1. Consider a deterministic version of the Rotemberg model, in which preferences are given by

\[ \sum_{t=0}^{\infty} \beta^t \left[ \log \left( \frac{h_t}{1 + \frac{\phi}{2} (\pi_t - 1)^2} \right) - \frac{\chi}{2} h_t^2 \right], \]

where \( \pi_t \) denotes the gross inflation rate and \( h_t \) denotes hours worked. The expression in the log function is consumption, \( C_t \), after the aggregate resource relation has been substituted out:

\[ C_t \left( 1 + \frac{\phi}{2} (\pi_t - 1)^2 \right) = h_t. \]

The variables, \( h_t \) and \( \pi_t \) are restricted by the equilibrium condition associated with firm price optimization:

\[ \left[ \left( 1 + \nu - \frac{\varepsilon}{\varepsilon - 1} \right) (1 - \varepsilon) + \varepsilon \left( \frac{\chi h_t^2}{1 + \frac{\phi}{2} (\pi_t - 1)^2} - 1 \right) \right] \left( 1 + \frac{\phi}{2} (\pi_t - 1)^2 \right) - \phi (\pi_t - 1) \pi_t + \beta \phi (\pi_{t+1} - 1) \pi_{t+1} = 0. \]

The Lagrangian representation of the Ramsey-optimal problem is:

\[ \max \sum_{t=0}^{\infty} \beta^t \left[ u(\pi_t, h_t) + \lambda_t f(h_t, \pi_t, \pi_{t+1}, \nu) \right], \]

where \( u \) represents the utility function and \( f \) represents the price setting equilibrium condition.

(a) Suppose the maximization is over \( \{\pi_t, h_t\}_{t=0}^{\infty} \) and \( \nu \). Derive explicit expressions for \( h_t, C_t, \pi_t \) in Ramsey equilibrium. Prove that the Ramsey problem is time consistent.

(b) Suppose \( \nu = 0 \) and the maximization is only over \( \{\pi_t, h_t\}_{t=0}^{\infty} \).
i. Prove that the solution to the Ramsey problem is not time consistent. Provide an intuitive explanation.

ii. Show that, in the steady state of the Ramsey-optimal problem, \( \pi = 1 \) and hours worked is less than what it was in (a). Provide intuition.

(c) Consider (i) the steady state of the Ramsey problem and (ii) the constant values of \( h \) and \( \pi \) which maximize steady state welfare, subject to the price-setting equilibrium condition. Prove that the concepts in (i) and (ii) are distinct.

2. A final good, \( Y \), is produced by a representative, competitive firm using the following production function:

\[
Y = \left[ \int_0^1 Y_i^{\frac{\varepsilon}{1-\varepsilon}} di \right]^{\frac{1}{\varepsilon}}, \quad \varepsilon > 1.
\]

Let \( P \) denote the price of the final good and let \( P_i \) denote the price of the \( i^{th} \) intermediate good, \( i \in (0, 1) \). The final good producer takes \( P \) and \( P_i \) as given, all \( i \). The \( i^{th} \) intermediate good, \( Y_{i,t} \), is produced by a monopoly producer with the following production function:

\[
Y_i = N_i.
\]

The \( i^{th} \) monopolist must supply whatever demand occurs at the posted price, \( P_i \). Consider a given distribution of \( P_i \) over \( i \in (0, 1) \).

(a) Carefully derive a closed form expression that maps \( N \) into \( Y \), where \( N \) denotes total employment:

\[
N \equiv \int_0^1 N_i di.
\]

Indicate precisely how this mapping depends on the distribution of \( P_i \) over \( i \).

(b) When, if ever, is the relationship between final output and aggregate employment simply \( Y = N ? \) Explain.
3. Suppose a representative household has preferences

$$\sum_{t=0}^{\infty} \beta^t u (c_t, n_t), \quad u (c_t, n_t) = \frac{c^{1-\sigma}}{1-\sigma} - \psi n.$$ 

The budget constraint is:

$$M_{t+1} = R_t (M_t - Q_t + X_t) + Q_t + W_t n_t - P_t C_t + D_t,$$

where $R_t$ denotes the gross nominal rate of interest, $M_t$ denotes the beginning of period $t$ stock of money, $Q_t$ denotes the household’s transactions balances, $X_t$ denotes a cash transfer from the government, $M_t - Q_t + X_t$ denotes deposits in a bank, and $D_t$ denotes lump sum taxes and profits from firms. The household is required to hold sufficient transactions balances so that, together with nominal wage earnings, the transactions balances are sufficient to cover consumption expenditures:

$$P_t C_t \leq Q_t + W_t n_t.$$ 

The household takes $P_t$, $R_t$, $D_t$, $X_t$ and $W_t$ as given. There is one representative, competitive, firm that produces output using labor, using the production function

$$Y_t = N_t.$$ 

The firm must borrow the wage bill in advance, so that when it hires one unit of labor, this costs the firm $W_t R_t$ in nominal units. The firm takes the price of output, $P_t$, and the wage rate, $W_t$, as given.

The resource constraint is $C_t = Y_t$. Finally, monetary policy is characterized by the following expression:

$$M_{t+1}^a = M_t^a + X_t.$$ 

Here, $M_t^a$ denotes the economy-wide average beginning-of-period stock of money. Monetary policy sets

$$\frac{X_t}{M_t^a} = 1 + x,$$

so that the money growth rate is constant. In equilibrium, $M_t^a = M_t$. 

3
(a) Set up the household problem in Lagrangian form, and derive the inter- and intra-temporal Euler equations of the household. Be sure to only include variables other than the multiplier in the two Euler equations.

(b) Derive the first order necessary condition for firm optimization.

(c) Suppose that the interest rate, $R_t$, is greater than unity. Show that the transactions balance constraint is strictly binding, in the sense that the multiplier on that constraint is positive.

(d) An interior (meaning, $R_t > 1$) equilibrium of this model can be expressed in terms of the following scalar first order difference equation in equilibrium hours alone:

$$\psi n_t = \beta \frac{n_{t+1}^{1-\sigma}}{1+x}.$$  \hfill (1)

Derive this result. (Hint: use the transactions balance equation constraint to substitute out for the price level in the intertemporal Euler equation; combine the household and firm intratemporal Euler equation, to obtain an equation expressing the nominal rate of interest as a function of current variables; use that equation to substitute out for $R_t$ in the household’s intertemporal equation.)

(e) Compute the model’s steady state level of employment. Log-linearize (1) about steady state. What restriction on the model parameters is necessary for the log-linearized system to have a unique convergent solution. Is that unique convergent solution a minimal state variable solution?

4. Consider a model in which households have access to a domestic bond with gross nominal return, $R_t$, and a foreign bond with gross nominal return, $R_f^t$. Let $S_t$ denote the domestic currency price of a unit of the foreign currency. Let $v_t$ denote the marginal utility of domestic currency enjoyed by domestic residents in period $t$, and let $m_{t+1}$ denote $\beta v_{t+1}/v_t$, where $\beta$ is the discount factor. Let $s_t = S_t/S_{t-1}$ denote the rate of depreciation of the domestic exchange rate. Suppose that the following conditions hold in steady state:

$$s R_f^t = R, \quad m_t = m.$$
(a) Display and explain intuitively, the relationship between the return on the domestic and foreign bonds which must hold with optimizing investors who have rational expectations.

(b) Log linearize the relationship in (a) about steady state, to obtain a linear expression relating the difference between the domestic and foreign rate of interest to the anticipated depreciation in the currency.

(c) Explain two sorts of evidence that go against the implication derived in (b).