

## 1 Introduction

This is a two-period economy with BGG financial frictions (Bernanke, Gertler and Gilchrist, 1999, The financial accelerator in a quantitative business cycle framework, in: Taylor, J.B., Woodford, M. (Eds.), Handbook of Macroeconomics, Vol. 1C. North Holland, Amsterdam, pp. 1341-1393.) There is no uncertainty in period 1 and the production function can be either high or low in period 2. In period 1, households face a work-leisure choice and they use their earnings to purchase consumption goods and purchase bonds from the bank. The bonds pay the same interest in period 2, regardless of the realization of aggregate uncertainty then.

Entrepreneurs build the capital in period 1 which gets used in period 2. In the first period, entrepreneurs are all alike and they own the stock of capital in that period. They rent the capital to firms in competitive markets and combine the proceeds with bank loans to finance the purchase of goods used in the production of capital. In constructing capital in period 1, entrepreneurs experience an idiosyncratic productivity shock that is observed by them, but outsiders (like the bank) must pay a monitoring cost to observe the shock. Banks offer the BGG standard debt contract to entrepreneurs, and they finance this with bonds issued to households in period 1. In period 2 banks pay off their state non-contingent debt to households. We follow BGG in assuming that banks do not have access to complete markets, so that their zero profit condition reduces to a state-by-state zero profit condition in period 2.

We derive the equilibrium conditions for the economy. There are 10 equilibrium conditions and 10 equilibrium variables to be determined. In period 1 there is no uncertainty and in period 2 there is an aggregate state variable that can take on values  $l$  and  $h$ .

## 2 Households and Goods-Producing Firms

Households are all alike and have the following intertemporal preferences:

$$U(c, l) + \pi U(c^h, l^h) + (1 - \pi) U(c^l, l^l). \quad (1)$$

Their budget constraints are:

$$\begin{aligned} c + B &\leq wl, \\ c^h &\leq w^h l^h + RB, \\ c^l &\leq w^l l^l + RB. \end{aligned}$$

Here,  $c$  and  $l$  denote period 1 consumption and employment, and the corresponding numbers with superscripts  $l$  and  $h$  indicate consumption and labor in the  $l$  and  $h$  state in period 2. Housholds have a consumption-leisure decision in each period. In period 1 they have a consumption-saving decision. The saving goes into a  $B$  which pays the same return in period 2, regardless of the realization of uncertainty then. The first order necessary conditions for household optimization are:

$$-\frac{U_l}{U_c} = w, \quad -\frac{U_l^h}{U_c^h} = w^h, \quad -\frac{U_l^l}{U_c^l} = w^l \quad (2)$$

$$1 = \beta \left[ \pi \frac{U_c^h}{U_c} + (1 - \pi) \frac{U_c^l}{U_c} \right] R, \quad (3)$$

Goods producing firms rent capital and labor in competitive markets and use these to produce output using the following technology:

$$\begin{aligned} y &= F(k, l) \\ y^h &= F^h(K, l^h) \\ y^l &= F^l(K, l^l), \end{aligned}$$

where  $y$  denotes period 1 output and  $y^h$  and  $y^l$  denote period 2 goods output in the high and low states, respectively. Competition ensures:

$$\begin{aligned} w &= F_l(k, l), \quad w^h = F_l^h(K, l^h), \quad w^l = F_l^l(K, l^l) \\ r &= F_k(k, l), \quad r^h = F_k^h(K, l^h), \quad r^l = F_k^l(K, l^l), \end{aligned}$$

where  $r$  denotes the period 1 rental rate of capital, while  $r^h$  and  $r^l$  denotes the period 2 rental rates in the high and low states, respectively.

When there is no risk of confusion, we will at times refer to a period 2 variable as  $r(s)$  where  $s$  indexes the peirod 2 aggregate state, instead of using  $r^h$  and  $r^l$ .

### 3 Entrepreneurs

Entrepreneurs are identical in period 1 and all own a per capita share of the capital stock,  $k$ , used in period 1. Their net worth at the end of period 1 is  $N = rk$ , where  $r$  is the period 1 competitively determined rental rate on capital,  $k$ . Capital is assumed to depreciate completely in the period. Entrepreneurs build capital,  $K$ , that they can rent out in period 2 using goods produced in period 1. Entrepreneurs want to build more capital than can be financed from their own assets, so they need to borrow. They borrow  $K - N$  from banks at the end of production in period 1, and banks in turn raise the funds by issuing bonds,  $B$ , to households.

Entrepreneurs who buy  $K$  units of goods in period 1 end up with  $\omega K$  units of capital, where each entrepreneur draws  $\omega$  independently from a cumulative

distribution,  $F(\omega)$ , which has mean unity. The contract that the entrepreneur has with the bank has the property that entrepreneurs with  $\omega > \bar{\omega}(s)$  in period 2 state  $s$  pay  $\bar{\omega}(s) r(s) K$  to the bank and they keep  $(\omega - \bar{\omega}(s)) r(s) K$ . Entrepreneurs with  $\omega \leq \bar{\omega}(s)$  receive nothing, while the bank receives  $(1 - \mu) \omega r(s) K$ . Here,  $\mu \omega r(s) K$  are monitoring expenses the bank incurs for verification, when an entrepreneur claims  $\omega \leq \bar{\omega}(s)$ .

The parameters of the contract are  $\bar{\omega}^h$ ,  $\bar{\omega}^l$  and  $K$ . They are chosen to maximize the expected utility of the entrepreneurs:

$$\pi [1 - \Gamma^h] r^h K + (1 - \pi) [1 - \Gamma^l] r^l K,$$

where

$$\Gamma(\bar{\omega}(s)) = \int_0^{\bar{\omega}(s)} \omega dF(\omega) + \bar{\omega}(s) \int_{\bar{\omega}(s)}^{\infty} dF(\omega),$$

subject to the state-by-state zero profit condition for banks in period 2:

$$[\Gamma(\bar{\omega}(s)) - \mu G(\bar{\omega}(s))] r(s) K = [K - N] R.$$

The Lagrangian representation of this problem is:

$$\begin{aligned} & \max_{\bar{\omega}(s), K} \pi [1 - \Gamma^h] r^h K + (1 - \pi) [1 - \Gamma^l] r^l K \\ & + \lambda^h \{ [\Gamma^h - \mu G^h] r^h K - [K - N] R \} \\ & + \lambda^l \{ [\Gamma^l - \mu G^l] r^l K - [K - N] R \}, \end{aligned}$$

where

$$G(\bar{\omega}(s)) = \int_0^{\bar{\omega}(s)} \omega f(\omega) d\omega.$$

Differentiate with respect to  $K$ :

$$\begin{aligned} & \pi [1 - \Gamma^h] r^h + (1 - \pi) [1 - \Gamma^l] r^l \\ & + \lambda^h \{ [\Gamma^h - \mu G^h] r^h - R \} + \lambda^l \{ [\Gamma^l - \mu G^l] r^l - R \} \end{aligned}$$

Differentiate with respect to  $\bar{\omega}^h$  and  $\bar{\omega}^l$ , respectively,

$$\begin{aligned} \lambda^h &= \frac{\pi \Gamma'^h}{\Gamma'^h - \mu G'^h} \\ \lambda^l &= \frac{(1 - \pi) [1 - \Gamma^l]}{\Gamma^l - \mu G^l}. \end{aligned}$$

Combine the two sets of first order conditions:

$$\begin{aligned} & \pi [1 - \Gamma^h] r^h + (1 - \pi) [1 - \Gamma^l] r^l \\ & + \frac{\pi \Gamma'^h}{\Gamma'^h - \mu G'^h} \{ [\Gamma^h - \mu G^h] r^h - R \} + \frac{(1 - \pi) [1 - \Gamma^l]}{\Gamma^l - \mu G^l} \{ [\Gamma^l - \mu G^l] r^l - R \} \end{aligned} \quad (4)$$

The zero cash flow condition must also be satisfied in equilibrium:

$$[\Gamma^h - \mu G^h] r^h K = [K - N] R \quad (5)$$

$$[\Gamma^l - \mu G^l] r^l K = [K - N] R. \quad (6)$$

Since the right side of (5) and (6) are the same, while most likely  $r^h$  and  $r^l$  are different, we expect that  $\bar{\omega}^h \neq \bar{\omega}^l$ .

The resource constraints are:

$$c + K \leq F(k, l) \quad (7)$$

$$c^h + \mu G(\bar{\omega}^h) r^h K \leq F(K, l^h) \quad (8)$$

$$c^l + \mu G(\bar{\omega}^l) r^l K \leq F(K, l^l) \quad (9)$$

## 4 Equilibrium

We now collect the equilibrium conditions. Combining the household and goods-producing firm first order conditions we have the following 4 equations:

$$-\frac{U_l}{U_c} = F_l(k, l), \quad -\frac{U_l^h}{U_c^h} = F_l^h(K, l^h), \quad -\frac{U_l^l}{U_c^l} = F_l^l(K, l^l)$$

$$1 = \beta \left[ \pi \frac{U_c^h}{U_c} + (1 - \pi) \frac{U_c^l}{U_c} \right] R,$$

Substituting out for the rental rate of capital in the equations that characterize the optimal entrepreneurial contract, (4) and (5)-(6), we obtain the following 3 equations:

$$\begin{aligned} & \pi [1 - \Gamma^h] F_k^h(K, l^h) + (1 - \pi) [1 - \Gamma^l] F_k^l(K, l^l) \\ & + \frac{\pi \Gamma^h}{\Gamma^h - \mu G^h} \{ [\Gamma^h - \mu G^h] F_k^h(K, l^h) - R \} + \frac{(1 - \pi) [1 - \Gamma^l]}{\Gamma^l - \mu G^l} \{ [\Gamma^l - \mu G^l] F_k^l(K, l^l) - R \} \end{aligned}$$

and,

$$\begin{aligned} [\Gamma^h - \mu G^h] r^h K &= [K - F_k(k, l) k] R \\ [\Gamma^l - \mu G^l] r^l K &= [K - F_k(k, l) k] R \end{aligned}$$

Finally, the 3 resource constraints are:

$$\begin{aligned} c + K &= F(k, l) \\ c^h + \mu G(\bar{\omega}^h) F_k^h(K, l^h) K &= F(K, l^h) \\ c^l + \mu G(\bar{\omega}^l) F_k^l(K, l^l) K &= F(K, l^l). \end{aligned}$$

We have 10 equilibrium conditions, total. There are 10 variables to be determined in equilibrium:

$$l, l(s), c, c(s), K, \bar{\omega}(s), R,$$

where, of course,  $l(s)$  is actually two variables.