Economics 416.
Advanced Macroeconomics
Christiano
Homework #5, due May 11.

A model popularized by Clarida Gali and Gertler has the following equations:

$$\begin{aligned}
\beta E_t \pi_{t+1} + \kappa x_t - \pi_t &= 0 \text{ (Calvo pricing equation)} \\
-\left[r_t - E_t \pi_{t+1} - rr_t^*\right] + E_t x_{t+1} - x_t &= 0 \text{ (intertemporal equation)} \\
\phi_\pi \pi_t + \phi_x x_t - r_t &= 0 \text{ (policy rule)} \\
rr_t^* - \rho \Delta a_t &= 0 \text{ (definition of natural rate)} \\
y_t^* &= a_t \text{ (natural output)} \\
x_t &= y_t - y_t^* \text{ (output gap)}
\end{aligned}$$

A detailed derivation of these equations can be found in example 5 in
http://faculty.wcas.northwestern.edu/~lchrist/course/solving_handout.pdf. Also, see Clarida-Gali-Gertler, 'The Sience of Monetary Policy,' or Gali, Lopez-Salido, and Valles, 2003, "Technology Shocks and Monetary Policy: Assessing the Fed's Performance." *Journal of Monetary Economics.* 50 (May): 723 - 743.

In the above model, x_t represents the percent deviation of equilibrium output from its 'first best' or 'natural' level: the one where inflation is zero (so that there are no distortions in the allocation of resources); consumption and employment are set to equate the marginal rate of substitution between consumption and leisure and the marginal product of labor; and where the rate of interest is the level that would just stop people from entering bond markets to try and deviate away from the first best intertemporal pattern of consumption. The latter interest rate is denoted rr_t^* in the above equations, and we'll refer to it as the 'natural rate'. This is the interest rate such that if the monetary authority set the actual interest rate, r_t, to rr_t^*, the equilibrium of the model involves $\pi_t = 0$, $x_t = 0$ (i.e., output is equal to its first-best). Put differently, rr_t^* is the interest rate that causes the household's intertemporal Euler equation to be satisfied conditional on consumption being set to its first best level.

The specification of the above model presumes a 'unit root' model for the law of motion for technology:

$$\Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t, \ E\varepsilon_t^2 = \sigma_\varepsilon^2, \ \Delta a_t \equiv a_t - a_{t-1}.$$

The 'trend stationary' law of motion for technology is:

$$a_t = \rho a_{t-1} + \varepsilon_t, \ E\varepsilon_t^2 = \sigma_\varepsilon^2.$$

Let the parameters of the model be given by:

$$\begin{aligned} \beta &= 0.97, \ \phi_x = 0., \ \phi_\pi = 1.5, \ \rho = 0.8, \\ \varphi &= 1, \ \theta = 0.75, \ \sigma_\varepsilon = 0.02, \end{aligned}$$

and

$$\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}.$$

1. What is the long run effect on a_t of a one-time jump in ε_t under the unit root and trend stationary representations?

2. Explain why the coefficient on Δa_t in the natural rate of interest equation is positive.

3. Re-derive the expression for the natural rate of interest under the trend stationary specification of technology. Note that the sign of the response of the natural rate to technology changes. Provide the economics for this.

4. Plot the response of hours, output and inflation, n_t, y_t and π_t, to a shock in ε_t for the unit root model (to understand how hours worked enters the picture here, have a look at the url I cited above). Also plot the dynamic response of the real rate of interest, $r_t - \pi_{t+1}$ (note the shift in timing on inflation). On the same graphs, plot the 'natural' responses of n_t, y_t, π_t and $r_t - \pi_{t+1}$. Provide intuition on the reason for the difference between the actual and natural responses.

5. Do the same experiment as in (4), except with ϕ_π set to a bigger number. Explain why this makes the actual and first best responses closer. How big does ϕ_π have to be for the actual and first best allocations to be similar? Is raising ϕ_x another way to make the actual and first best responses more similar? Explain.

6. Redo (4) and (5) with the trend stationary representation. How do your results change, both in terms of the comparison of actual and first best responses and in terms of inference about the best interest rate rule parameters?

7. Adopt the following time series representation for technology:

$$a_t = \rho a_{t-1} + \varepsilon_t + \xi_{t-1},$$

   where ε_t and ξ_{t-1} are mutually uncorrelated at all leads and lags and uncorrelated with a_{t-s}, $s > 0$. The statistical innovation in a_t is still an iid process. It's just that now the time t innovation is represented as $\varepsilon_t + \xi_{t-1}$. The point of this extra complication is that part of the period t innovation, namely ξ_{t-1}, is observed in period $t-1$ by agents.[1] Derive the expression for the natural rate and explain why the sign of the impact of ε_t and ξ_t on the natural rate of interest are opposite.

8. Most people take it for granted that a jump in news about the future (i.e., in ξ_t) will cause an expansion in the economy, and will lead to an immediate rise in inflation because the expansion is fundamentally demand driven (remember, the signal is about *future* technology and involves no change in current technology).

   (a) Simulate the response of model variables to ξ_t. Show that there are parameter values under which π_t falls and y_t rises in the period of a jump in ξ_t. Provide the economic intuition for these responses.

[1] The formal definition of the 'period t' innovation in a stochastic process, a_t, is the surprise in a_t relative to the past history on a_t :

$$a_t - P\left[a_t | a_{t-1}, a_{t-2}, ...\right],$$

which is $\varepsilon_t + \xi_{t-1}$ in this setting. The point is that agents in the model are assumed to observe a piece of the period t innovation in a_t one period in advance.

According to the model, what is the flaw in the conventional wisdom in terms of the response of inflation to good news about the future?

(b) What is the first best contemporaneous response in π_t and y_t to ξ_t? Why are the responses in the equilibrium with the interest rate rule so different? Can you explain this in terms of a deficiency in the interest rate rule?