Fiscal Policy in a DSGE Model:

The Fiscal Multiplier in 'Normal times' and When the Zero

Lower Bound on the Interest Rate is Binding

Based on work by:

Eggertsson and Woodford, 2003, 'The Zero Interest-Rate Bound and Optimal Monetary Policy,' Brookings Panel on Economic Activity.

Christiano, 2004, 'The Zero-Bound, Zero-Inflation Targeting, and Output Collapse'

Christiano, Eichenbaum, Rebelo (in progress)

Background

- Policy makers anxious to fight the current recession.
- Perception: because interest rates are low, traditional monetary policy can't do much more.
  - Must turn to fiscal policy.
- How much can we hope to get from fiscal policy?
  - Empirical data tends to be ambiguous because
    - movements in *G* tend to be accompanied by other shocks.
    - a priori considerations suggest effects of G depends on state of economy.
  - Approach taken here: investigate what the equilibrium models which fit the data well have to say.

Findings

- What is the size of the fiscal multiplier?
  - Some prominent economists argue (without support) that the multiplier is near zero.
  - We will see that a standard equilibrium model implies:
    - in ‘normal times’ multiplier may be bigger than unity, but depends on the nature of monetary policy.
    - When lower bound on nominal interest rate is binding, multiplier may be quite large.

Outline

- Fiscal multiplier in normal times.

- Fiscal multiplier when non-negativity constraint on nominal rate of interest is binding.

Derivation of Model Equilibrium Conditions

- Households
  - First order conditions
- Firms:
  - final goods and intermediate goods
  - marginal cost of intermediate good firms
- Aggregate resources
- Monetary policy
- Three linearized equilibrium conditions:
  - Intertemporal, Pricing, Monetary policy
- Results

Model

King-Plosser-Rebelo (KPR) preferences.

- Household preferences and constraints:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left[C_t^{\gamma}(1-N_t)^{1-\gamma}\right]^{1-\sigma}-1}{1-\sigma} + v(G_t) \right]$$

$$P_t C_t + B_{t+1} \leq W_t N_t + (1+R_t)B_t + T_t,\ T_t \sim \text{lump sum taxes and profits}$$

- Optimality conditions

$$\overbrace{u_{c,t}}^{\text{marginal cost of giving up one unit of consumption to save}} = \overbrace{E_t\ \beta u_{c,t+1} \overbrace{\frac{1+R_{t+1}}{1+\pi_{t+1}}}^{\text{extra goods tomorrow from saving more today}}}^{\text{marginal benefit tomorrow from saving more today}},$$

$$\overbrace{\frac{-u_{N,t}}{u_{c,t}}}^{\text{marginal cost (in units of goods) of labor effort}} = \overbrace{\frac{W_t}{P_t}}^{\text{marginal benefit of labor effort}}$$

Linearized Intertemporal Equation

- Inter-temporal Euler equation

$$E_t\left[u_{c,t} - \beta u_{c,t+1}\frac{1+R_{t+1}}{1+\pi_{t+1}}\right] = 0$$

- In zero inflation no growth steady state:

$$1 = \beta(1+R)$$

- Totally differentiate:

$$du_{c,t} - [\beta(1+R)du_{c,t+1} + \beta u_c dR_{t+1} - \beta u_c(1+R)d\pi_{t+1}] = 0$$

  - Log-differentiation:

$$u_c\hat{u}_{c,t} - \beta(1+R)u_c\left[\hat{u}_{c,t+1} + \frac{1}{1+R}dR_{t+1} - d\pi_{t+1}\right] = 0$$

  - Finally:

$$\hat{u}_{c,t} - [\hat{u}_{c,t+1} + \beta dR_{t+1} - d\pi_{t+1}] = 0$$

Linearized intertemporal , cnt'd

- Repeat:

$$\hat{u}_{c,t} - [\hat{u}_{c,t+1} + \beta dR_{t+1} - d\pi_{t+1}] = 0$$

$$u = \frac{\left[C_t^{\gamma}(1-N_t)^{1-\gamma}\right]^{1-\sigma} - 1}{1-\sigma} \rightarrow u_{c,t} = \gamma C_t^{\gamma(1-\sigma)-1}(1-N_t)^{(1-\gamma)(1-\sigma)}$$

$$\hat{u}_{c,t} = [\gamma(1-\sigma) - 1]\hat{C}_t - \frac{(1-\gamma)(1-\sigma)N}{1-N}\hat{N}_t$$

Firms

- Final, homogeneous good

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}, \ \varepsilon > 1$$

  – Efficiency condition:

$$P_t(i) = P_t \left(\frac{Y_t}{Y_t(i)}\right)^{\frac{1}{\varepsilon}}$$

- i-th intermediate good

$$Y_t(i) = N_t(i)$$

  – Optimize price with probability *1*-θ, otherwise

$$P_t(i) = P_{t-1}(i)$$

Intermediate Good Firm Marginal Cost

- Marginal cost:

$$MC_t = \frac{\frac{dCost_t}{dWorker_t}}{\frac{dOutput_t}{dWorker_t}} = \frac{W_t \overbrace{(1-\nu)}^{\text{subsidy to undo effects of monopoly power }=(\varepsilon-1)/\varepsilon}}{MP_{L,t}}$$

$$= W_t(1-\nu) = P_t \overbrace{\frac{-u_{N,t}}{u_{c,t}}}^{\text{household first order condition}} (1-\nu)$$

- Real marginal cost

$$s_t \equiv \frac{MC_t}{P_t} = \frac{-u_{N,t}}{u_{c,t}}(1-\nu) \overbrace{=}^{\text{in steady state}} \frac{\varepsilon-1}{\varepsilon}$$

$$\overbrace{\frac{-u_{N,t}}{u_{c,t}}}^{\text{marginal cost to household of providing one more unit of labor}} \overbrace{=}^{\text{in steady state}} \overbrace{1}^{\text{marginal benefit of one extra unit of labor}}$$

Aggregate Resources

- Resource relation:

$$C_t + G_t = Y_t = p_t^* N_t$$

  – p_t^* is 'Tak Yun' distortion

  – recall, distortion = 1 to first order:

$$\hat{Y}_t = \hat{N}_t$$

- Log-linear expansion:

$$(1-g)\hat{C}_t + g\hat{G}_t = \hat{Y}_t,\ g \equiv \frac{G}{Y}$$

- Consumption:

$$\hat{C}_t = \frac{1}{1-g}\hat{Y}_t - \frac{g}{1-g}\hat{G}_t$$

Simplifying Marginal Utility of C

$$\frac{-u_{N,t}}{u_{c,t}} \overbrace{=}^{\text{in steady state}} 1 \to \frac{1-\gamma}{1-N} = \frac{\gamma}{C}$$

$$\hat{u}_{c,t} = [\gamma(1-\sigma)-1]\hat{C}_t - \frac{(1-\gamma)(1-\sigma)N}{1-N}\hat{N}_t$$

$$= [\gamma(1-\sigma)-1]\hat{C}_t - \frac{\gamma(1-\sigma)N}{C}\hat{N}_t$$

$$= [\gamma(1-\sigma)-1]\hat{C}_t - \frac{\gamma(1-\sigma)}{1-g}\hat{N}_t$$

$$= [\gamma(1-\sigma)-1]\left[\frac{1}{1-g}\hat{Y}_t - \frac{g}{1-g}\hat{G}_t\right] - \frac{\gamma(1-\sigma)}{1-g}\hat{Y}_t$$

$$= -\frac{1}{1-g}\hat{Y}_t + \frac{g}{1-g}\hat{G}_t$$

Simplify Intertemporal Equation

- Intertemporal Euler equation:

$$\hat{u}_{c,t} - [\hat{u}_{c,t+1} + \beta dR_{t+1} - d\pi_{t+1}] = 0$$

- Substitute out marginal utility of consumption:

$$-\tfrac{1}{1-g}\hat{Y}_t + \tfrac{g}{1-g}\hat{G}_t - \left[-\tfrac{1}{1-g}\hat{Y}_{t+1} + \tfrac{g}{1-g}\hat{G}_{t+1} + \beta dR_{t+1} - d\pi_{t+1}\right] = 0$$

- Rearranging:

$$\boxed{\hat{Y}_t - g\hat{G}_t = \hat{Y}_{t+1} - g\hat{G}_{t+1} - (1-g)(\beta dR_{t+1} - d\pi_{t+1})}$$

Phillips Curve

- Equilibrium condition associated with price setting just like before:

$$\pi_t = \beta\pi_{t+1} + \kappa\widehat{s}_t,$$

$$\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

- Marginal cost:

$$\widehat{s}_t = \widehat{\frac{(1-\gamma)C_t}{\gamma(1-N_t)}} = \hat{C}_t - \widehat{(1-N_t)} = \hat{C}_t + \frac{N}{1-N}\hat{N}_t$$

$$\overbrace{=}^{\left(\hat{C}_t = \frac{1}{1-g}\hat{Y}_t - \frac{g}{1-g}\hat{G}_t,\ \hat{N}_t = \hat{Y}_t\right)} \left[\frac{1}{1-g} + \frac{N}{1-N}\right]\hat{Y}_t - \frac{g}{1-g}\hat{G}_t$$

Monetary Policy

- Monetary policy rule (after linearization)

$$dR_{t+1} = \rho_R dR_t + (1-\rho_R)\left[\frac{\phi_1}{\beta}\pi_{t+k} + \frac{\phi_2}{\beta}\hat{Y}_{t+l}\right]$$

$$dR_{t+1} \equiv R_{t+1} - R,\ R = \frac{1}{\beta} - 1$$

$$\hat{Y}_t \equiv \frac{Y_t - Y}{Y}$$

$$k, l = 0, 1.$$

Pulling All the Equations Together

- IS equation:

$$\hat{Y}_t - g\hat{G}_t = \hat{Y}_{t+1} - g\hat{G}_{t+1} - (1-g)(\beta dR_{t+1} - d\pi_{t+1})$$

- Phillips curve:

$$\pi_t = \beta\pi_{t+1} + \kappa\left[\left(\frac{1}{1-g} + \frac{N}{1-N}\right)\hat{Y}_t - \frac{g}{1-g}\hat{G}_t\right]$$

- Monetary policy rule:

$$dR_{t+1} = \rho_R dR_t + (1-\rho_R)\left[\frac{\phi_1}{\beta}\pi_{t+k} + \frac{\phi_2}{\beta}\hat{Y}_{t+l}\right]$$

The Equations in Matrix Form

$$
\begin{bmatrix} -\frac{1}{1-g} & -1 & 0 \\ 0 & \beta & 0 \\ l(1-\rho_R)\frac{\phi_2}{\beta} & k(1-\rho_R)\frac{\phi_1}{\beta} & 0 \end{bmatrix}
\begin{pmatrix} \hat{Y}_{t+1} \\ \pi_{t+1} \\ dR_{t+2} \end{pmatrix}
$$

$$
+\begin{bmatrix} \frac{1}{1-g} & 0 & \beta \\ \kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right) & -1 & 0 \\ (1-l)(1-\rho_R)\frac{\phi_2}{\beta} & (1-k)(1-\rho_R)\frac{\phi_1}{\beta} & -1 \end{bmatrix}
\begin{pmatrix} \hat{Y}_{t} \\ \pi_{t} \\ dR_{t+1} \end{pmatrix}
+\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \rho_R \end{bmatrix}
\begin{pmatrix} \hat{Y}_{t-1} \\ \pi_{t-1} \\ dR_{t} \end{pmatrix}
$$

$$
+\begin{pmatrix} \frac{g[\gamma(\sigma-1)+1]}{1-g} \\ 0 \\ 0 \end{pmatrix}\hat{G}_{t+1}
+\begin{pmatrix} -\frac{g[\gamma(\sigma-1)+1]}{1-g} \\ -\frac{\kappa g}{1-g} \\ 0 \end{pmatrix}\hat{G}_{t},
$$

- or, $\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t = 0.$

$$
s_t = P s_{t-1} + \varepsilon_t,\ s_t \equiv \hat{G}_t,\ P = \rho
$$

Solution:

- Undetermined coefficients, *A* and *B*:

$$z_t = Az_{t-1} + Bs_t$$

- *A* and *B* must satisfy:

$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 = 0$$

$$\alpha_0(AB + BP) + \alpha_1 B + \beta_0 P + \beta_1 = 0.$$

- When $\rho_R = 0,\ \alpha_2 = 0 \rightarrow A = 0$ works .

Results

- Fiscal spending multiplier small, but can easily be bigger than unity (i.e., *C* rises in response to *G* shock)

- Contrasts with standard results in which multiplier is less than unity
  - Typical preferences in estimated models:

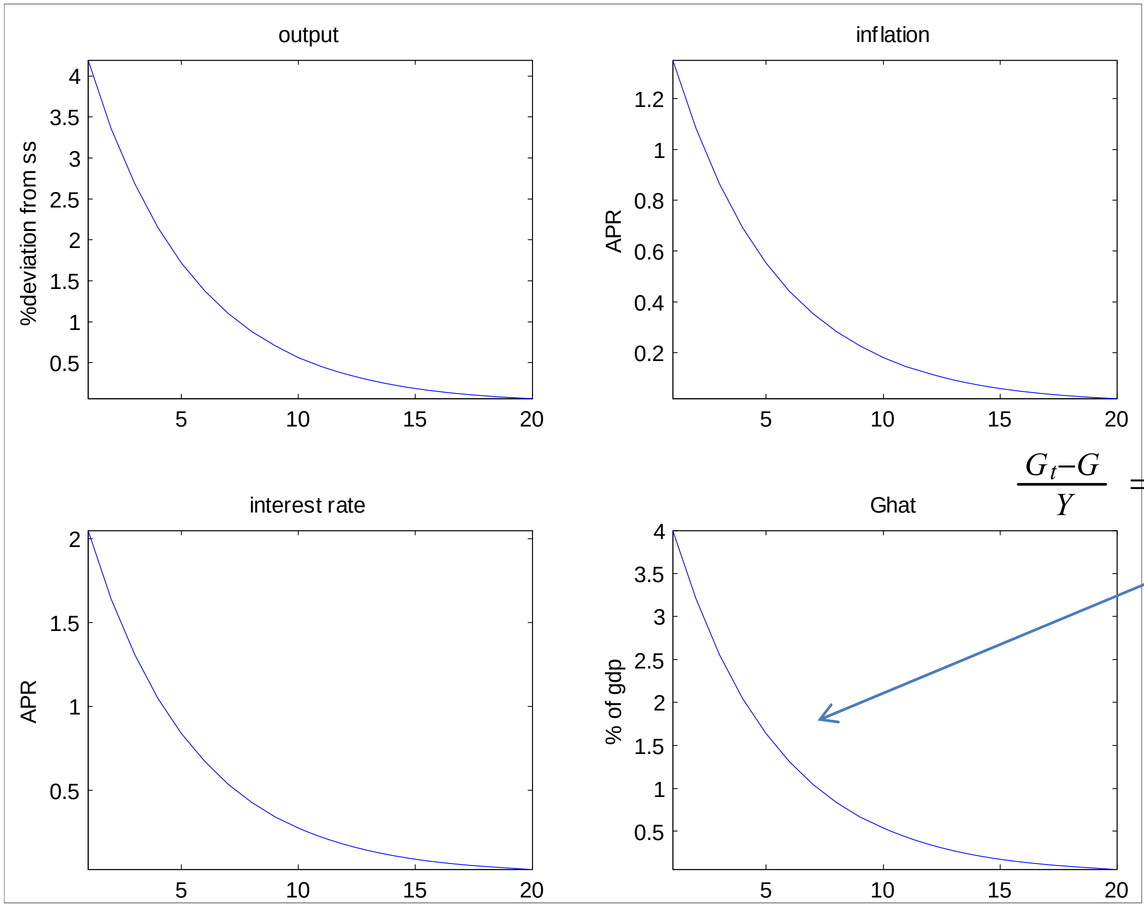
$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1+\gamma}}{1+\gamma} + v(G_t) \right],\ \psi, \gamma, \sigma > 0.$$

  - Marginal utility of *C* independent of *N* for CGG
  - Martinal utility of *C* increases in *N* for KPR.

Simulation Results

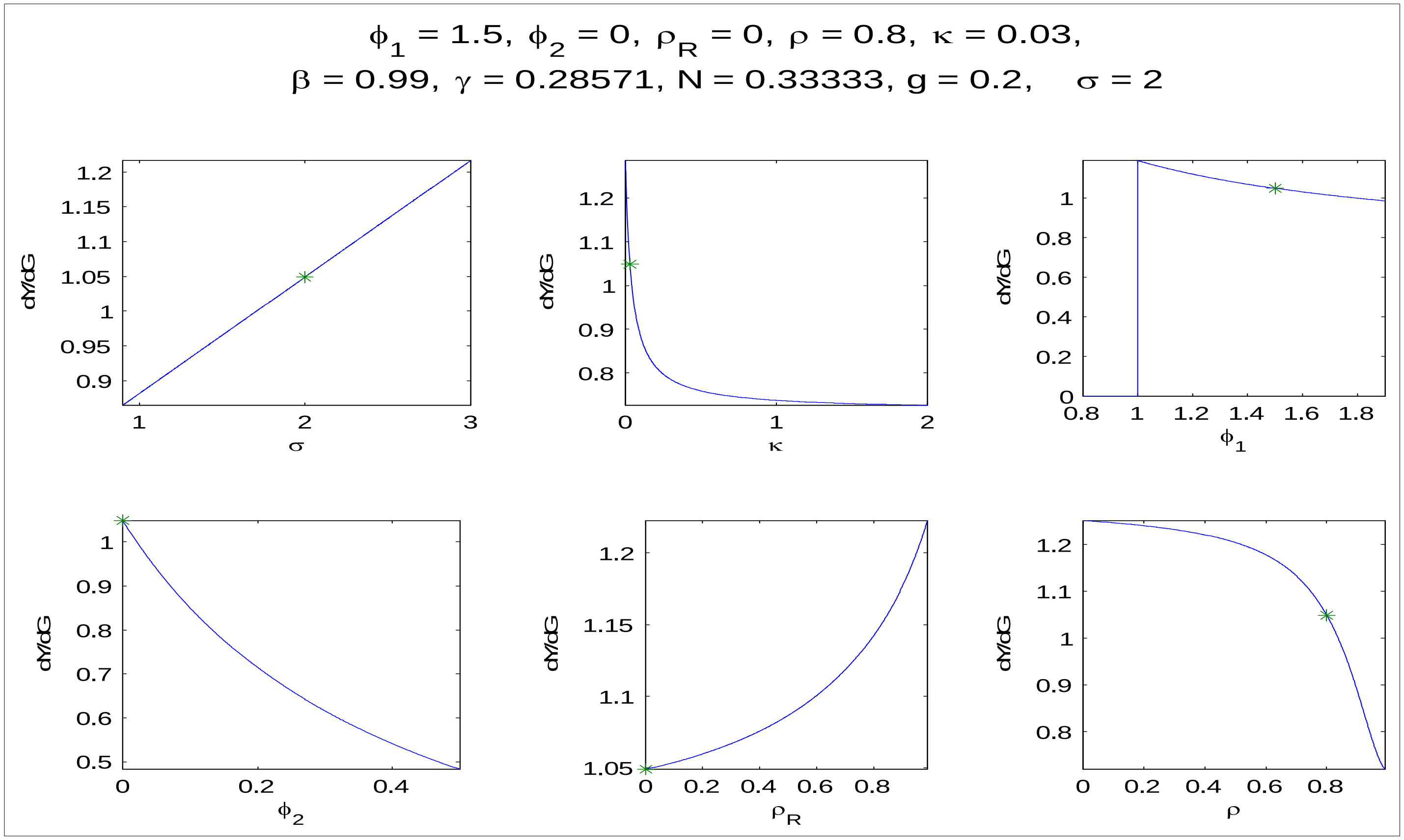
- Benchmark parameter values:

$$\kappa = 0.035,\ \beta = 0.99,\ \phi_1 = 1.5,\ \phi_2 = 0,\ N = 0.23,\ g = 0.2,\ \sigma = 2,\ \rho = 0.8,\ \rho_R = 0$$



$$\frac{G_t - G}{Y} = \frac{G_t - G}{G}\frac{G}{Y} = \hat{G}_t g$$

Multiplier for Alternative Parameter Values



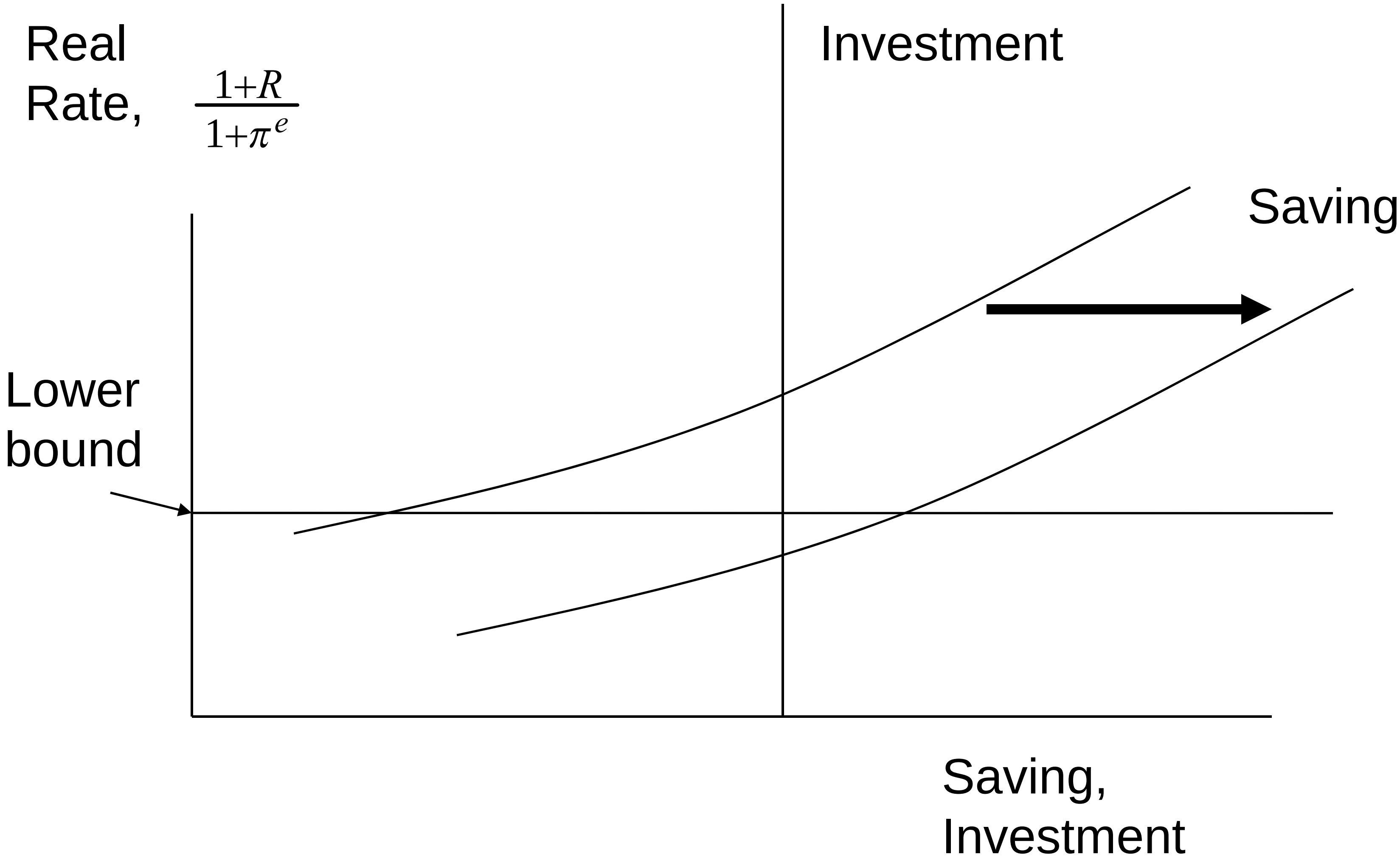
- Results: multiplier bigger
  - the less monetary policy allows *R* to rise.
  - the smoother people want consumption to be.
  - the smaller the negative income effect on consumption.
  - smaller values of κ (i.e., more sticky prices)

Analysis of Case when the Non-negativity Constraint on the Nominal Interest Rate is Binding

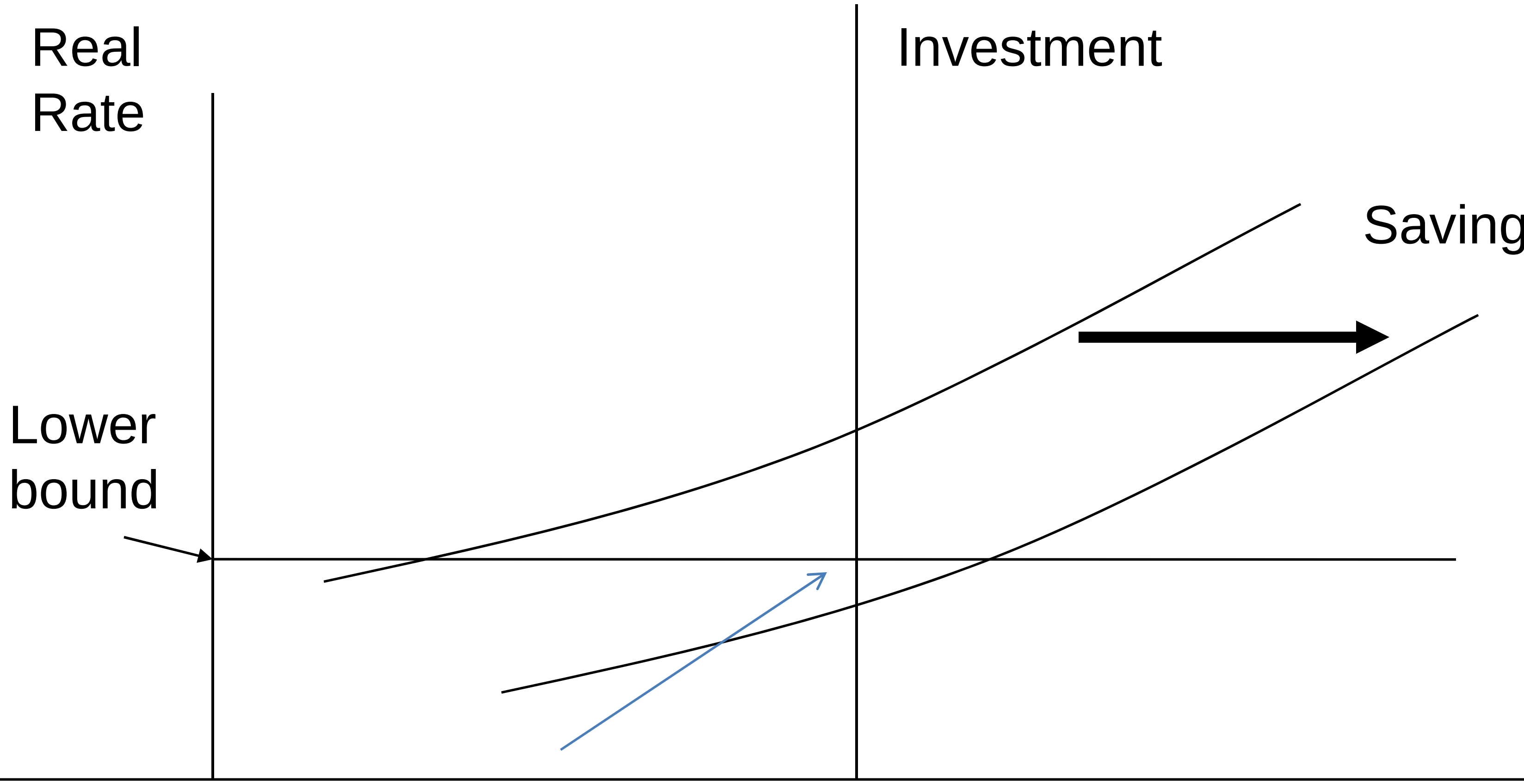
Intuition

- Why can't nominal interest rate be negative?
  - Lending would be zero.
  - Desired borrowing would be infinite.
- The 'zero bound melt-down scenario':
  - Suppose the nominal rate of interest is zero (i.e., it cannot go lower)
  - For some reason, people cut back on spending.
  - Reduced spending creates reduced expected inflation, raising real rate of interest.
  - Motive to cut back on spending is reinforced, raising real rate of interest even more.
- Downward spiral could be arrested by an increase in government spending
  - Scenario emerges naturally in New Keynesian models
  - 'Paradox of Thrift'

Consequence of Increase in Saving When there is Lower Bound on Real Interest Rate

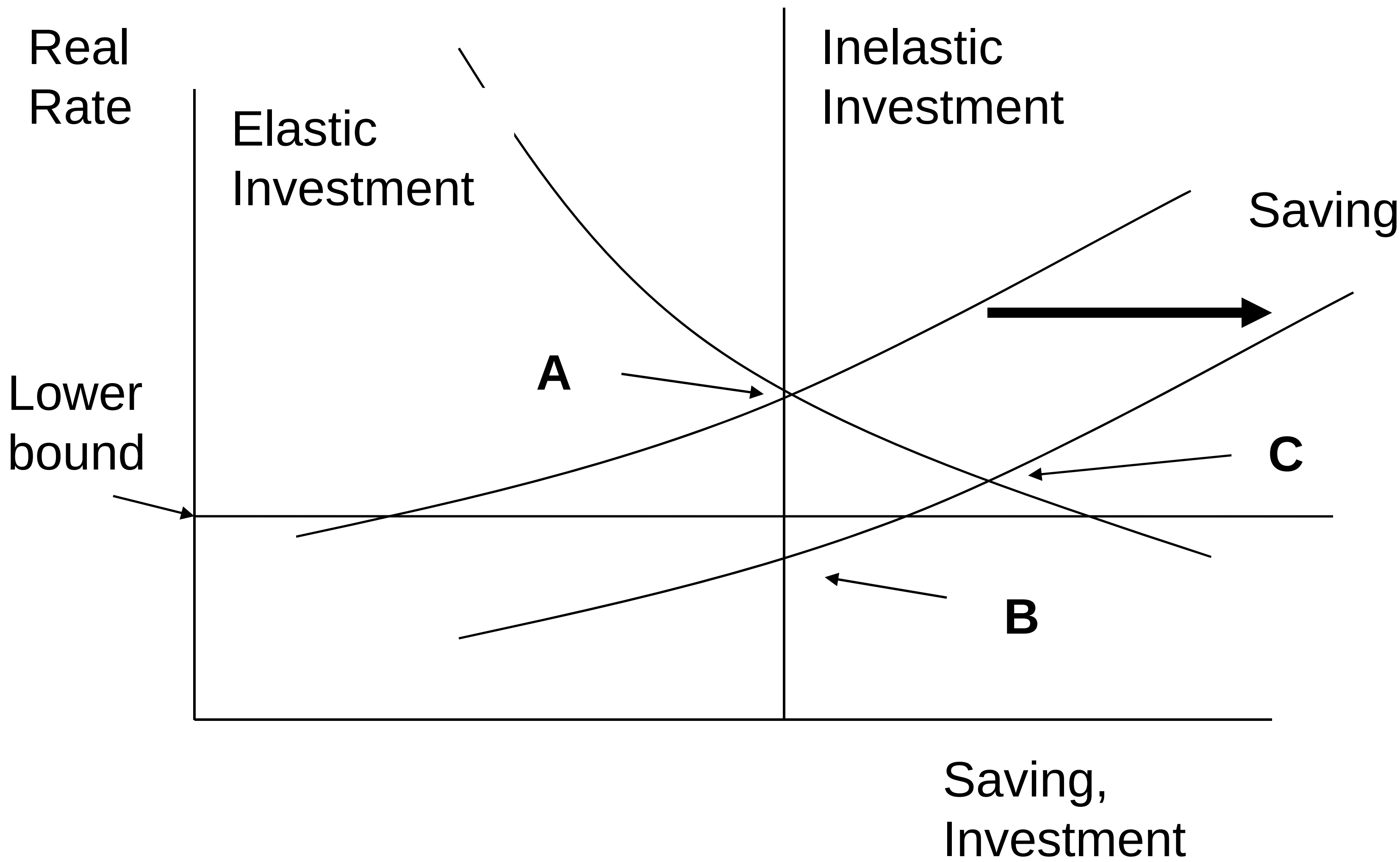


Consequence of Increase in Saving When there is Lower Bound on Real Interest Rate



Loan market cannot be cleared by a reduction in real interest rate. Equilibrium must be achieved by some other mechanism. In New Keynesian model, equilibrium restored by a fall in income, which causes a reduction in desired saving. Same as in textbook `Paradox of Thrift' discussion.

Figure 1: Consequence of Increase in Saving When there is Lower Bound on Real Interest Rate, For Two Investment Elasticities



Turning to the formal analysis….

- Need a shock that puts us into the lower bound.

- Natural (not exclusive) candidate: increased desire to save.

- Saving is influenced by discount rate.

Monetary Policy

- Definition of monetary policy, to take into account the possibility that lower bound is binding.
- Let

$$Z_t = \frac{1}{\beta} - 1 + \rho_R dR_t + (1-\rho_R)\left[\frac{\phi_1}{\beta}\pi_t + \frac{\phi_2}{\beta}\hat{Y}_t\right]$$

- Monetary policy:

$$R_t = \begin{cases} Z_t & \text{if } Z_t \geq 0 \\ 0 & Z_t < 0 \end{cases}.$$

Eggertsson-Woodford Model of a Saving Shock

- Preferences:

$$u(C_0,N_0,G_0)+\tfrac{1}{1+r_1}E_0\left\{u(C_1,N_1,G_1)+\tfrac{1}{1+r_2}u(C_2,N_2,G_1)+\tfrac{1}{1+r_2}\tfrac{1}{1+r_3}u(C_3,N_3,G_3)\ldots\right\}$$

- Before *t<0*
  - System was in non-stochastic, zero inflation steady state,

$$r_t = r \equiv \frac{1}{\beta} - 1,\ \text{for all } t$$

$$R_t = r,\ \text{all } t$$

$$\hat{G}_t = 0,\ \text{all } t$$

Saving Shock, cnt'd

- At time *t=0,*

$$r_1 = r^l < 0$$

$$\Pr ob[r_{t+1} = r | r_t = r^l] = 1 - p$$

$$\Pr ob[r_{t+1} = r^l | r_t = r^l] = p$$

$$\Pr ob[r_{t+1} = r^l | r_t = r] = 0$$

- "Discount rate drops in *t=0* and is expected to return to its 'normal' level with constant probability, *1-p.* When it returns to normal, it is expected to stay there."

Equations that Characterize Equil

- Conjecture (later verified)
  - Equilibrium characterized by
    - $\pi^l, \hat{Y}^l \neq 0, R = 0, Z^l \leq 0$ as long as discount rate is low
    - $\pi_t, \hat{Y}_t = 0,\ R = r$ as soon as discount rate returns to normal
- Equations that must be satisfied, for the candidate equilibrium to be an actual equilibrium…..

Must Modify IS Equation

- Household inter-temporal Euler equation

$$u_{c,t} = \beta_{t+1} E_t u_{c,t+1} \frac{1+R_{t+1}}{1+\pi_{t+1}}$$

$$\hat{u}_{c,t} = \hat{\beta}_{t+1} + \hat{u}_{c,t+1} + \widehat{1+R_{t+1}} - \widehat{1+\pi_{t+1}}$$

$$= \hat{\beta}_{t+1} + \hat{u}_{c,t+1} + \beta dR_{t+1} - d\pi_{t+1}$$

- Result:

$$\hat{\beta}_{t+1} = \widehat{\frac{1}{1+r_{t+1}}} = \frac{d\frac{1}{1+r_{t+1}}}{\frac{1}{1+r}} = \frac{-\frac{1}{(1+r)^2} dr_{t+1}}{\frac{1}{1+r}} = -\frac{\beta^2 dr_{t+1}}{\beta} = -\beta dr_{t+1}$$

- Then:

$$\hat{u}_{c,t} = \hat{u}_{c,t+1} + \beta \overbrace{(R_{t+1} - r_{t+1})}^{\text{note:} = dR_{t+1} - dr_{t+1}} - d\pi_{t+1}$$

Fiscal Policy

- Government spending is set to a constant deviation from steady state, during the zero bound.

- That is,

$\hat{G}_t$ may be nonzero while $r_{t+1} = r^l$, $\hat{G}_t = 0$ when $r_{t+1} = r$

Equations With Discount Shock

- IS equation:

$$\hat{Y}_t - g[\gamma(\sigma-1)+1]\hat{G}_t = -(1-g)[\beta(R_{t+1}-r_{t+1}) - E_t\pi_{t+1}] + E_t\hat{Y}_{t+1} - g[\gamma(\sigma-1)+1]E_t\hat{G}_{t+1}$$

$$\hat{Y}^l - g[\gamma(\sigma-1)+1]\hat{G}^l = -(1-g)[\beta(0-r^l) - p\pi^l] + p\hat{Y}^l - g[\gamma(\sigma-1)+1]p\hat{G}$$

- Phillips curve:

$$\pi_t = \beta E_t\pi_{t+1} + \kappa\left[\left(\frac{1}{1-g}+\frac{N}{1-N}\right)\hat{Y}_t - \frac{g}{1-g}\hat{G}_t\right]$$

$$\pi^l = \beta p\pi^l + \kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right)\hat{Y}^l - \frac{g}{1-g}\kappa\hat{G}^l$$

- Monetary Policy:

$$Z_t = R + \rho_R(R_t - R) + (1-\rho_R)\left[\frac{\phi_1}{\beta}\pi_t + \frac{\phi_2}{\beta}\hat{Y}_t\right]$$

$$R_{t+1} = \begin{cases} Z_t & \text{if } Z_t > 0 \\ 0 & \text{if } Z_t \le 0 \end{cases},$$

$$Z^l = R + \rho_R(0-R) + (1-\rho_R)\left[\frac{\phi_1}{\beta}\pi^l + \frac{\phi_2}{\beta}\hat{Y}^l\right] \le 0$$

Solving for the Zero Bound Allocations

- Is equation:

$$\hat{Y}^l - g[\gamma(\sigma-1)+1]\hat{G}^l = -(1-g)[\beta(0-r^l) - p\pi^l] + p\hat{Y}^l - g[\gamma(\sigma-1)+1]p\hat{G}$$

- Phillips curve:

$$\pi^l = \beta p \pi^l + \kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right)\hat{Y}^l - \frac{g}{1-g}\kappa\hat{G}^l$$

- Two equations in two unknowns!

  – Solve for $\hat{Y}^l, \pi^l$ and verify that $Z^l \leq 0$

Solution

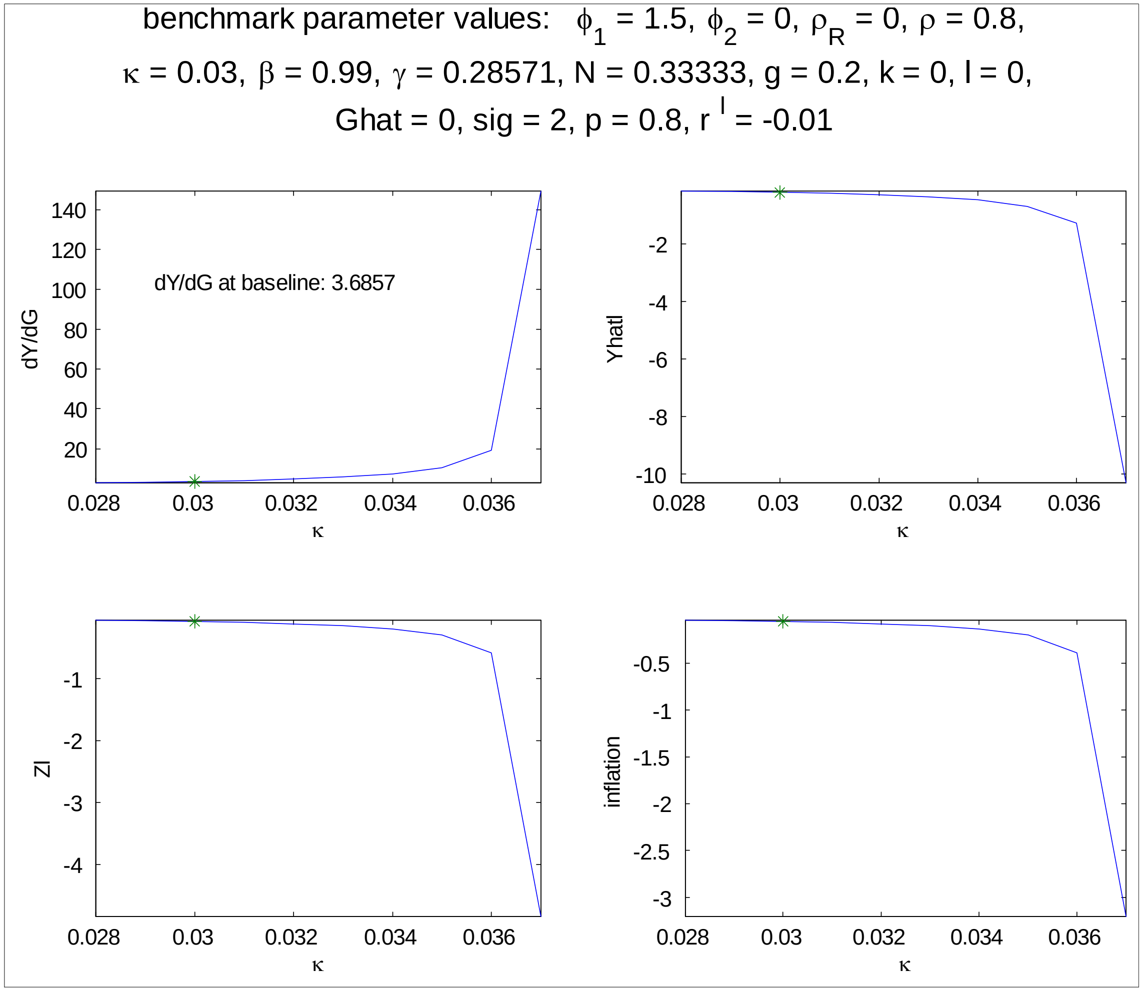
- Inflation:

$$\pi^l = \frac{\kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right)\left[g[\gamma(\sigma-1)+1]\hat{G}^l+\frac{1-g}{1-p}\beta r^l\right]-\frac{g}{1-g}\kappa\hat{G}^l}{1-\beta p-\kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right)p\frac{1-g}{1-p}}$$

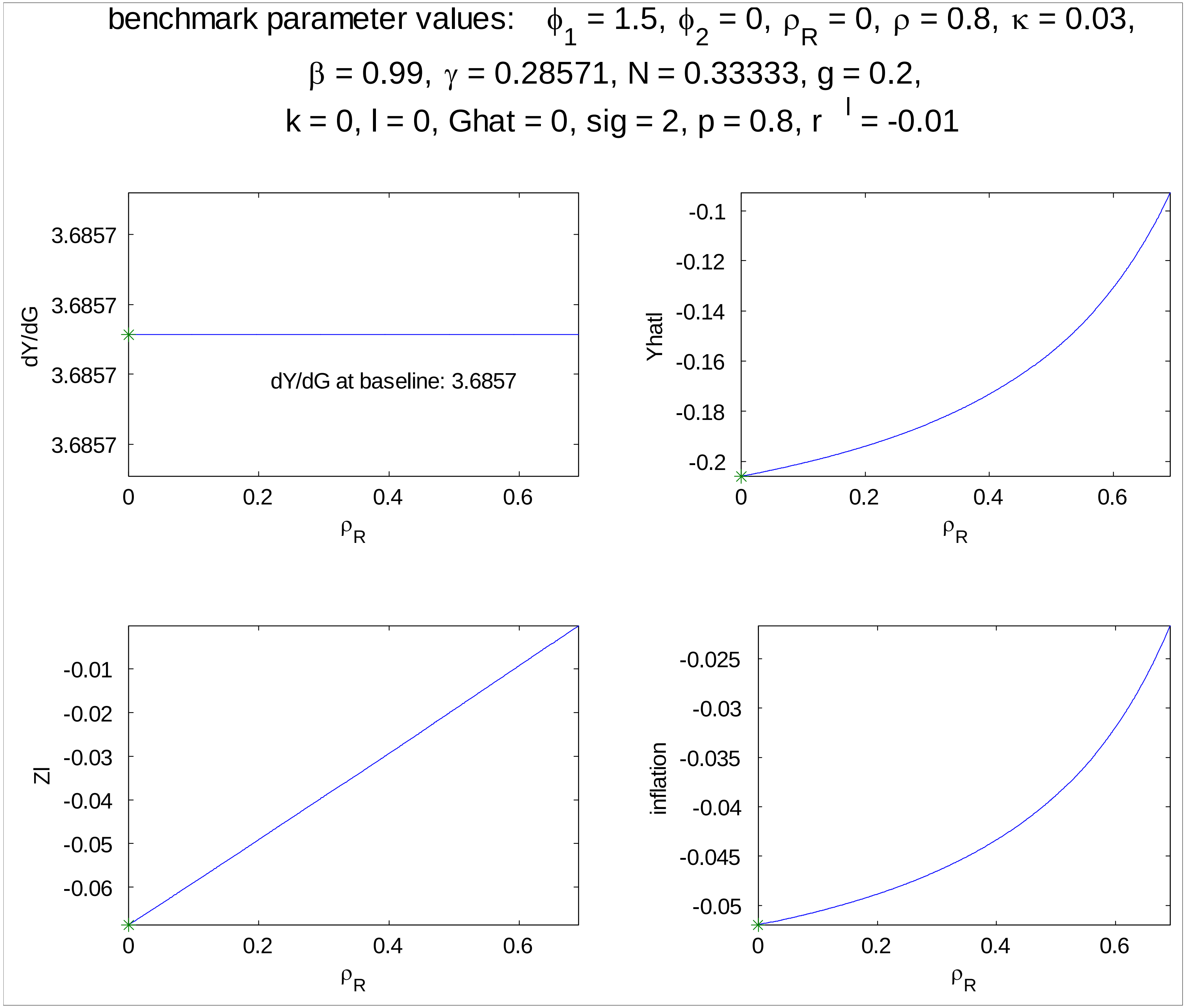
- Output:

$$\hat{Y}^l = g[\gamma(\sigma-1)+1]\hat{G}^l + \frac{1-g}{1-p}[\beta r^l + p\pi^l]$$

Numerical Simulations



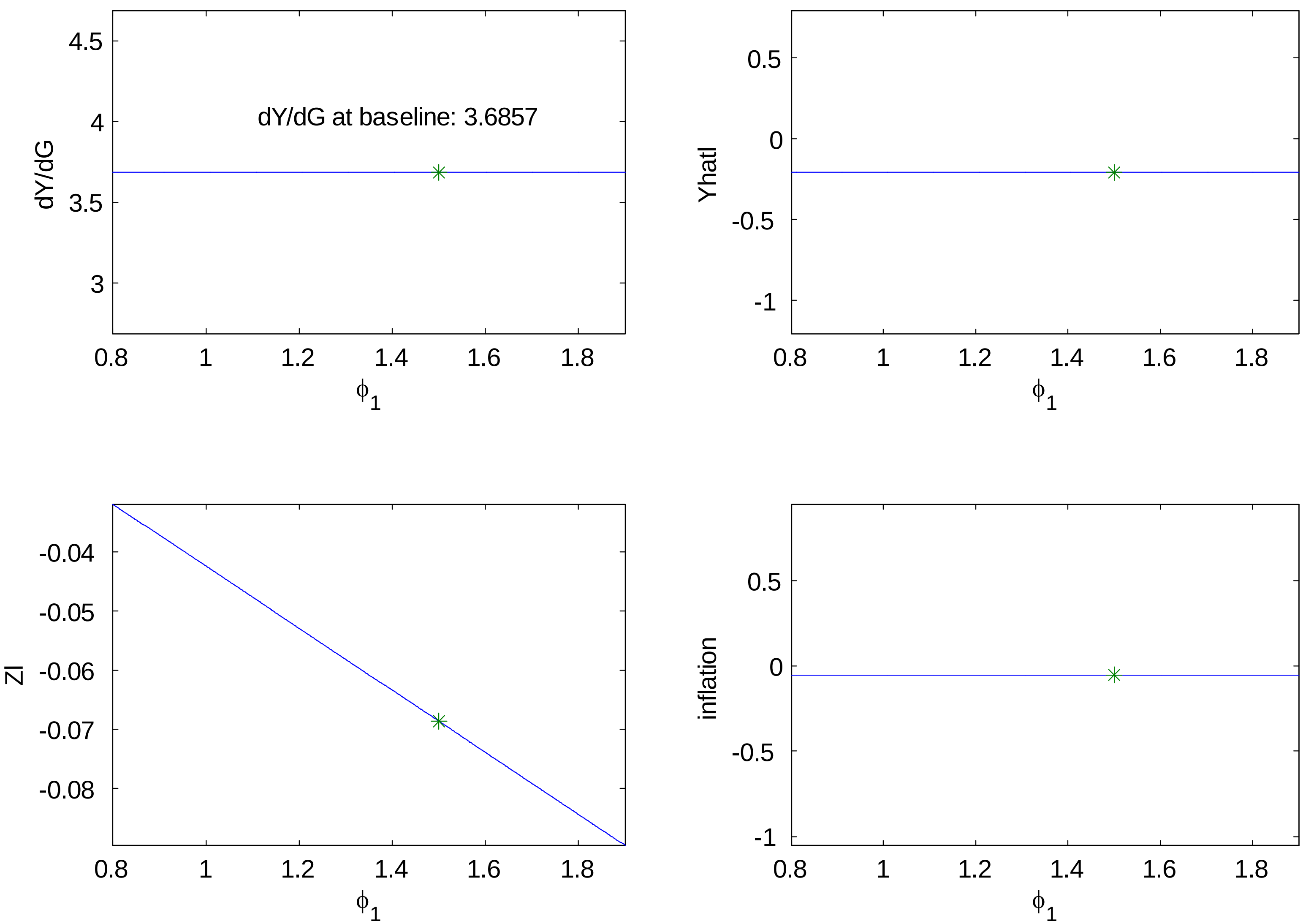
- Results: multiplier 3.7 at benchmark parameter values and may be gigantic


benchmark parameter values: ϕ_1 = 1.5, ϕ_2 = 0, ρ_R = 0, ρ = 0.8, κ = 0.03,
β = 0.99, γ = 0.28571, N = 0.33333, g = 0.2,
k = 0, l = 0, Ghat = 0, sig = 2, p = 0.8, r^l = -0.01
dY/dG
3.6857
3.6857
3.6857
3.6857
dY/dG at baseline: 3.6857
0
0.2
0.4
0.6
ρ_R
Yhatl
-0.1
-0.12
-0.14
-0.16
-0.18
-0.2
0
0.2
0.4
0.6
ρ_R
Zl
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
0
0.2
0.4
0.6
ρ_R
inflation
-0.025
-0.03
-0.035
-0.04
-0.045
-0.05
0
0.2
0.4
0.6
ρ_R

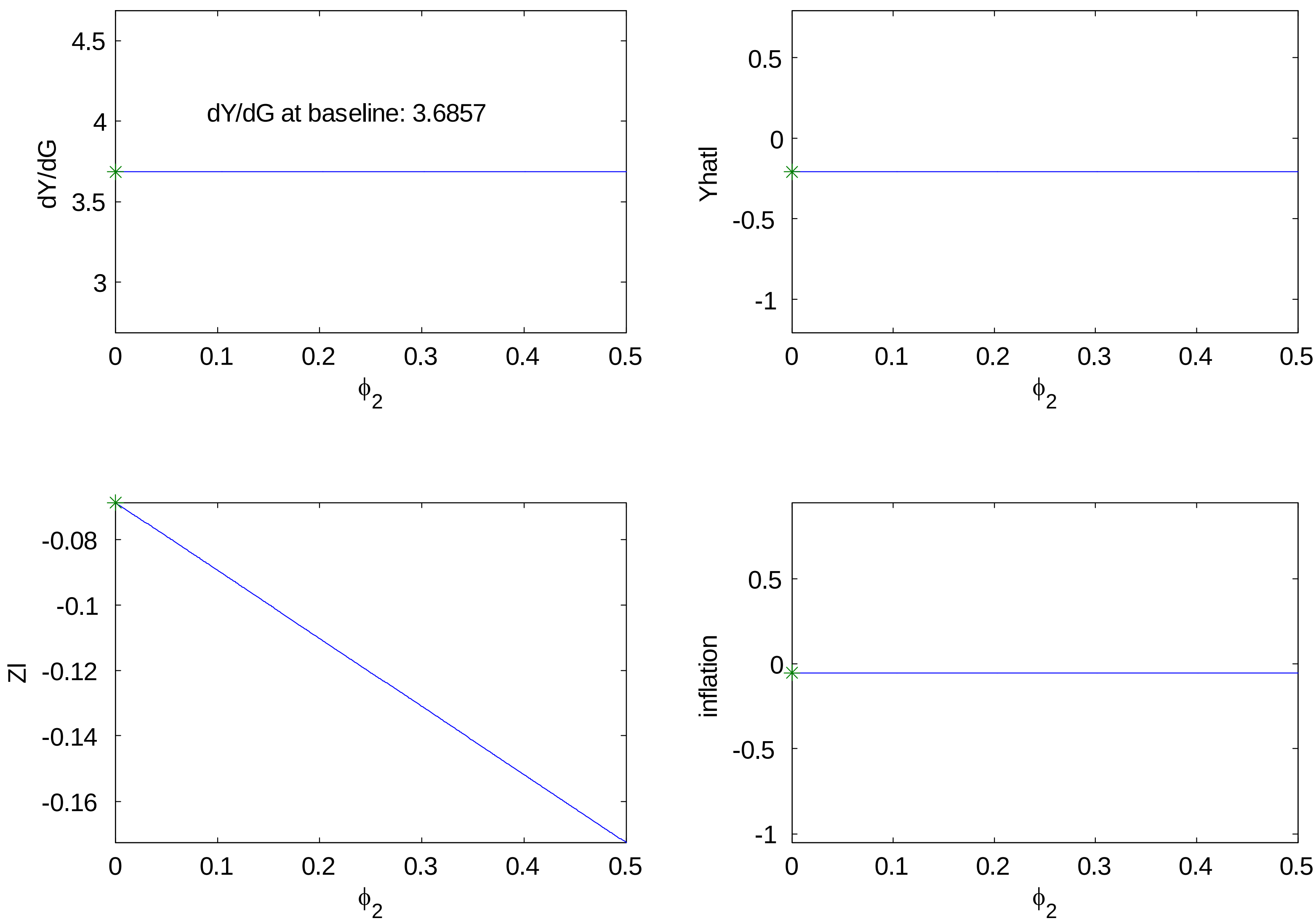
benchmark parameter values: ϕ_1 = 1.5, ϕ_2 = 0, ρ_R = 0, ρ = 0.8, κ = 0.03,

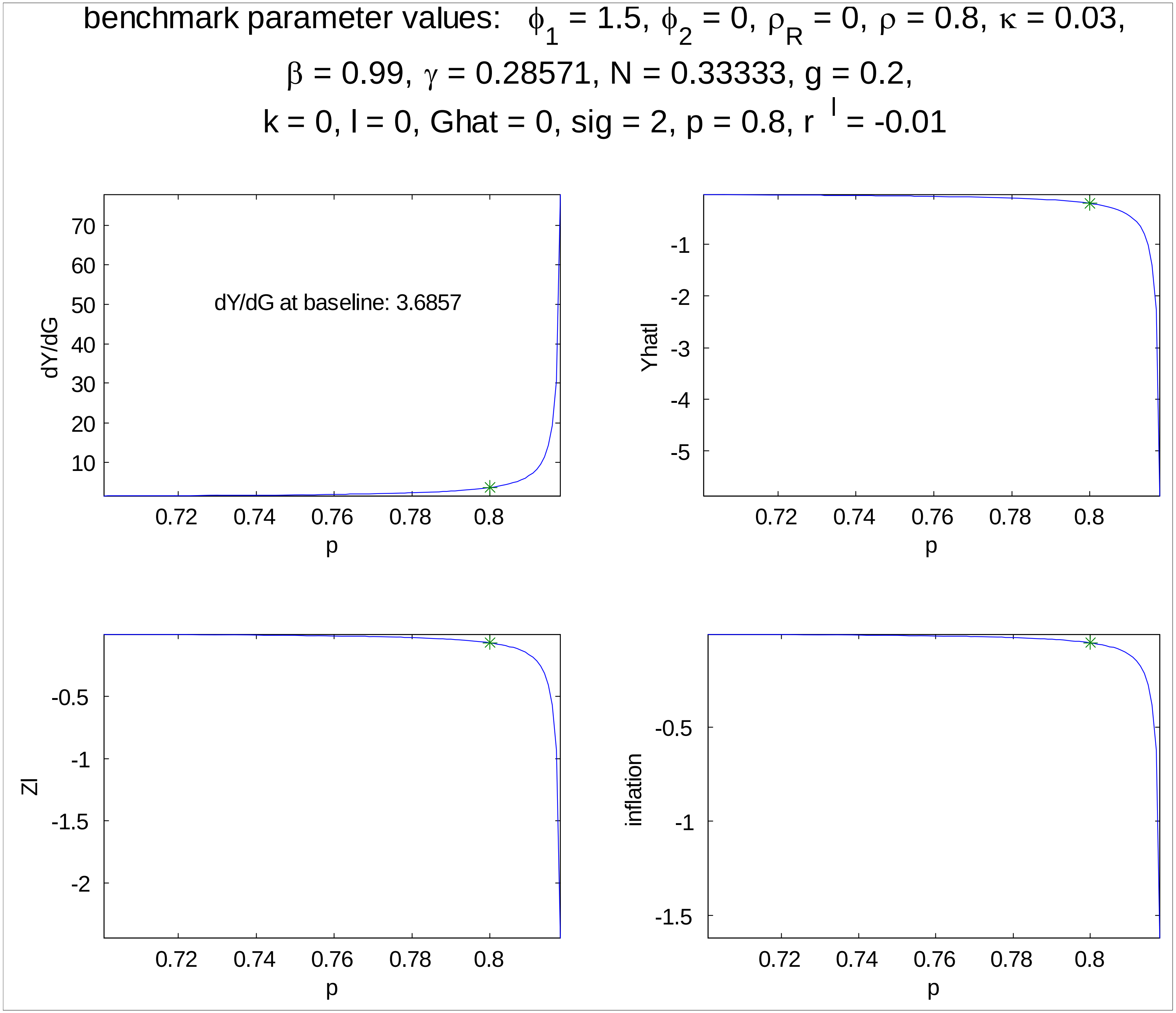
β = 0.99, γ = 0.28571, N = 0.33333, g = 0.2,

k = 0, l = 0, Ghat = 0, sig = 2, p = 0.8, r^l = -0.01



benchmark parameter values: $\phi_1 = 1.5$, $\phi_2 = 0$, $\rho_R = 0$, $\rho = 0.8$, $\kappa = 0.03$,
$\beta = 0.99$, $\gamma = 0.28571$, N = 0.33333, g = 0.2,
k = 0, l = 0, Ghat = 0, sig = 2, p = 0.8, $r^l = -0.01$




benchmark parameter values: ϕ_1 = 1.5, ϕ_2 = 0, ρ_R = 0, ρ = 0.8, κ = 0.03,
β = 0.99, γ = 0.28571, N = 0.33333, g = 0.2,
k = 0, l = 0, Ghat = 0, sig = 2, p = 0.8, r^l = -0.01
dY/dG at baseline: 3.6857
dY/dG
Yhatl
Zl
inflation
p
p
p
p

Fiscal Expansion in Zero Bound Highly Effective, But is it *Desirable*?

- Preferences

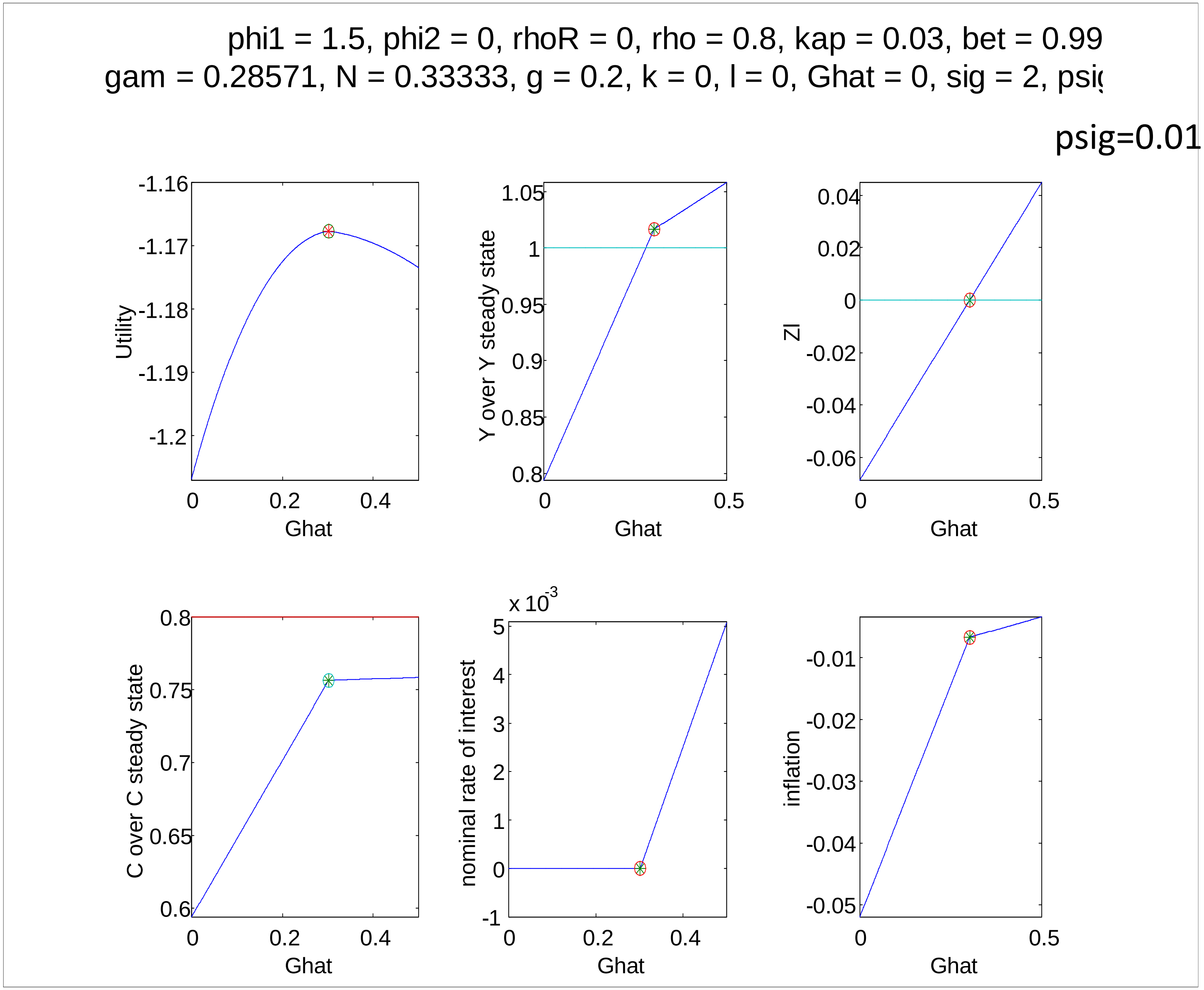
$$\sum_{t=0}^{\infty}(p\beta)^t\left[\frac{\left[(C^l)^{\gamma}(1-N^l)^{1-\gamma}\right]^{1-\sigma}-1}{1-\sigma}+v(G^l)\right]$$

$$=\frac{1}{1-p\beta}\left[\frac{\left[(C^l)^{\gamma}(1-N^l)^{1-\gamma}\right]^{1-\sigma}-1}{1-\sigma}+v(G^l)\right]$$

$$=\frac{1}{1-p\beta}\frac{\left([N(\hat{Y}^l+1)-Ng(\hat{G}^l+1)]^{\gamma}(1-N(\hat{Y}^l+1))^{1-\gamma}\right)^{1-\sigma}-1}{1-\sigma}+v(Ng(\hat{G}^l+1))$$

- Compute optimal $\hat{G}^l$
  - (i) $v(G^l)$ = 0,
  - (ii) $v(G)=\psi_g\frac{G^{1-\sigma}}{1-\sigma}$, ψ_g chosen to rationalize $g=0.2$ as optimal in steady state

Case Where Gov't Spending is Desirable



psig=0.015226

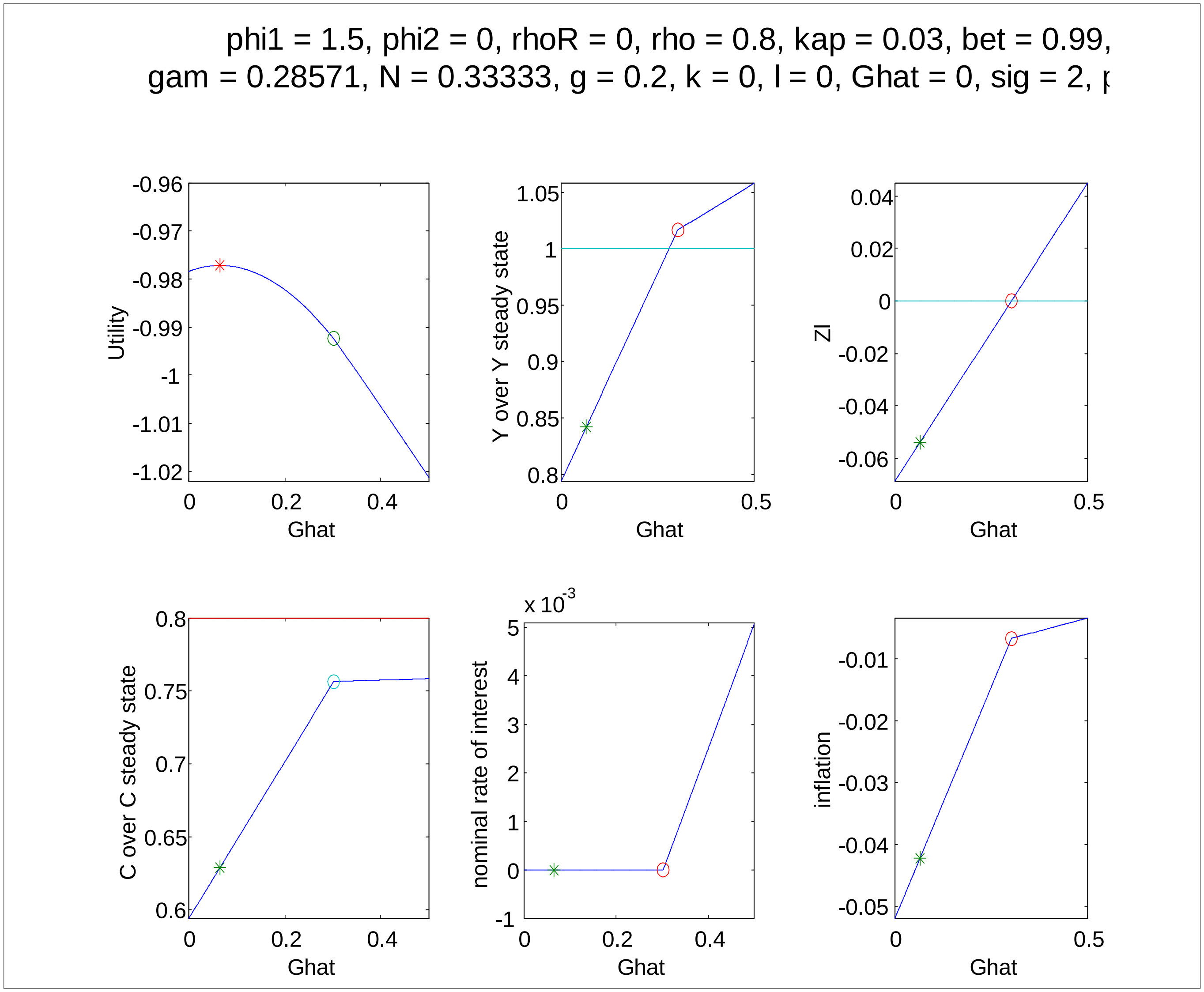
Results When G is Desirable

- Exercise G stimulus to the point where output is higher than in the non-zero bound steady state.

- Consumption is still a little low in this case, because the higher G in effect taxes away output.

- The higher G raises welfare because it helps to undo the inefficiency of the lower bound, and G is valued.

Case Where G is not Valued



Results when G is not Valued

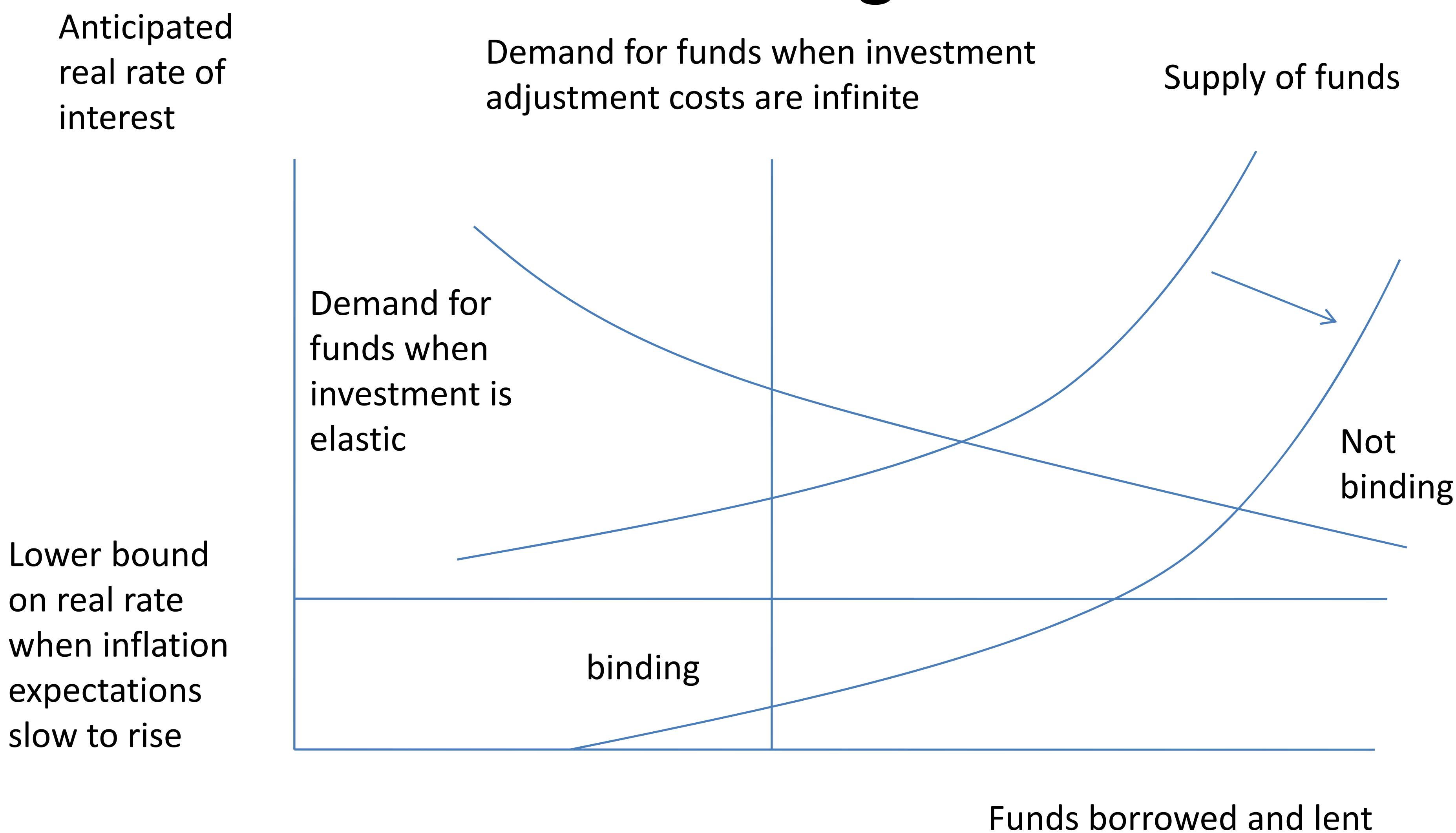
- It is still optimal to apply a stimulus during the lower bound.

- However, it is not optimal to put output at its non-zero bound steady state.

Introducing Investment

- Important to check robustness to investment
- Could greatly reduce the likelihood of zero bound shock (as real rate falls with rise in saving, investment expands to absorb rise in saving).
- Simulations that follow provide preliminary evidence that results are in fact robust to introduction of investment.

Impact of Adjustment Costs on Likelihood of Binding Zero bound



Model with Investment

- Preferences same as before:

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{[C_t^{\gamma}(1-N_t)^{1-\gamma}]^{1-\sigma}-1}{1-\sigma} + v(G_t) \right]$$

- Capital accumulation

$$K_{t+1} = (1-\delta)K_t + \left[1 - \frac{\sigma_I}{2}\left(\frac{I_t}{K_t} - \delta\right)^2\right]I_t$$

- Resources:

$$Y_t = C_t + G_t + I_t$$

$$Y_t = p_t^* K_t^{\alpha} N_t^{1-\alpha}$$

The household Lagrangian problem is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \left[\frac{[C_t^{\gamma}(1-N_t)^{1-\gamma}]^{1-\sigma} - 1}{1-\sigma} + v(G_t) \right]$$
$$+ \lambda_t [B_t(1+R_t) + W_t N_t + P_t r_t^k K_t + T_t - P_t(C_t + e^{-\psi_t} I_t) - B_{t+1}]$$
$$+ \mu_t \left[(1-\delta)K_t + \left[1 - \frac{\sigma_I}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 \right] I_t - K_{t+1} \right] \Bigg\},$$

$$\begin{aligned}
u_{c,t} &= \lambda_t P_t \\
\lambda_t e^{-\psi_t} P_t &= \mu_t \left[-\sigma_I \left(\frac{I_t}{K_t} - \delta \right) \frac{I_t}{K_t} + 1 - \frac{\sigma_I}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 \right] \\
\mu_t &= \beta \left[\lambda_{t+1} P_{t+1} r_{t+1}^k + \mu_{t+1} \left(1 - \delta + \sigma_I \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \left(\frac{I_{t+1}}{K_{t+1}} \right)^2 \right) \right] \\
u_{Nt} + \lambda_t W_t &= 0 \\
\lambda_t &= \beta \lambda_{t+1} (1 + R_{t+1})
\end{aligned}$$

Note that

$$\mu_t = \frac{d\text{utility}}{dK_{t+1}}$$

$$\lambda_t P_t = \frac{d\text{utility}}{dc_t}$$

so that

$$\frac{\mu_t}{\lambda_t P_t} = \frac{\frac{d\text{utility}}{dK_{t+1}}}{\frac{d\text{utility}}{dc_t}} = \frac{dc_t}{dK_{t+1}} = P_{k',t}.$$

The first order condition for K_{t+1} can be rewritten

$$\frac{\mu_t}{P_t\lambda_t} = \beta\frac{P_{t+1}\lambda_{t+1}}{P_t\lambda_t}\left[r_{t+1}^k + \frac{\mu_{t+1}}{P_{t+1}\lambda_{t+1}}\left(1-\delta+\sigma_I\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)\left(\frac{I_{t+1}}{K_{t+1}}\right)^2\right)\right],$$

or,

$$P_{k',t} = \beta\frac{P_{t+1}\lambda_{t+1}}{P_t\lambda_t}\left[r_{t+1}^k + P_{k',t+1}\left(1-\delta+\sigma_I\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)\left(\frac{I_{t+1}}{K_{t+1}}\right)^2\right)\right],$$

or,

$$P_{k',t} = \beta\frac{u_{c,t+1}}{u_{c,t}}\left[r_{t+1}^k + P_{k',t+1}\left(1-\delta+\sigma_I\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)\left(\frac{I_{t+1}}{K_{t+1}}\right)^2\right)\right],$$

Calibration of Adjustment Cost

- Price of capital in competitive market = marginal cost:

$$P_{k',t} = \frac{P_{I,t}}{MP_{I,t}} = \frac{1}{-\sigma_I\left(\frac{I_t}{K_t}-\delta\right)\frac{I_t}{K_t}+1-\frac{\sigma_I}{2}\left(\frac{I_t}{K_t}-\delta\right)^2}$$

- Elasticity of investment to price, in steady state:

$$\frac{d\log\frac{I_t}{K_t}}{d\log P_{k',t}} = \frac{1}{\sigma_I\delta^2}$$

$$\simeq 1 \text{ in data}$$

$$\rightarrow \sigma_I = \frac{1}{\delta^2} = 2,500.$$

Experiment

- Discount rate drops in periods 1 to 10, and switches to steady state starting in period 11.

- Deterministic, to simplify calculations.

Monetary Policy

$$Z_t = \frac{1}{\beta} - 1 + \rho_R dR_t + (1-\rho_R)\left[\frac{\phi_1}{\beta}\pi_t + \frac{\phi_2}{\beta}\hat{Y}_t\right]$$

$$R_{t+1} = \begin{cases} Z_t & Z_t \geq 0 \quad \text{‘zero bound not binding’} \\ 0 & Z_t < 0 \quad \text{‘binding zero bound’} \end{cases}$$

Model Equations In *t* When Zero-Bound Not Binding

- Period t endogenous variables:

$$z_t = \begin{pmatrix} dN_t & d\pi_t & dK_{t+1} & dR_{t+1} & dI_t & dZ_t \end{pmatrix}'$$

$$s_t = \begin{pmatrix} dr_{t+1} & \hat{G}_t \end{pmatrix}$$

- System outside of lower bound:

$$\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t = 0,$$

- Bottom equation corresponds to $R_{t+1} = Z_t$:
  - bottom row of $\alpha_0,\ \alpha_2,\ \beta_0,\ \beta_1$ composed of zeros.
  - bottom row of α_1:

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & -1 \end{pmatrix}$$

System When Zero Bound is Binding

- In this case,

$$dR_t = -\left(\frac{1}{\beta} - 1\right)$$

- The difference equation has to be modified to accommodate the change

System when lower bound is binding

$$\alpha_0 z_{t+1} + \tilde{\alpha}_1 z_t + \alpha_2 z_{t-1} + d_t + \beta_0 s_{t+1} + \beta_1 s_t = 0,$$

$$\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + d_t + \beta_0 s_{t+1} + \beta_1 s_t = 0$$

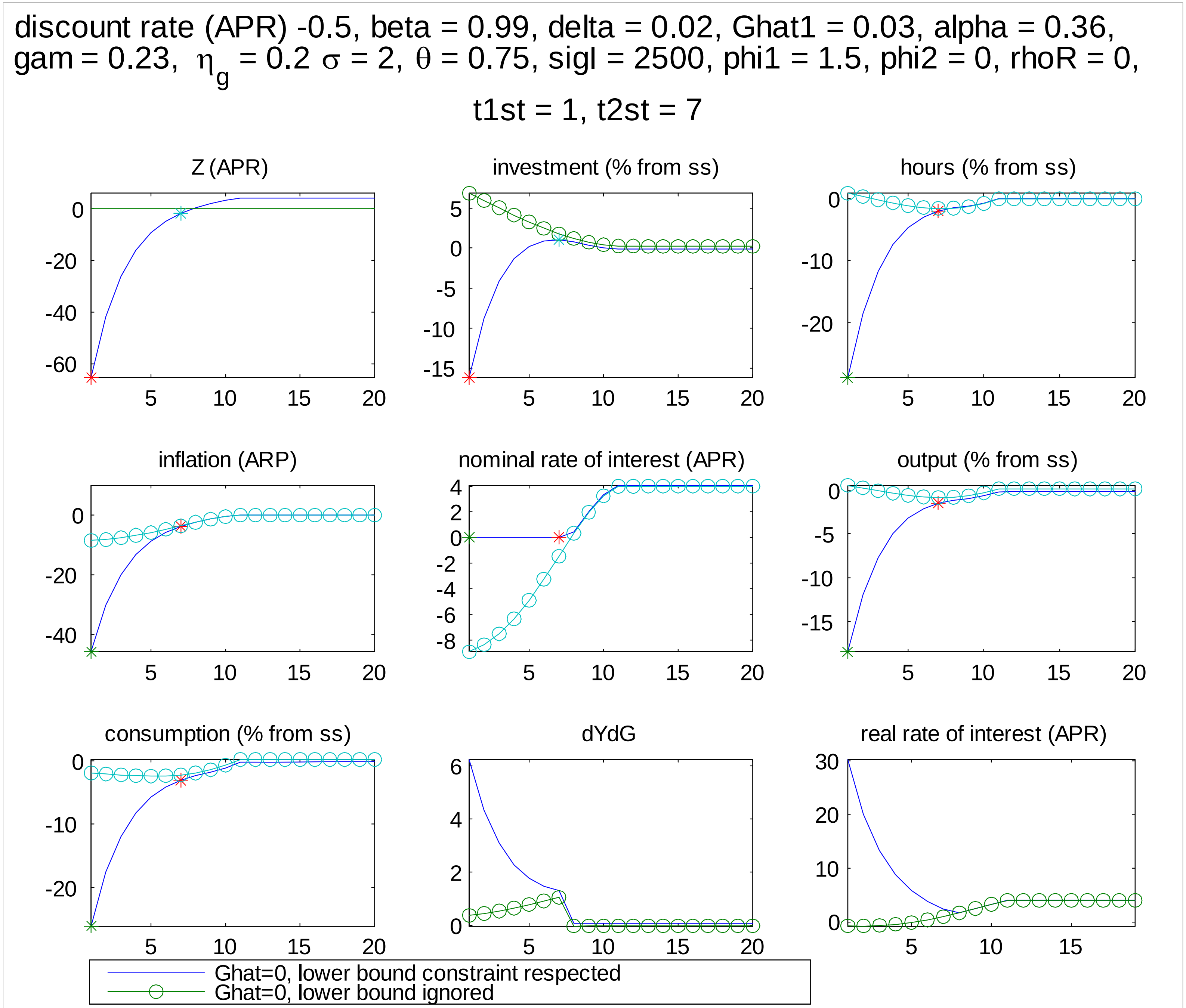
System when lower bound is non-binding

- Search over initial z(0)

- Posit first and last dates when zero bound is binding.

- Simulate the system to some last date.

- Shoot until the last and second-to-last dates are consistent with policy rule.


discount rate (APR) -0.5, beta = 0.99, delta = 0.02, Ghat1 = 0.03, alpha = 0.36,
gam = 0.23, η_g = 0.2 σ = 2, θ = 0.75, sigl = 2500, phi1 = 1.5, phi2 = 0, rhoR = 0,
t1st = 1, t2st = 7
Z (APR)
investment (% from ss)
hours (% from ss)
inflation (ARP)
nominal rate of interest (APR)
output (% from ss)
consumption (% from ss)
dYdG
real rate of interest (APR)
Ghat=0, lower bound constraint respected
Ghat=0, lower bound ignored

Conclusion

- Government spending multiplier can be bigger than unity in ‘normal times’, though the multiplier is not very large.
- Multiplier can be enormous when the zero bound is binding. It is welfare improving at that time to apply fiscal stimulus.
- Further explorations: introduction of capital, open economy.