient to compute the lower triangular Choleski factorization of $\hat{V}$, $C$, where

$$CC' = \hat{V},$$

and the diagonal elements of $C$ are non-negative (the MATLAB program, chol, computes the upper triangular decomposition of a matrix). The dynamic response to, say, the first orthogonalized shock in the system is computed as follows. The contemporaneous impact on $y_t$ of a one-standard deviation orthogonalized shock to the first variable is given by $C_1$, the first column of $C$. One way to compute the dynamic effects of the shock is to express the VAR($p$) as a VAR(1). Let

$$\tilde{y}_t = \begin{pmatrix} y_t \\ y_{t-1} \\ \vdots \\ y_{t-(p-1)} \end{pmatrix} = A \begin{pmatrix} y_t \\ y_{t-1} \\ \vdots \\ y_{t-(p-1)} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ 0 \\ \vdots \\ 0 \end{pmatrix},$$

where the matrix $A$ is composed of zero’s, an identity matrix and $\hat{\phi}_1, \ldots, \hat{\phi}_p$ which can be recovered from $\hat{\Pi}$. For computing impulse response functions, the constant term in the VAR may be ignored. Let

$$\tilde{C}_1 = \begin{pmatrix} C_1 \\ 0 \end{pmatrix}.$$
The contemporaneous effect on $\tilde{y}_t$ of an orthogonalized shock to the first variable is given by $\tilde{C}_1$. The dynamic effects are given by:

$$\tilde{y}_{t+j} = A^j \tilde{C}_1, \quad j = 0, 1, 2, ....$$

The dynamic effects on the individual components of $y_{t+j}$ can be recovered from $\tilde{y}_{t+j}$.

Modify the open economy model to include government spending. You may suppose that government purchases the homogeneous domestic good, $Y$, directly and that in steady state government purchases are a given fraction of $Y$, $G = \eta_g Y$, where $\eta_g = 0.20$. This is a good number for US data. The dynamics of government spending should be given by

$$\log \left( \frac{G_t}{G} \right) = 0.9 \log \log \left( \frac{G_{t-1}}{G} \right) + \varepsilon_t + \xi_{t-2},$$

where both shocks are iid over time and independent of each other.

Generate two sets of 5,500 observations from this model and disregard the first 500 observations. In the first set of 5,500 observations, set $\sigma^2_{\varepsilon}$ very small and set $\sigma^2_{\xi} = 0.028$. In the second set of observations, set these the other way around (data are generated in DYNARE using the stoch_simul command, see the manual). Make sure there is a total of at least four shocks in the system. Note that the second set of observations is consistent with the identification assumption underlying the VAR. Let $y_t = \begin{pmatrix} G_t/G & Y_t/Y & C_t/C & I_t/I \end{pmatrix}$, where all variables are measured in the same units. Estimate the impulse response functions in the two data sets. The variables in $y_t$ may be highly correlated, leading to singularity in $X'X$. If this is the case, make sure there are shocks affecting net exports, so that there isn’t a tight link between $G_t$, $Y_t$, $C_t$ and $I_t$. Do your results confirm Ramey’s idea?

2. Make sure to include the $\sigma_t$ shock in the open economy model. Compute the response of domestic homogeneous output and $\pi^c_t$ to a $\sigma_t$ shock in the Ramsey equilibrium. Compare the response to what is implied by the Taylor rule that is in the model now, and to a version of the Taylor rule in which the risk premium appears with a negative sign in the Taylor rule. What can you conclude about the appropriate response to a risk shock?