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Application of Log-linearization Methods: Optimal Policy

Log-linear Methods

- Equilibrium conditions:

$$v\left(k_t, k_{t+1}, k_{t+2}\right) = 0,$$

  $t = 0, 1, 2, ...$

- Solution:
  – compute steady state, k^* such that $v\left(k^*, k^*, k^*\right) = 0.$
  – expansion about steady state: $V_0\tilde{k}_t + V_1\tilde{k}_{t+1} + V_2\tilde{k}_{t+2} = 0.$
  – solve linearized system.

- Last time:
  – v equilibrium conditions of a monetary model.
  – included a monetary policy rule.

- This time:
  - what is optimal monetary policy?
  - drop monetary policy rule
  - now we’re short one equation!
  - system underdetermined....‘many solutions’
  - pick the best one.

- Why *optimal* policy?
  - important benchmark for policy analysis.
  - answers important questions....

- Potential problem with optimal monetary policy:
  - time inconsistency
  - implication: equilibrium conditions, v, not time invariant.
  - methods potentially not appropriate.
  - use Kydland and Prescott ‘trick’....set up optimal problem in Lagrangian form.

Example #1: Optimal Monetary Policy - Toy Example

- Setup
  - Model
    - One equation characterizing private sector behavior:

$$\pi_t - \beta\pi_{t+1} - \gamma y_t = 0, \; t = 0, 1, 2, ... \tag{1}$$

    - Another equation characterizes policy.
  - Want to do *optimal* policy, so throw away policy equation.
  - System is now under-determined: one equation in two variables, π_t and y_t.

– Optimization delivers the other equations.

  * optimize objective:

$$\sum_{t=0}^{\infty} \beta^t u\left(\pi_t, y_t\right)$$

    subject to (1).

  * If objective corresponds to social welfare function, this is called *Ramsey* optimal problem

  * Objective may be preferences of policy maker.

Example #1: Optimal Monetary Policy - Toy Example ...

- Lagrangian representation of problem:

$$\max_{\{\pi_t, y_t; t=0,1,...\}} \sum_{t=0}^{\infty} \beta^t \left\{ u\left(\pi_t, y_t\right) + \lambda_t \left[\pi_t - \beta \pi_{t+1} - \gamma y_t\right]\right\}$$

$$= \max_{\{\pi_t, y_t; t=0,1,...\}} \{ u\left(\pi_0, y_0\right) + \lambda_0 \left[\pi_0 - \beta \pi_1 - \gamma y_0\right]$$

$$+ \beta u\left(\pi_1, y_1\right) + \beta \lambda_1 \left[\pi_1 - \beta \pi_2 - \gamma y_1\right] + ...\}$$

- First order necessary conditions for optimization:

$$\begin{aligned}
u_\pi\left(\pi_0, y_0\right) + \lambda_0 &= 0 \text{ (*)} \\
u_\pi\left(\pi_1, y_1\right) + \lambda_1 - \lambda_0 &= 0 \\
&... \\
u_y(\pi_0, y_0) - \gamma \lambda_0 &= 0 \\
u_y(\pi_1, y_1) - \gamma \lambda_1 &= 0 \\
&... \\
\pi_0 - \beta \pi_1 - \gamma y_0 &= 0 \\
\pi_1 - \beta \pi_2 - \gamma y_1 &= 0 \\
&...
\end{aligned}$$

Example #1: Optimal Monetary Policy - Toy Example ...

- These equations 'look' different than the ones we've seen before
  - They are not stationary, (*) is different from the others.
    - reflects that at time 0 there is a constraint 'missing'
    - no need to respect what people were expecting you to do as of time -1
    - do need to respect what they expect you to do in the future, because that affects current behavior.
    - that's the source of the 'time inconsistency of optimal plans'.
- Can trick the problem into being stationary (see, e.g., Kydland and Prescott (JEDC, 1990s) and Levin, Onatski, Williams, and Williams, Macro Annual, 2005). Then, apply standard log-linearization solution method.

Example #1: Optimal Monetary Policy - Toy Example ...

- Consider:

$$v\left(\pi_t, \pi_{t+1}, y_t, \lambda_t, \lambda_{t-1}\right) = \begin{bmatrix} u_\pi\left(\pi_t, y_t\right) + \lambda_t - \lambda_{t-1} \\ u_y(\pi_t, y_t) - \gamma\lambda_t \\ \pi_t - \beta\pi_{t+1} - \gamma y_t \end{bmatrix}, \text{ for all } t.$$

  – time t ‘endogenous variables’: λ_t, π_t, y_t

  – time t ‘state variable’: λ_{t-1}.

  – ‘solution’:

$$\lambda_t = \lambda\left(\lambda_{t-1}\right), \ \pi_t = \pi\left(\lambda_{t-1}\right), \ y_t = y\left(\lambda_{t-1}\right),$$

  such that

$$v\left(\pi\left(\lambda_{t-1}\right), \pi\left(\lambda\left(\lambda_{t-1}\right)\right), y\left(\lambda_{t-1}\right), \lambda\left(\lambda_{t-1}\right), \lambda_{t-1}\right) = 0, \text{ for all possible } \lambda_{t-1}.$$

- In general, solving this problem exactly is intractable.
- But, can log-linearize!

  – **Step 1**: find π^*, y^*, λ^* such that following three equations are satisfied:

$$v\left(\pi^*, \pi^*, y^*, \lambda^*, \lambda^*\right) = \underbrace{0}_{3\times 1}.$$

  – **Step 2**: log-linearly expand v about steady state

$$v\left(\pi_t, \pi_{t+1}, y_t, \lambda_t, \lambda_{t-1}\right) \simeq v_1\pi^*\hat{\pi}_t + v_2\pi^*\hat{\pi}_{t+1} + v_3 y^*\hat{y}_t + v_4\Delta\hat{\lambda}_t + v_5\Delta\hat{\lambda}_{t-1},$$

  where

$\Delta\hat{\lambda}_t \equiv \lambda_t - \lambda^*$ (play it safe, don't divide by something that could be zero!)

  – **Step 3**: Posit

$$\Delta\hat{\lambda}_t = A_\lambda\Delta\hat{\lambda}_{t-1},\ \hat{\pi}_t = A_\pi\Delta\hat{\lambda}_{t-1},\ \hat{y}_t = A_y\Delta\hat{\lambda}_{t-1},$$

  and find A_λ, A_π, A_y that solve

$$\left[v_1\pi^* A_\pi + v_2\pi^* A_\pi A_\lambda + v_3 y^* A_y + v_4 A_\lambda + v_5\right]\Delta\hat{\lambda}_{t-1} = \underbrace{0}_{3\times 1}$$

  for all $\Delta\hat{\lambda}_{t-1}$.

- What does the stationary solution have to do with the original non-stationary problem?

  – Do we have a solution to the period 0 problem, (*)?

$$u_\pi \left(\pi_0, y_0\right) + \lambda_0 = 0.$$

  – Yes! Just pretend that this equation really has the following form:

$$u_\pi \left(\pi_0, y_0\right) + \lambda_0 - \lambda_{-1} = 0.$$

  Expression (*) does have this form, if we set $\lambda_{-1} = 0$. Then,

$$\pi_0 = \pi \left(0\right),\ y_0 = y \left(0\right),\ \lambda_0 = \lambda \left(0\right).$$

- The situation is exactly what it is in the neoclassical model when we want to know what happens when initial capital is away from steady state.

  – Plug k_0 into the stationary rule

$$k_1 = g\left(k_0\right).$$

- Possible computational pitfall: if $\lambda_{-1} = 0$ is far from λ^*, then linearized solution might be highly inaccurate (see LOWW).

Example #1: Optimal Monetary Policy - Toy Example ...

- Optimal policy in real time.

- Suppose today is date zero.

  – Solve for $\lambda(\cdot), y(\cdot), \pi(\cdot)$

  – set $\lambda_{-1} = 0$

  – Compute and present in charts:

$$\begin{aligned}
\lambda_0 &= \lambda(\lambda_{-1}),\ y_0 = y(\lambda_{-1}),\ \pi_0 = \pi(\lambda_{-1}) \\
\lambda_1 &= \lambda(\lambda_0),\ y_1 = y(\lambda_0),\ \pi_1 = \pi(\lambda_0) \\
&\ldots \\
\lambda_t &= \lambda(\lambda_{t-1}),\ y_t = y(\lambda_{t-1}),\ \pi_t = \pi(\lambda_0) \\
&\ldots.
\end{aligned}$$

Example #1: Optimal Monetary Policy - Toy Example ...

- One could take the perspective that date 0 is actually a continuation of optimal policy that started up long ago. Under these assumptions, λ_{-1}, is an unobserved variable that you have to extract from the data.

- Warning, for this real time optimal policy strategy to make sense, you must ‘always respect your multipliers’.

  – Avoid temptation to set them to zero.

  – Time inconsistency of optimal plans.

- What does ‘always respect your multipliers’ mean in practice?

  – Make sure the charts you present from one meeting to the next are consistent

Example #1: Optimal Monetary Policy - Toy Example ...

– Example:

date 0 meeting : $y_0 = y(0),\ y_1 = y(\lambda(\lambda_{-1})), y_2 = y(\lambda(\lambda(\lambda_{-1}))), \ldots$

date 1 meeting :
YES - $y_1 = y(\lambda(\lambda_{-1})),\ y_2 = y(\lambda(\lambda(\lambda_{-1}))), \ldots$
NO - $y_1 = y(0),\ y_2 = y(\lambda_1(0)), \ldots$

– If Central Bank selects the bad **('NO')** option people will see the temporal inconsistency of policy, and CB will lose credibility.

– Any differences in charts from one meeting to the next must be fully explicable in terms of new information.

Example #2: Optimal Monetary Policy - More General Discussion

- The equilibrium conditions of a model

$$E_t \underbrace{f\left(z_{t-1}, z_t, z_{t+1}, s_t, s_{t+1}\right)}_{(N-1)\times 1} = 0, \text{ for all } \underbrace{z_{t-1}}_{N\times 1} \text{ (endogenous), } s_t \text{ (exogenous)}$$

$$s_t = P s_{t-1} + \varepsilon_t.$$

- Preferences:

$$E_t \sum_{t=0}^{\infty} \beta^t U\left(z_t, s_t\right).$$

  – Could include discounted utility in f :

$$v\left(z_{t-1}, z_t, s_t\right) = U\left(z_t, s_t\right) + \beta E_t v\left(z_t, z_{t+1}, s_{t+1}\right)$$

Example #2: Optimal Monetary Policy - More General Discussion ...

- Optimum problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U\left(z_t, s_t\right) + \underbrace{\lambda_t'}_{1\times(N-1)} E_t \underbrace{f\left(z_{t-1}, z_t, z_{t+1}, s_t, s_{t+1}\right)}_{(N-1)\times 1} \right\}.$$

- N first order conditions:

$$\underbrace{U_1\left(z_t, s_t\right)}_{1\times N} + \underbrace{\lambda_t'}_{1\times(N-1)} E_t \underbrace{f_2\left(z_{t-1}, z_t, z_{t+1}, s_t, s_{t+1}\right)}_{(N-1)\times N}$$
$$+\beta^{-1} \underbrace{\lambda_{t-1}'}_{1\times(N-1)} \underbrace{f_3\left(z_{t-2}, z_{t-1}, z_t, s_{t-1}, s_t\right)}_{(N-1)\times N}$$
$$+\beta \underbrace{\lambda_{t+1}'}_{1\times(N-1)} E_t \underbrace{f_1\left(z_t, z_{t+1}, z_{t+2}, s_{t+1}, s_{t+2}\right)}_{(N-1)\times N} = \underbrace{0}_{1\times N}$$

– Endogenous variables: $z_t\ (N)$, $\lambda_t\ (N-1)$

– Equations: Ramsey optimality conditions (N), equilibrium condition $(N-1)$

Example #2: Optimal Monetary Policy - More General Discussion ...

- First order conditions of optimum problem have exactly the same form as the type of problem we solved using linearization methods.

- Seem much more cumbersome:

  - must differentiate f (includes private first order conditions that have already involved differentiation!)

  - good news: LOWW wrote a program that takes U, f as input and writes Dynare code for solving the system

  - solving policy optimum problem is no harder than solving original problem.

Example #3: Optimal Monetary Policy - Rotemberg Model

- Household preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t\right) - \frac{\chi}{2} h_t^2 \right].$$

- Household budget constraint:

$$P_t C_t + B_t = \left(1 + R_{t-1}\right) B_{t-1} + W_t h_t + \Pi_t.$$

- First order conditions:

$$\overbrace{\chi h_t C_t = \frac{W_t}{P_t}}^{\text{intratemporal fonc}}, \qquad \overbrace{\frac{1}{1+R_t} = \beta E_t \frac{P_t C_t}{P_{t+1} C_{t+1}}}^{\text{intertemporal fonc}}, \qquad t = 0, 1, 2, \ldots.$$

Example #3: Optimal Monetary Policy - Rotemberg Model ...

- Final good firms

$$\overbrace{Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}}^{\text{production function}},\ \varepsilon \geq 1, \qquad \overbrace{Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} Y_t}^{\text{first order condition (demand curve for } j^{th} \text{ intermediate good producer)}}$$

- j^{th} intermediate good firm:

$$E_t \sum_{l=0}^{\infty} \beta^l \upsilon_{t+l}[(1+\nu) P_{j,t+l} Y_{j,t+l} - \overbrace{s_{t+l} P_{t+l} Y_{j,t+l}}^{\text{labor costs of production}}$$

$$- \overbrace{\frac{\phi}{2}\left(\frac{P_{j,t+l}}{P_{j,t+l-1}} - 1\right)^2 P_{t+l} C_{t+l}}^{\text{cost (in terms of final goods) of adjusting prices}} \quad],$$

$$Y_{j,t} = \overbrace{A_t h_{j,t}}^{\text{production function}},\ a_t \equiv \log(A_t),\ s_t = \overbrace{\frac{W_t}{P_t A_t}}^{\text{real marginal cost}} = \overbrace{\frac{\chi h_t C_t}{A_t}}^{\text{substitute out household fonc}}.$$

Example #3: Optimal Monetary Policy - Rotemberg Model ...

- Substitute out j^{th} firm's demand curve $\upsilon_t = 1/(P_t C_t)$:

$$\max_{\{P_{j,l}\}_{l=0}^{\infty}} E_t \sum_{l=0}^{\infty} \beta^l \frac{1}{P_{t+l}C_{t+l}} [(1+\nu)\, P_{j,t+l} \left(\frac{P_{j,t+l}}{P_{t+l}}\right)^{-\varepsilon} Y_{t+l}$$
$$-P_{t+l}s_{t+l}\left(\frac{P_{j,t+l}}{P_{t+l}}\right)^{-\varepsilon} Y_{t+l} - \frac{\phi}{2}\left(\frac{P_{j,t+l}}{P_{j,t+l-1}} - 1\right)^2 P_{t+l}C_{t+l}]$$

- Differentiate with respect to $P_{j,t}$:

$$\left[(1+\nu)(1-\varepsilon)\left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon}\frac{1}{P_t} + s_t\varepsilon\left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon-1}\frac{1}{P_t}\right]\frac{Y_t}{C_t} - \phi\left(\frac{P_{j,t}}{P_{j,t-1}} - 1\right)\frac{1}{P_{j,t-1}}$$
$$+\beta\phi E_t\left(\frac{P_{j,t+1}}{P_{j,t}} - 1\right)\frac{P_{j,t+1}}{\left(P_{j,t}\right)^2} = 0.$$

- Rearrange firm efficiency condition:

$$\overbrace{(1+\nu)\frac{P_{j,t}}{P_t}}^{\text{real price received}} = \overbrace{\frac{\varepsilon}{\varepsilon-1}}^{\text{markup}} \times \overbrace{s_t}^{\text{real marginal cost (exclusive of price adjustment costs)}}$$

$$+\frac{1}{\varepsilon-1}\phi\left(\frac{P_{j,t}}{P_t}\right)^{\varepsilon}\frac{C_t}{Y_t}[\quad -\overbrace{\left(\frac{P_{j,t}}{P_{j,t-1}}-1\right)\frac{P_{j,t}}{P_{j,t-1}}}^{\text{if static considerations entail a rise in price, adj costs imply rising by less}}$$

$$+\beta E_t \overbrace{\left(\frac{P_{j,t+1}}{P_{j,t}}-1\right)\frac{P_{j,t+1}}{P_{j,t}}}^{\text{if contemplating a rise in future price, then raise price today by more}} \quad].$$

- When $\phi = 0$:
  - get ‘normal’ efficiency condition, ‘price equals markup over marginal cost’
  - get ‘normal’ monopoly power correction: $1+\nu = \varepsilon/\left(\varepsilon-1\right)$.

Example #3: Optimal Monetary Policy - Rotemberg Model ...

- Impose $P_{j,t} = P_{i,t} = P_t$ for all i, j :

$$(\pi_t - 1)\,\pi_t = \frac{1}{\phi}\left(1 + \nu - \frac{\varepsilon}{\varepsilon - 1}\right)(1 - \varepsilon)\frac{Y_t}{C_t} + \frac{\varepsilon}{\phi}(s_t - 1)\frac{Y_t}{C_t} + \beta E_t(\pi_{t+1} - 1)\,\pi_{t+1}.$$

- Resource constraint:

$$C_t \left[\overbrace{1}^{\text{consumption}} + \overbrace{\frac{\phi}{2}(\pi_t - 1)^2}^{\text{price adjustment costs}} \right] = \overbrace{A_t h_t}^{\text{total output}} = Y_t, \;\; a_t = \rho a_{t-1} + u_t, \;\; a_t \equiv \log A_t$$

- Substitute out the resource constraint:

$$\begin{aligned}(\pi_t - 1)\,\pi_t \; &= \; \frac{1}{\phi}\left[\left(1 + \nu - \frac{\varepsilon}{\varepsilon - 1}\right)(1 - \varepsilon) + \varepsilon(s_t - 1)\right]\left[1 + \frac{\phi}{2}(\pi_t - 1)^2\right] \\ &\quad + \beta E_t(\pi_{t+1} - 1)\,\pi_{t+1}.\end{aligned}$$

- Looks ‘sort of’ like Calvo equilibrium relation for inflation.

- Log-linearize around 'efficient steady state' $(\pi_t = 1, 1+\nu = \varepsilon/(\varepsilon-1),$ $s_t = 1)$:

$$(\pi_t - 1)\pi_t = \frac{1}{\phi}\left[\left(1+\nu-\frac{\varepsilon}{\varepsilon-1}\right)(1-\varepsilon)+\varepsilon(s_t-1)\right]\left[1+\frac{\phi}{2}(\pi_t-1)^2\right]$$
$$+\beta E_t(\pi_{t+1}-1)\pi_{t+1}.$$

$$d\left[(\pi_t-1)\pi_t\right] \overbrace{=}^{\text{totally differentiate}} \pi_t d\pi_t + (\pi_t - 1)\,d\pi_t \overbrace{=}^{\text{evaluate in steady state, } \pi=1} \pi d\pi_t \equiv \pi^2\hat{\pi}_t = \hat{\pi}_t.$$

- Doing the log-linearization:

$$\hat{\pi}_t = \frac{\varepsilon}{\phi}\hat{s}_t + \beta E_t \hat{\pi}_{t+1}.$$

'marginal cost affects price less the bigger is ϕ'

so Rotemberg *IS* Calvo, up to linear approximation and with a different interpretation of slope on marginal cost.

Example #3: Optimal Monetary Policy - Rotemberg Model ...

- Summarizing equilibrium conditions:

  – household intertemporal efficiency condition:

$$\frac{1}{1+R_t} = \beta E_t \frac{P_t C_t}{P_{t+1} C_{t+1}}$$

  – firm efficiency condition for prices (after rearranging) -

$$\left[\left(1+\nu-\frac{\varepsilon}{\varepsilon-1}\right)(1-\varepsilon)+\varepsilon\left(s_t-1\right)\right]\left(1+\frac{\phi}{2}\left(\pi_t-1\right)^2\right)-\phi\left(\pi_t-1\right)\pi_t$$

$$+\beta\phi E_t\left(\pi_{t+1}-1\right)\pi_{t+1} \;=\; 0$$

  – marginal cost:

$$s_t = \frac{\overbrace{\chi h_t C_t}^{\text{uses household intratemporal efficiency condition}}}{A_t}$$

  – Resource constraint:

$$C_t\left[1+\frac{\phi}{2}\left(\pi_t-1\right)^2\right] = A_t h_t,\ a_t = \rho a_{t-1} + u_t$$

- Ramsey optimal problem:

$$\max_{\{\nu, C_t, h_t, \pi_t, R_t\}} E_0 \sum_{t=0}^{\infty} \beta_n^t \{ \left(\log (C_t) - \frac{\chi}{2} h_t^2 \right) +$$

$$\lambda_{1t} \left(\frac{1}{1+R_t} - \beta E_t \frac{C_t}{\pi_{t+1} C_{t+1}} \right)$$

$$+\lambda_{2t} [\left(\left(1 + \nu - \frac{\varepsilon}{\varepsilon - 1} \right) (1 - \varepsilon) + \varepsilon \left(\frac{\chi h_t C_t}{A_t} - 1 \right) \right) \left(1 + \frac{\phi}{2} (\pi_t - 1)^2 \right)$$

$$-\phi (\pi_t - 1) \pi_t + \beta \phi E_t (\pi_{t+1} - 1) \pi_{t+1}]$$

$$+\lambda_{3t} \left[C_t \left(1 + \frac{\phi}{2} (\pi_t - 1)^2 \right) - A_t h_t \right] \}$$

- Conjecture about solution to Ramsey problem:

$$\lambda_{1t} = \lambda_{2t} = 0.$$

- We will implement, and then verify the conjecture formally.

- Intuition:

  – intertemporal equation non-binding from the point of view of maximizing utility, because R_t (a variable of no direct interest in utility) can always be chosen to enforce intertemporal Euler equation).

  – price equation non-binding from the point of view of maximizing utility, because ν (a variable of no direct interest in utility) can always be chosen to enforce the price equation.

Example #3: Optimal Monetary Policy - Rotemberg Model ...

- Simplified Ramsey problem (ν, R_t are a matter of indifference here)

$$\max_{\{C_t, h_t, \pi_t\}} E_0 \sum_{t=0}^{\infty} \beta_n^t \{ \left(\log\left(C_t\right) - \frac{\chi}{2} h_t^2 \right) + \lambda_{3t} \left[C_t \left(1 + \frac{\phi}{2} \left(\pi_t - 1\right)^2 \right) - A_t h_t \right] \}.$$

- – first order necessary condition for π_t:

$$\lambda_{3t} C_t \phi \left(\pi_t - 1\right) = 0 \rightarrow \pi_t = 1 \text{ (obviously, } \lambda_{3t} = 0, \text{ or } C_t = 0 \text{ is not the solution!)}$$

– first order condition for h_t, C_t :

$$-\chi h_t - \lambda_{3t} A_t = 0, \ \frac{1}{C_t} + \lambda_{3t} \left(1 + \frac{\phi}{2} \left(\pi_t - 1\right)^2 \right) = 0 \rightarrow \lambda_{3t} = -\frac{1}{C_t}.$$

or

$$\chi h_t = \frac{1}{C_t} A_t, \ \rightarrow \overbrace{\chi h_t C_t}^{MRS} = \overbrace{A_t}^{MPL}$$

so solution to Ramsey problem achieves ‘first best’:

$$\max_{C_t, h_t} E_t \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t\right) - \frac{\chi}{2} h_t^2 \right], \text{ subject to } C_t = A_t h_t.$$

Example #3: Optimal Monetary Policy - Rotemberg Model ...

- Solution to Ramsey problem:

$$
\begin{aligned}
C_t &= A_t h_t \\
\chi h_t C_t &= A_t \rightarrow h_t^2 = \left[\frac{1}{\chi}\right]^{1/2}, \\
\pi_t &= 1, \\
R_t &= \frac{1}{\beta E_t \frac{P_t C_t}{P_{t+1} C_{t+1}}} - 1, \\
1+\nu &= \frac{\varepsilon}{\varepsilon - 1}.
\end{aligned}
$$

- Notice:
  - this is first best, and there is no time inconsistency problem!
  - treatment of ν crucial here (reason that price equation is non-binding)

Example #3: Optimal Monetary Policy - Rotemberg Model ...

- Ramsey problem with ν fixed, $\neq \varepsilon / (\varepsilon - 1)$

$$\max_{\{C_t, h_t, \pi_t, R_t\}} E_0 \sum_{t=0}^{\infty} \beta_n^t \{ \left(\log(C_t) - \frac{\chi}{2} h_t^2 \right) +$$

$$\lambda_{1t} \left(\frac{1}{1+R_t} - \beta E_t \frac{C_t}{\pi_{t+1} C_{t+1}} \right)$$

$$+\lambda_{2t} [\left(\left(1 + \nu - \frac{\varepsilon}{\varepsilon - 1} \right) (1 - \varepsilon) + \varepsilon \left(\frac{\chi h_t C_t}{A_t} - 1 \right) \right) \left(1 + \frac{\phi}{2} (\pi_t - 1)^2 \right)$$

$$-\phi (\pi_t - 1) \pi_t + \beta \phi E_t (\pi_{t+1} - 1) \pi_{t+1}]$$

$$+\lambda_{3t} \left[C_t \left(1 + \frac{\phi}{2} (\pi_t - 1)^2 \right) - A_t h_t \right] \},$$

- Note: intertemporal condition still non-binding.

Example #3: Optimal Monetary Policy - Rotemberg Model ...

- First order conditions for Ramsey problem (impose $\lambda_{1t} \equiv 0$, β_n is discount rate in planner objective, $\lambda_{i,-1} = 0$) for h_t, π_t, C_t :

$$\boxed{-\chi h_t + \lambda_{2t}\varepsilon\frac{\chi C_t}{A_t}\left(1+\frac{\phi}{2}(\pi_t-1)^2\right) - \lambda_{3,t}A_t = 0.}$$

$$\boxed{\begin{array}{l}\lambda_{2t}[\left(\left(1+\nu-\frac{\varepsilon}{\varepsilon-1}\right)(1-\varepsilon)+\varepsilon\left(\frac{\chi h_t C_t}{A_t}-1\right)-1\right)\phi(\pi_t-1)-\phi\pi_t] \\ \\ +\lambda_{2,t-1}\beta_n^{-1}\beta\phi\left[(\pi_t-1)+\pi_t\right]+\lambda_{3t}C_t\phi(\pi_t-1)=0\end{array}}$$

$$\boxed{\frac{1}{C_t} + \lambda_{2t}\varepsilon\frac{\chi h_t}{A_t}\left(1+\frac{\phi}{2}(\pi_t-1)^2\right) + \lambda_{3t}\left(1+\frac{\phi}{2}(\pi_t-1)^2\right) = 0}$$

- Plus, intertemporal household equation, price equation and resource constraint yields 6 equations in $R_t, h_t, C_t, \pi_t, \lambda_{2t}, \lambda_{3t}$.

- Conclusion of example #3
  - When labor market is treated optimally, policy achieves first best and there is no time consistency problem.
  - When labor market not treated optimally, there may be a time consistency problem, but must solve model numerically to find out.
  - Can use same log-linear methods discussed before!
  - Bad news: Ramsey first order conditions painful to compute in practice.
  - Good news: there exists computer code for deriving the Ramsey first order conditions symbolically (we will explore this in a homework with Dynare).

Example #4: Optimal Monetary Policy, Calvo-Sticky Price Model

- Model:
  - Households choose consumption and labor.
  - Monopolistic firms produce and sell output using labor, subject to sticky prices

Household

- Preferences & budget constraint:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \exp\left(\tau_t\right) \frac{N_t^{1+\varphi}}{1+\varphi} - \upsilon \frac{\left(\frac{P_t C_t}{M_t^d}\right)^{1+\sigma_q}}{1+\sigma_q} \right),\ \tau_t = \lambda \tau_{t-1} + \varepsilon_t^{\tau}.$$

$$P_t C_t + M_t^d + B_{t+1} \leq M_{t-1}^d + B_t R_{t-1} + W_t N_t + Transfers\ and\ profits_t$$

- Household efficiency conditions (ignore money because υ is small):

$$\begin{aligned} C_t^{-1} &= \beta E_t C_{t+1}^{-1} R_t / \bar{\pi}_{t+1}, \\ \bar{\pi}_{t+1} &\equiv \frac{P_{t+1}}{P_t}, \\ MRS_t &= \exp\left(\tau_t\right) N_t^{\varphi} C_t = \frac{W_t}{P_t}. \end{aligned}$$

Firms

- Final Good Firms (simple!)
  - Buy $Y_{i,t}$, $i \in [0,1]$ at prices $P_{i,t}$ and sell Y_t at price P_t
  - Technology:

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \ \varepsilon \geq 1. \tag{1}$$

  - Demand for intermediate good (fonc for optimization of $Y_{i,t}$):

$$Y_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \tag{2}$$

  - Eqs (1) and (2) imply:

$$P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}} \tag{3}$$

Firms ...

- Intermediate Good Firms (an astounding amount of algebra!)
  - Technology:

$$Y_{i,t} = A_t N_{i,t}, \;\; a_t = \log A_t.$$

Firms ...

- Intermediate Good Firms (an astounding amount of algebra!)

  – Technology:

  $$Y_{i,t} = A_t N_{i,t}, \;\; a_t = \log A_t.$$

  – Marginal cost of production for i^{th} firm (with subsidy, ν_t) :

$$s_t = \overbrace{(1-\nu_t)}^{\text{net of subsidy}} \frac{\overbrace{W_t}^{\text{`Normal' Marginal Cost}}}{A_t P_t} \overbrace{(1-\psi+\psi R_t)}^{\text{fraction, } \psi \text{, of labor costs requires bank finance}}$$

Firms ...

- Intermediate Good Firms (an astounding amount of algebra!)

  – Technology:

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  – Calvo price-setting frictions:

    * A fraction, θ, of intermediate good firms cannot change price:

$$P_{i,t} = P_{i,t-1}$$

    * A fraction, $1-\theta$, set price optimally:

$$P_{i,t} = \tilde{P}_t$$

Firms ...

- Decision of intermediate good firm

  - Only choice problem: optimize price, $P_{i,t}$, whenever opportunity arises.

  - Otherwise, always produce the quantity dictated by demand.

Firms ...

- Decision of intermediate good firm

  - Only choice problem: optimize price, $P_{i,t}$, whenever opportunity arises.

  - Otherwise, always produce the quantity dictated by demand.

- The firm's periodic optimization gives rise to equilibrium conditions needed to solve the model.

  - Ultimate equilibrium conditions simple.

  - Lot's of (simple) algebra to get them.

Firms ...

- Solving intermediate good firm optimization problem
  - Discounted profits:

$$E_t \sum_{j=0}^{\infty} \beta^j \overbrace{\upsilon_{t+j}}^{\text{Lagrange multiplier on household budget constraint}} \overbrace{\left[\overbrace{P_{i,t+j} Y_{i,t+j}}^{\text{revenues}} - \overbrace{P_{t+j} s_{t+j} Y_{i,t+j}}^{\text{total cost}} \right]}^{\text{period } t+j \text{ profits sent to household}}$$

Firms ...

- Solving intermediate good firm optimization problem
  - Discounted profits:

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  - Each of the $1-\theta$ firms that have opportunity to reoptimize price, $P_{i,t}$, select $\tilde{P}_t$ so maximize:

$$E_t \sum_{j=0}^{\infty} \beta^j \overbrace{\theta^j}^{\substack{\text{in selecting price, firm only cares about} \\ \text{future states in which it can't reoptimize}}} \upsilon_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right].$$

Firms ...

- Substitute out for intermediate good firm output using demand curve:

$$
\begin{aligned}
& E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, \upsilon_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right] \\
= \; & E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, \upsilon_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[\tilde{P}_t^{1-\varepsilon} - P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon} \right].
\end{aligned}
$$

Firms ...

- Substitute out for intermediate good firm output using demand curve:

$$
\begin{aligned}
& E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, \upsilon_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right] \\
= \ & E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, \upsilon_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[\tilde{P}_t^{1-\varepsilon} - P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon} \right].
\end{aligned}
$$

- Differentiate with respect to $\tilde{P}_t$:

$$
E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, \upsilon_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[(1-\varepsilon) \left(\tilde{P}_t \right)^{-\varepsilon} + \varepsilon P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon-1} \right] = 0,
$$

Firms ...

- Substitute out for intermediate good firm output using demand curve:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, \upsilon_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right]$$
$$= \; E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, \upsilon_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[\tilde{P}_t^{1-\varepsilon} - P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon} \right].$$

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or,

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, \upsilon_{t+j} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j} \right] = 0.$$

- When $\theta = 0$, get standard result - price is fixed markup over marginal cost.

Firms ...

- Substitute out the multiplier:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \qquad \overbrace{\frac{u'\left(C_{t+j}\right)}{P_{t+j}}}^{\text{marginal utility of a dollar} = \upsilon_{t+j}} \qquad Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j} \right] = 0.$$

Firms ...

- Substitute out the multiplier:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \overbrace{\frac{u'(C_{t+j})}{P_{t+j}}}^{\text{marginal utility of a dollar} = v_{t+j}} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j} \right] = 0.$$

- Use utility functional form and goods market clearing condition, $C_{t+j} = Y_{t+j}$:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j P_{t+j}^{\varepsilon} \left[\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j} \right] = 0.$$

Firms ...

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$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \overbrace{\frac{u'(C_{t+j})}{P_{t+j}}}^{\text{marginal utility of a dollar} = v_{t+j}} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j} \right] = 0.$$

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or,

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j (X_{t,j})^{-\varepsilon} \left[\tilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j} \right] = 0,$$

$$\tilde{p}_t = \frac{\tilde{P}_t}{P_t},\ X_{t,j} = \begin{cases} \frac{1}{\bar{\pi}_{t+j}\bar{\pi}_{t+j-1}\cdots\bar{\pi}_{t+1}},\ j \geq 1 \\ 1,\ j = 0. \end{cases},\ X_{t,j} = X_{t+1,j-1}\frac{1}{\bar{\pi}_{t+1}},\ j > 0$$

Firms ...

- Want $\tilde{p}_t$ in:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{-\varepsilon} \left[\tilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j}\right] = 0$$

- Solve for $\tilde{p}_t$:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon}} = \frac{K_t}{F_t},$$

Firms ...

- Want $\tilde{p}_t$ in:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{-\varepsilon} \left[\tilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j}\right] = 0$$

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- We've almost finished solving the intermediate firm problem!

- But, still need expressions for K_t, F_t.

Firms ...

$$
\begin{aligned}
K_t &= E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+j} \\
&= \frac{\varepsilon}{\varepsilon - 1} s_t + \beta\theta E_t \sum_{j=1}^{\infty} (\beta\theta)^{j-1} \left(\frac{1}{\bar{\pi}_{t+1}} X_{t+1,j-1} \right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+j}
\end{aligned}
$$

Firms ...

$$
\begin{aligned}
K_t &= E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j} \\
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&= \frac{\varepsilon}{\varepsilon-1} s_t + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} \sum_{j=0}^{\infty} (\beta\theta)^j X_{t+1,j}^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+1+j}
\end{aligned}
$$

Firms ...

$$
\begin{aligned}
K_t &= E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j} \\
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&= \frac{\varepsilon}{\varepsilon-1} s_t + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} \sum_{j=0}^{\infty} (\beta\theta)^j X_{t+1,j}^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+1+j} \\
&= \frac{\varepsilon}{\varepsilon-1} s_t + \beta\theta \overbrace{E_t E_{t+1}}^{=E_t \text{ by LIME}} \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} \sum_{j=0}^{\infty} (\beta\theta)^j X_{t+1,j}^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+1+j}
\end{aligned}
$$

Firms ...

$$
\begin{aligned}
K_t &= E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j} \\
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&= \frac{\varepsilon}{\varepsilon-1} s_t + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} \sum_{j=0}^{\infty} (\beta\theta)^j X_{t+1,j}^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+1+j} \\
&= \frac{\varepsilon}{\varepsilon-1} s_t + \beta\theta \overbrace{E_t E_{t+1}}^{=E_t \text{ by LIME}} \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} \sum_{j=0}^{\infty} (\beta\theta)^j X_{t+1,j}^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+1+j} \\
&= \frac{\varepsilon}{\varepsilon-1} s_t + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} \overbrace{E_{t+1} \sum_{j=0}^{\infty} (\beta\theta)^j X_{t+1,j}^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+1+j}}^{\text{exactly } K_{t+1}!}
\end{aligned}
$$

Firms ...

$$
\begin{aligned}
K_t &= E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j} \\
&= \frac{\varepsilon}{\varepsilon-1} s_t + \beta\theta E_t \sum_{j=1}^{\infty} (\beta\theta)^{j-1} \left(\frac{1}{\bar{\pi}_{t+1}} X_{t+1,j-1}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j} \\
&= \frac{\varepsilon}{\varepsilon-1} s_t + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} \sum_{j=0}^{\infty} (\beta\theta)^j X_{t+1,j}^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+1+j} \\
&= \frac{\varepsilon}{\varepsilon-1} s_t + \beta\theta \overbrace{E_t E_{t+1}}^{=E_t \text{ by LIME}} \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} \sum_{j=0}^{\infty} (\beta\theta)^j X_{t+1,j}^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+1+j} \\
&= \frac{\varepsilon}{\varepsilon-1} s_t + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} \overbrace{E_{t+1} \sum_{j=0}^{\infty} (\beta\theta)^j X_{t+1,j}^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+1+j}}^{\text{exactly } K_{t+1}!} \\
&= \frac{\varepsilon}{\varepsilon-1} s_t + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} K_{t+1}
\end{aligned}
$$

Firms ...

- So,

$$K_t = \frac{\varepsilon}{\varepsilon - 1} s_t + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} K_{t+1}.$$

- Simplify marginal cost term:

$$\begin{aligned} \frac{\varepsilon}{\varepsilon - 1} s_t &= \frac{\varepsilon}{\varepsilon - 1}(1 - \nu_t)\frac{W_t}{A_t P_t}(1 - \psi + \psi R_t) \\ &= \frac{\varepsilon}{\varepsilon - 1}(1 - \nu_t)\overbrace{\exp(\tau_t) N_t^{\varphi} C_t}^{=\frac{W_t}{P_t} \text{ by household optimization}} \frac{1 - \psi + \psi R_t}{A_t}. \end{aligned}$$

- Conclude:

  – Optimal price:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon}} = \frac{K_t}{F_t},$$

where

$$K_t = (1-\nu_t) \frac{\varepsilon}{\varepsilon-1} \frac{\exp(\tau_t) N_t^{\varphi} C_t}{A_t} (1-\psi+\psi R_t) + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} K_{t+1}.$$

Similarly,

$$F_t \equiv E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} = 1 + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{1-\varepsilon} F_{t+1}$$

Aggregate Conditions

- We now have optimization conditions for households and firms.

- Need some aggregate conditions:

  – Relationship among prices

  – Relationship between aggregate inputs (e.g., technology, A_t, and labor, N_t) and aggregate output (e.g., Y_t).

- Aggregate Price Relationship

$$P_t = \left[\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right]^{\frac{1}{1-\varepsilon}}$$

$$= \left[\int_{\text{firms that reoptimize price}} P_{i,t}^{(1-\varepsilon)} di + \int_{\text{firms that don't reoptimize price}} P_{i,t}^{(1-\varepsilon)} di \right]^{\frac{1}{1-\varepsilon}}$$

Aggregate Conditions ...

- Aggregate Price Relationship

$$P_t = \left[\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right]^{\frac{1}{1-\varepsilon}}$$

$$= \left[\int_{\text{firms that reoptimize price}} P_{i,t}^{(1-\varepsilon)} di + \int_{\text{firms that don't reoptimize price}} P_{i,t}^{(1-\varepsilon)} di \right]^{\frac{1}{1-\varepsilon}}$$

$$\overbrace{=}^{\text{all reoptimizers choose same price}} \left[(1-\theta)\, \tilde{P}_t^{(1-\varepsilon)} + \int_{\text{firms that don't reoptimize price}} P_{i,t}^{(1-\varepsilon)} di \right]^{\frac{1}{1-\varepsilon}}$$

Aggregate Conditions ...

- Rewrite integral of prices of intermediate good firms that do not reoptimize:

$$\int_{\text{firms that don't reoptimize price in } t} P_{i,t}^{(1-\varepsilon)} di$$

$$\overbrace{=}^{\text{add over prices, weighted by \# of firms posting that price}}$$

$$\int \left[\overbrace{f_{t-1,t}(\omega)}^{\text{`number' of firms that had price, } P(\omega) \text{, in } t-1 \text{ and were not able to reoptimize in } t} P(\omega)^{(1-\varepsilon)} \right] d\omega$$

In principle, HARD integral to evaluate!

Aggregate Conditions ...

- By Calvo randomization assumption:

$$f_{t-1,t}(\omega) = \theta \times \overbrace{f_{t-1}(\omega)}^{\text{total 'number' of firms with price } P(\omega) \text{ in } t-1}, \text{ for all } \omega$$

Aggregate Conditions ...

- By Calvo randomization assumption:

$$f_{t-1,t}(\omega) = \theta \times \overbrace{f_{t-1}(\omega)}^{\text{total ‘number' of firms with price } P(\omega) \text{ in } t-1}, \text{ for all } \omega$$

- Substituting:

$$\begin{aligned}\int_{\text{firms that don't reoptimize price}} P_{i,t}^{(1-\varepsilon)} di &= \int f_{t-1,t}(\omega) P(\omega)^{(1-\varepsilon)} d\omega \\ &= \theta \int f_{t-1}(\omega) P(\omega)^{(1-\varepsilon)} d\omega \\ &= \theta P_{t-1}^{(1-\varepsilon)}\end{aligned}$$

- Trivial!

Aggregate Conditions ...

- Conclude that the following relationship holds between prices:

$$P_t = \left[(1-\theta)\, \tilde{P}_t^{(1-\varepsilon)} + \theta P_{t-1}^{(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}.$$

- Divide by P_t :

$$1 \;=\; \left[(1-\theta)\, \tilde{p}_t^{(1-\varepsilon)} + \theta \left(\frac{1}{\bar{\pi}_t} \right)^{(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}$$

- Rearrange:

$$\tilde{p}_t \;=\; \left[\frac{1-\theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}}$$

Aggregate Conditions ...

- Aggregate inputs and outputs

  – Technically, there is no ‘aggregate production function’:

    i.e., simple relationship between output, Y_t, and aggregate inputs, N_t, A_t

  – Aggregate output, Y_t, is not only a function of total labor input, N_t, and A_t, but also of the *distribution* of labor input among intermediate goods.

  – Tak Yun (JME) developed a simple characterization of the connection between N, A, Y and the distribution of resources.

Aggregate Conditions ...

– Define Y_t^*:

$$Y_t^* = \int_0^1 Y_{i,t} di \left(= \int_0^1 A_t N_{i,t} di \overbrace{=}^{\text{labor market clearing}} A_t N_t \right)$$

Aggregate Conditions ...

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$$Y_t^* = \int_0^1 Y_{i,t} di \left(= \int_0^1 A_t N_{i,t} di \overbrace{=}^{\text{labor market clearing}} A_t N_t \right)$$

$$\overbrace{=}^{\text{demand curve}} Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} di$$

Aggregate Conditions ...

– Define Y_t^*:

$$Y_t^* = \int_0^1 Y_{i,t} di \left(= \int_0^1 A_t N_{i,t} di \overbrace{=}^{\text{labor market clearing}} A_t N_t \right)$$

$$\overbrace{=}^{\text{demand curve}} Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} di$$

$$= Y_t P_t^{\varepsilon} \int_0^1 (P_{i,t})^{-\varepsilon} di$$

Aggregate Conditions ...

– Define Y_t^*:

$$Y_t^* = \int_0^1 Y_{i,t} di \left(= \int_0^1 A_t N_{i,t} di \overbrace{=}^{\text{labor market clearing}} A_t N_t \right)$$

$$\overbrace{=}^{\text{demand curve}} Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} di$$

$$= Y_t P_t^{\varepsilon} \int_0^1 \left(P_{i,t} \right)^{-\varepsilon} di$$

$$= Y_t P_t^{\varepsilon} \left(P_t^* \right)^{-\varepsilon}$$

where

$$P_t^* \equiv \left[\int_0^1 P_{i,t}^{-\varepsilon} di \right]^{\frac{-1}{\varepsilon}} = \left[(1-\theta) \tilde{P}_t^{-\varepsilon} + \theta \left(P_{t-1}^* \right)^{-\varepsilon} \right]^{\frac{-1}{\varepsilon}}$$

Aggregate Conditions ...

– Relationship between aggregate inputs and outputs:

$$Y_t = \left(\frac{P_t^*}{P_t}\right)^{\varepsilon} Y_t^*$$

$$= p_t^* A_t N_t,$$

where

$$p_t^* \equiv \left(\frac{P_t^*}{P_t}\right)^{\varepsilon}.$$

– 'Efficiency distortion', p_t^*:

$$p_t^* : \begin{cases} \leq 1 \\ = 1 \quad P_{i,t} = P_{j,t}, \text{ all } i, j \end{cases}$$

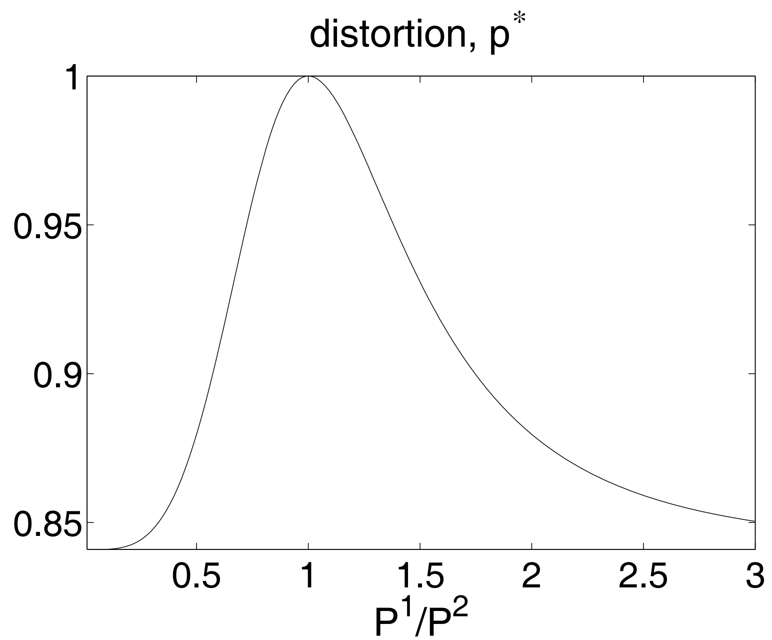
– When prices of different intermediate goods differ, then resources allocated inefficiently.

Aggregate Conditions ...

– Example:

$$P_{j,t} = \begin{cases} P^1 & 0 \leq j \leq \alpha \\ P^2 & \alpha \leq j \leq 1 \end{cases}, \; p_t^* = \left(\frac{P_t^*}{P_t}\right)^{\varepsilon} = \left(\frac{\left[\alpha + (1-\alpha)\left(\frac{P^2}{P^1}\right)^{-\varepsilon}\right]^{\frac{-1}{\varepsilon}}}{\left[\alpha + (1-\alpha)\left(\frac{P^2}{P^1}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}} \right)^{\varepsilon}$$

$$\alpha = 0.5, \varepsilon = 5$$



Summary of Equilibrium Conditions

- Combining efficiency condition of intermediate firms with household static efficiency:

$$K_t = (1-\nu_t)\frac{\varepsilon}{\varepsilon-1}\frac{\exp(\tau_t)\,N_t^{\varphi}C_t}{A_t}(1-\psi+\psi R_t)+\beta\theta E_t\bar{\pi}_{t+1}^{\varepsilon}K_{t+1} \text{ (1)}$$

$$F_t = 1+\beta\theta E_t\bar{\pi}_{t+1}^{\varepsilon-1}F_{t+1} \text{ (2)}$$

- Intermediate good firm optimality and restriction across prices:

$$\overbrace{\frac{K_t}{F_t}}^{=\tilde{p}_t \text{ by firm optimality}} = \overbrace{\left[\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}}}^{=\tilde{p}_t \text{ by restriction across prices}} \quad (3)$$

Summary of Equilibrium Conditions ...

- Law of motion for efficiency distortion:

$$p_t^* = \left[(1-\theta) \left(\frac{1-\theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \quad (4)$$

- Household intertemporal condition:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5)$$

- Aggregate inputs and outputs:

$$C_t = p_t^* A_t N_t \quad (6)$$

- 8 unknowns - $\nu_t, C_t, p_t^*, N_t, \bar{\pi}_t, K_t, F_t, R_t$ - 6 equations.

- Need two more equations to close the model!

Example #4: Optimal Monetary Policy - Clarida-Gali-Gertler Model ...

– Lagrangian:

$$\max_{\nu_t,p_t^*,N_t,R_t,\bar{\pi}_t,F_t,K_t} E_0 \sum_{t=0}^{\infty} \beta^t \{ \left(\log N_t + \log p_t^* - \exp\left(\tau_t\right) \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

$$+\lambda_{1t} \left[\frac{1}{p_t^* N_t} - E_t \frac{A_t \beta}{p_{t+1}^* A_{t+1} N_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \right]$$

$$+\lambda_{2t} \left[\frac{1}{p_t^*} - \left((1-\theta) \left(\frac{1-\theta\left(\bar{\pi}_t\right)^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right) \right]$$

$$+\lambda_{3t} \left[1 + E_t \bar{\pi}_{t+1}^{\varepsilon-1} \beta\theta F_{t+1} - F_t \right]$$

$$+\lambda_{4t} \left[(1-\nu_t) \frac{\varepsilon}{\varepsilon-1} \exp\left(\tau_t\right) N_t^{1+\varphi} p_t^* \left(1-\psi+\psi R_t\right) + E_t \bar{\pi}_{t+1}^{\varepsilon} \beta\theta K_{t+1} - K_t \right]$$

$$+\lambda_{5t} \left[F_t \left[\frac{1-\theta\bar{\pi}_t^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} - K_t \right] \}$$

– The problem looks like a normal public finance problem.

– ‘two degree of freedom’ 7 variables, 5 equilibrium conditions

Example #4: Optimal Monetary Policy - Clarida-Gali-Gertler Model ...

- Optimal inflation rate depends on ν_t.
  - two cases: ν_t constant, and ν_t can vary.
- Consider the variable ν_t case.
  - Conjecture: restrictions 1, 3, 4, 5 nonbinding (i.e., $\lambda_{1t} = \lambda_{3t} = \lambda_{4t} = \lambda_{5t} = 0$)
    - * Step 1: Optimize w.r.t. p_t^*, $\bar{\pi}_t$, N_t ignoring restrictions 1, 3, 4, 5.
    - * Step 2: Solve for ν_t, R_t, F_t, K_t, to satisfy restrictions 1, 3, 4, 5.
  - If this can be done, then the conjecture is verified.

Example #4: Optimal Monetary Policy - Clarida-Gali-Gertler Model ...

- Simplified problem under conjecture:

$$\max_{\bar{\pi}_t, p_t^*, N_t} E_0 \sum_{t=0}^{\infty} \beta^t \{ \left(\log N_t + \log p_t^* - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$
$$+\lambda_{2t} \left[\frac{1}{p_t^*} - \left((1-\theta) \left(\frac{1-\theta(\bar{\pi}_t)^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right) \right] \}$$

first order conditions with respect to p_t^*, $\bar{\pi}_t$, N_t (after rearranging):

$$p_t^* + \beta \lambda_{2,t+1} \theta \bar{\pi}_{t+1}^{\varepsilon} = \lambda_{2t}, \; \bar{\pi}_t = \left[\frac{\left(p_{t-1}^*\right)^{\varepsilon-1}}{1-\theta+\theta\left(p_{t-1}^*\right)^{\varepsilon-1}} \right]^{\frac{1}{\varepsilon-1}}, \; N_t = \exp\left(-\frac{\tau_t}{\varphi+1} \right)$$

– Substituting the solution for $\bar{\pi}_t$ into the law of motion for p_t^* :

$$p_t^* = \left[(1-\theta) + \theta \left(p_{t-1}^*\right)^{(\varepsilon-1)} \right]^{\frac{1}{(\varepsilon-1)}}.$$

- Can the other constraints be satisfied?

  – Choose R_t so the intertemporal constraint is satisfied:

$$R_t = \frac{\frac{1}{p_t^* N_t}}{E_t \frac{A_t \beta}{p_{t+1}^* A_{t+1} N_{t+1} \bar{\pi}_{t+1}}}.$$

  – Remaining constraints: three price-setting conditions.

- Price setting conditions:

$$1 + E_t \bar{\pi}_{t+1}^{\varepsilon-1} \beta\theta F_{t+1} = F_t \text{ (1)}$$

$$(1-\nu_t)\frac{\varepsilon}{\varepsilon-1}\exp(\tau_t)\,N_t^{1+\varphi}p_t^*\,(1-\psi+\psi R_t) + E_t\bar{\pi}_{t+1}^{\varepsilon}\beta\theta K_{t+1} = K_t \text{ (2)}$$

$$F_t\left[\frac{1-\theta\bar{\pi}_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \overbrace{=}^{\text{(making use of the expression for optimal inflation)}} F_t p_t^* = K_t \text{ (3)}$$

- Divide (2) by p_t^*, impose (3) and use $\bar{\pi}_{t+1} = p_t^*/p_{t+1}^*$:

$$\overbrace{\frac{\varepsilon}{\varepsilon-1}}^{\text{markup}} \times (1-\nu_t) \times \overbrace{\overbrace{\exp(\tau_t)\,N_t^{1+\varphi}}^{=\frac{W_t}{P_tA_t}=\frac{MRS}{A}} \overbrace{=}^{\text{by optimality of } N_t} 1 \quad (1-\psi+\psi R_t)}^{\text{=firm marginal cost}/P_t} + E_t\bar{\pi}_{t+1}^{\varepsilon-1}\beta\theta F_{t+1} = F_t$$

- Subtract from (1) (subsidy must cancel markup and interest rate distortion):

$$(1-\nu_t)\frac{\varepsilon}{\varepsilon-1}(1-\psi+\psi R_t) = 1.$$

- Bottom line. Optimality under state-contingent ν_t implies:

$$
\begin{aligned}
p_t^* &= \left[(1-\theta)+\theta\left(p_{t-1}^*\right)^{(\varepsilon-1)}\right]^{\frac{1}{(\varepsilon-1)}} \\
\bar{\pi}_t &= \frac{p_{t-1}^*}{p_t^*} \\
N_t &= \exp\left(-\frac{\tau_t}{1+\varphi}\right) \\
1-\nu_t &= \frac{\varepsilon-1}{\varepsilon\left(1-\psi+\psi R_t\right)} \\
C_t &= p_t^* A_t N_t .
\end{aligned}
$$

- Ramsey-optimal policy is time consistent (no forward-looking constraints on core problem).
- If $\psi > 0$ and ν_t not state-contingent must work out Ramsey solution numerically.

  (For further discussion, see Christiano-Motto-Rostagno, 'Two Reasons Why Money Might be Useful in Monetary Policy', 2007 NBER WP.)

Example #4: Optimal Monetary Policy - Clarida-Gali-Gertler Model ...

- Example - no working capital channel (no lending channel):

$$\theta = 0.5,\ \varepsilon = 2,\ \beta = 0.99,\ \psi = 0,\ \rho = 0.5,\ \varphi = 1.$$

- In this case:

$$\begin{aligned}
N_t &= 1 + 0.45(\lambda_{1t-1} - \lambda_1) + .06(\lambda_{3,t-1} - \lambda_3) + 0.63(\lambda_{4,t-1} - \lambda_4) \\
r_t &= 0.01 - 0.50(\lambda_{1t-1} - \lambda_1) + 0.10(\lambda_{3,t-1} - \lambda_3) - 0.02(\lambda_{4,t-1} - \lambda_4) + 0.25a_{t-1} \\
&\quad +0.51u_t \\
\pi_t &= 1 + 0.07(\lambda_{1t-1} - \lambda_1) + 0.09(\lambda_{3,t-1} - \lambda_3) + 0.31(\lambda_{4,t-1} - \lambda_4) + 0.25(p^*_{t-1} - 1) \\
\lambda_{1t} &= 0, \\
\lambda_{2,t} &= 3.88 + 0.82(\lambda_{1t-1} - \lambda_1) + 1.46(\lambda_{3,t-1} - \lambda_3) + 3.65(\lambda_{4,t-1} - \lambda_4) \\
&\quad +4.13(p^*_{t-1} - 1) \\
\lambda_{3,t} &= 0.05(\lambda_{1t-1} - \lambda_1) + 0.69(\lambda_{3,t-1} - \lambda_3) + 0.12(\lambda_{4,t-1} - \lambda_4) \\
\lambda_{4,t} &= -0.05(\lambda_{1t-1} - \lambda_1) + 0.06(\lambda_{3,t-1} - \lambda_3) + 0.63(\lambda_{4,t-1} - \lambda_4) \\
\lambda_{5,t} &= 0.05(\lambda_{1t-1} - \lambda_1) - 0.06(\lambda_{3,t-1} - \lambda_3) + 0.12(\lambda_{4,t-1} - \lambda_4)
\end{aligned}$$

$$\lambda_1 = \lambda_3 = \lambda_4 = \lambda_5 = 0, \lambda_2 = 3.88$$

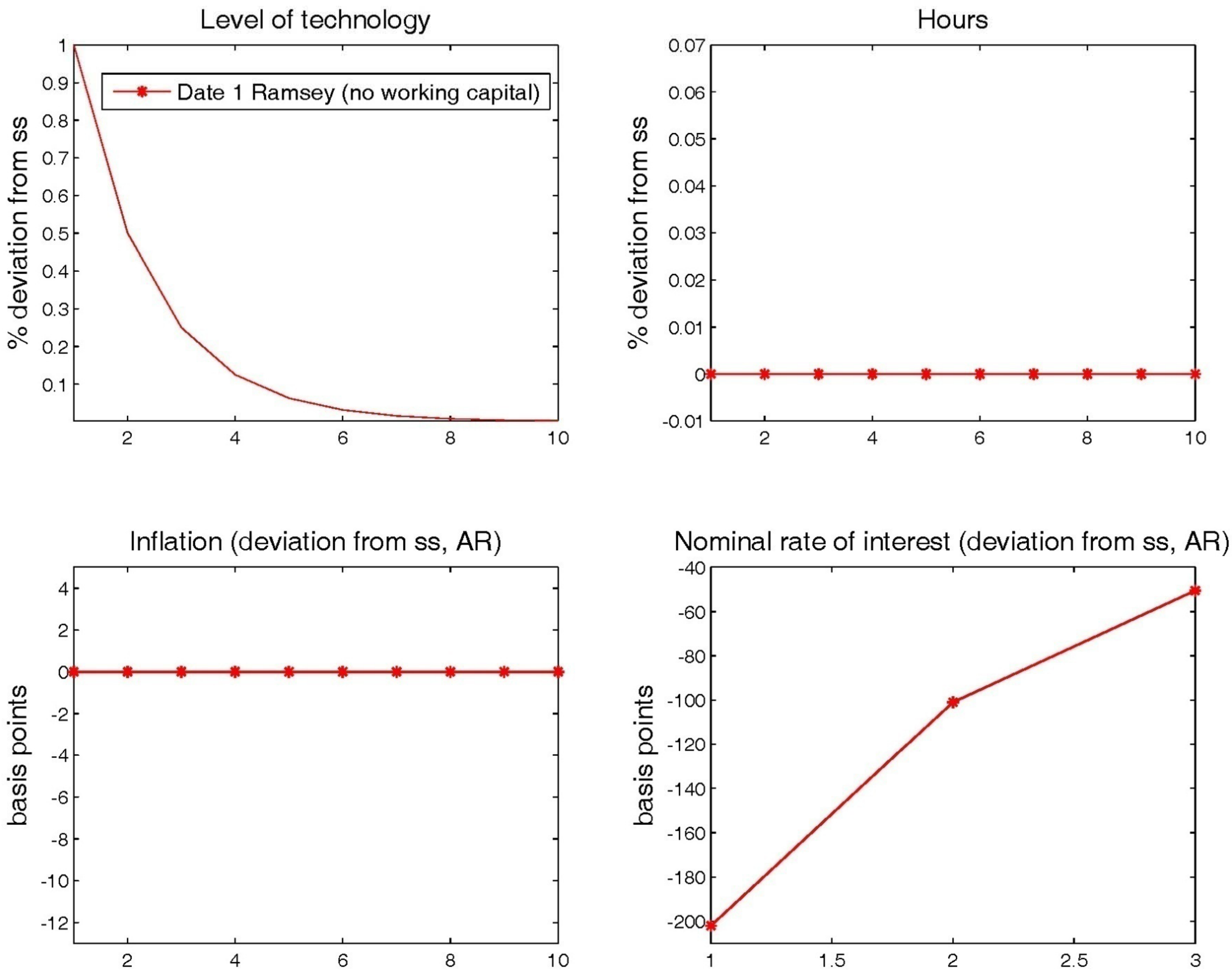
- ‘Resetting multipliers’ makes no difference, no time inconsistency problem.

- Example with $\psi = 0.7$:

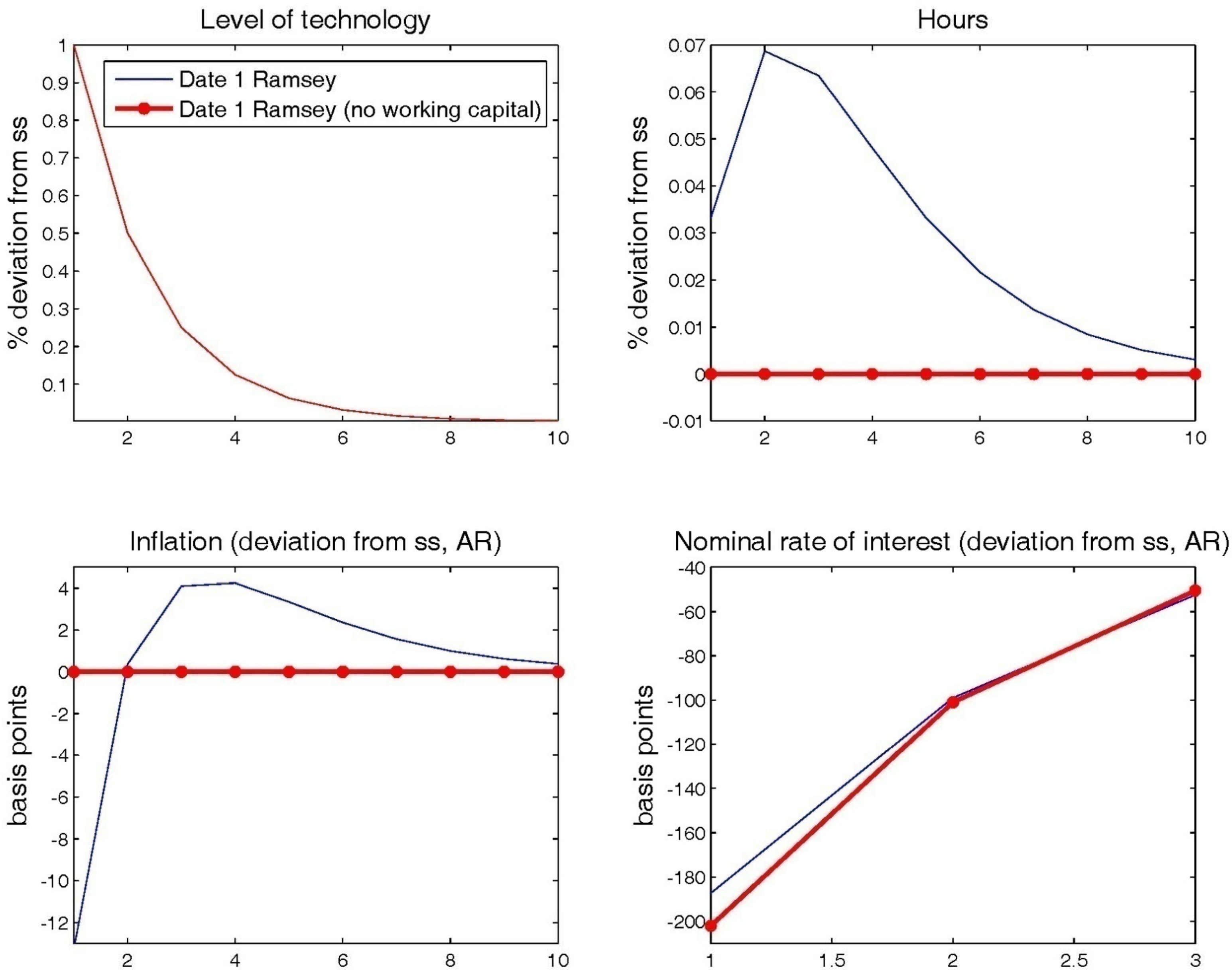
$$
\begin{aligned}
N_t &= 1 + 0.50\lambda_{1t-1} + .03\lambda_{3,t-1} + 0.40\lambda_{4,t-1} - 0.02a_{t-1} - 0.03u_t \\
r_t &= 0.01 - 0.51\lambda_{1t-1} + 0.12\lambda_{3,t-1} + 0.30\lambda_{4,t-1} + 0.24a_{t-1} + 0.49u_t \\
\pi_t &= 1 + 0.05\lambda_{1t-1} + 0.10\lambda_{3,t-1} + 0.31\lambda_{4,t-1} + 0.01a_{t-1} + 0.25(p^*_{t-1} - 1) + 0.02u_t \\
p^*_t &= 1 + .75(p^*_{t-1} - 1) \\
\lambda_{1t} &= -0.01\lambda_{1t-1} + 0.04\lambda_{3,t-1} + 0.44\lambda_{4,t-1} - 0.02a_{t-1} - 0.03u_t \\
\lambda_{2,t} &= 3.88 + 0.95\lambda_{1t-1} + 1.42\lambda_{3,t-1} + 3.63\lambda_{4,t-1} - 0.09a_{t-1} - 0.18u_t + 4.13(p^*_{t-1} - 1) \\
\lambda_{3,t} &= 0.01\lambda_{1t-1} + 0.70\lambda_{3,t-1} + 0.13\lambda_{4,t-1} + 0.02a_{t-1} + 0.04u_t \\
\lambda_{4,t} &= -0.01\lambda_{1t-1} + 0.05\lambda_{3,t-1} + 0.62\lambda_{4,t-1} - 0.02a_{t-1} - 0.05u_t \\
\lambda_{5,t} &= 0.01\lambda_{1t-1} - 0.05\lambda_{3,t-1} + 0.13\lambda_{4,t-1} + 0.02a_{t-1} + 0.05u_t
\end{aligned}
$$

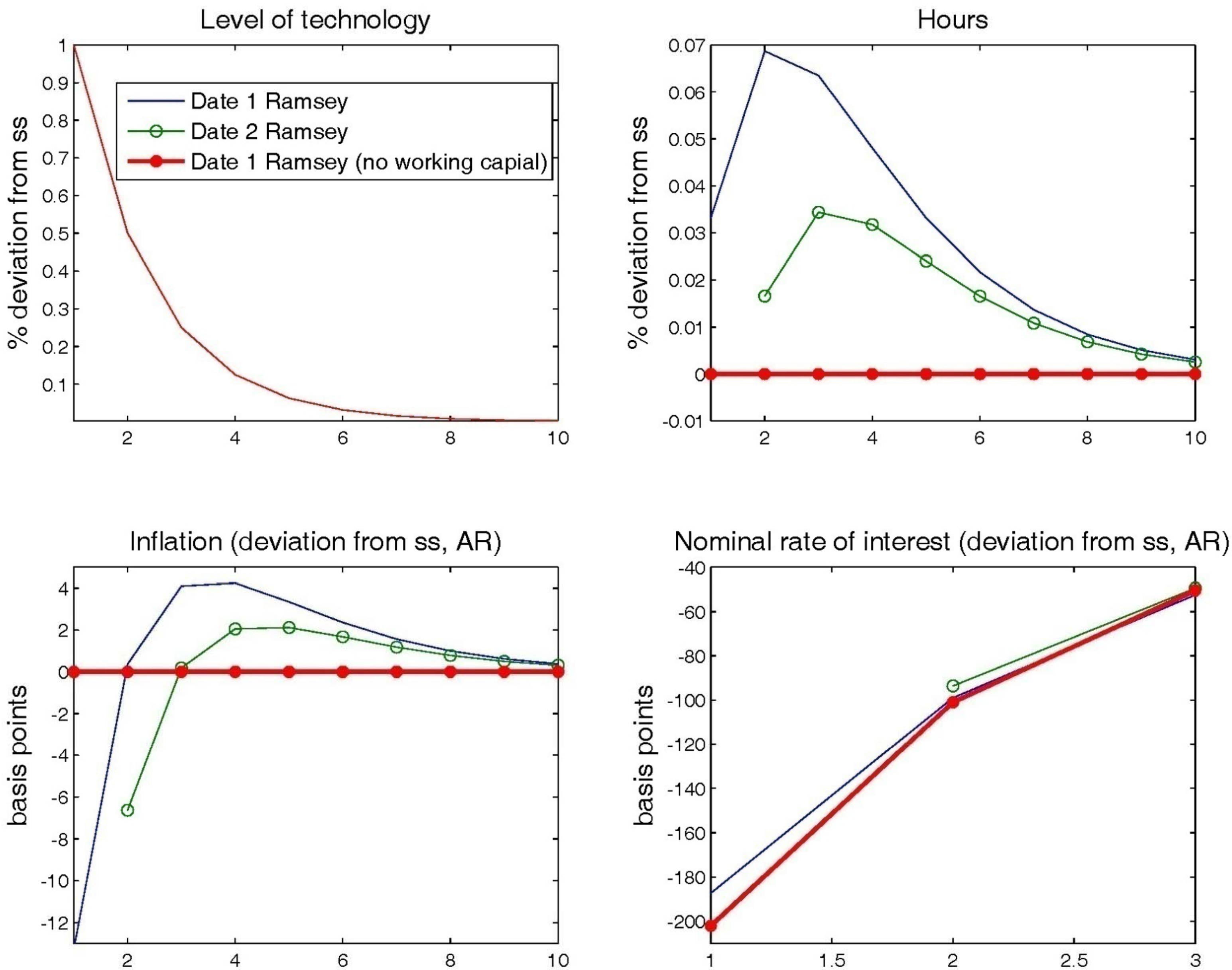
- Properties: all multipliers response to u_t; optimal plan not time consistent; employment and inflation respond to u_t; r_t drops a little less than before (it's a tax now); N_t falls somewhat because of the interest rate 'tax'.

- Experiment:
  - Economy is in steady state of optimal plan up to period t.
  - A positive shock to technology occurs.
  - Monetary authority computes optimal policy and displays it in a set of charts.
  - Redo charts one period later.



Level of technology
% deviation from ss
Date 1 Ramsey (no working capital)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
2
4
6
8
10
Hours
% deviation from ss
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
2
4
6
8
10
Inflation (deviation from ss, AR)
basis points
4
2
0
-2
-4
-6
-8
-10
-12
2
4
6
8
10
Nominal rate of interest (deviation from ss, AR)
basis points
-40
-60
-80
-100
-120
-140
-160
-180
-200
1
1.5
2
2.5
3


Level of technology
Date 1 Ramsey
Date 1 Ramsey (no working capital)
% deviation from ss
Hours
% deviation from ss
Inflation (deviation from ss, AR)
basis points
Nominal rate of interest (deviation from ss, AR)
basis points



Level of technology
% deviation from ss
Date 1 Ramsey
Date 2 Ramsey
Date 1 Ramsey (no working capial)
Hours
% deviation from ss
Inflation (deviation from ss, AR)
basis points
Nominal rate of interest (deviation from ss, AR)
basis points

Example #4: Optimal Monetary Policy - Clarida-Gali-Gertler Model ...

- Discussion of the results
  - In the absence of a working capital channel (i.e., $\psi = 0$) it is optimal to cut the interest rate, to encourage households not smooth consumption away from what is optimal.
  - In the presence of a working capital channel, (i.e., $\psi > 0$), the cut in the interest rate reduces the marginal cost of labor and expands output and employment. By reducing marginal cost, inflation drops.
  - The rise in employment and fall in inflation are both costly, and so:
    - it is optimal when $\psi > 0$ to cut the interest rate by less.
    - it is optimal to manage expectations so that the incentive to cut prices in the present is reduced.
      - announce inflation close to zero in the next period
      - announce relatively small interest rate drop in the next period.

Example #4: Optimal Monetary Policy - Clarida-Gali-Gertler Model ...

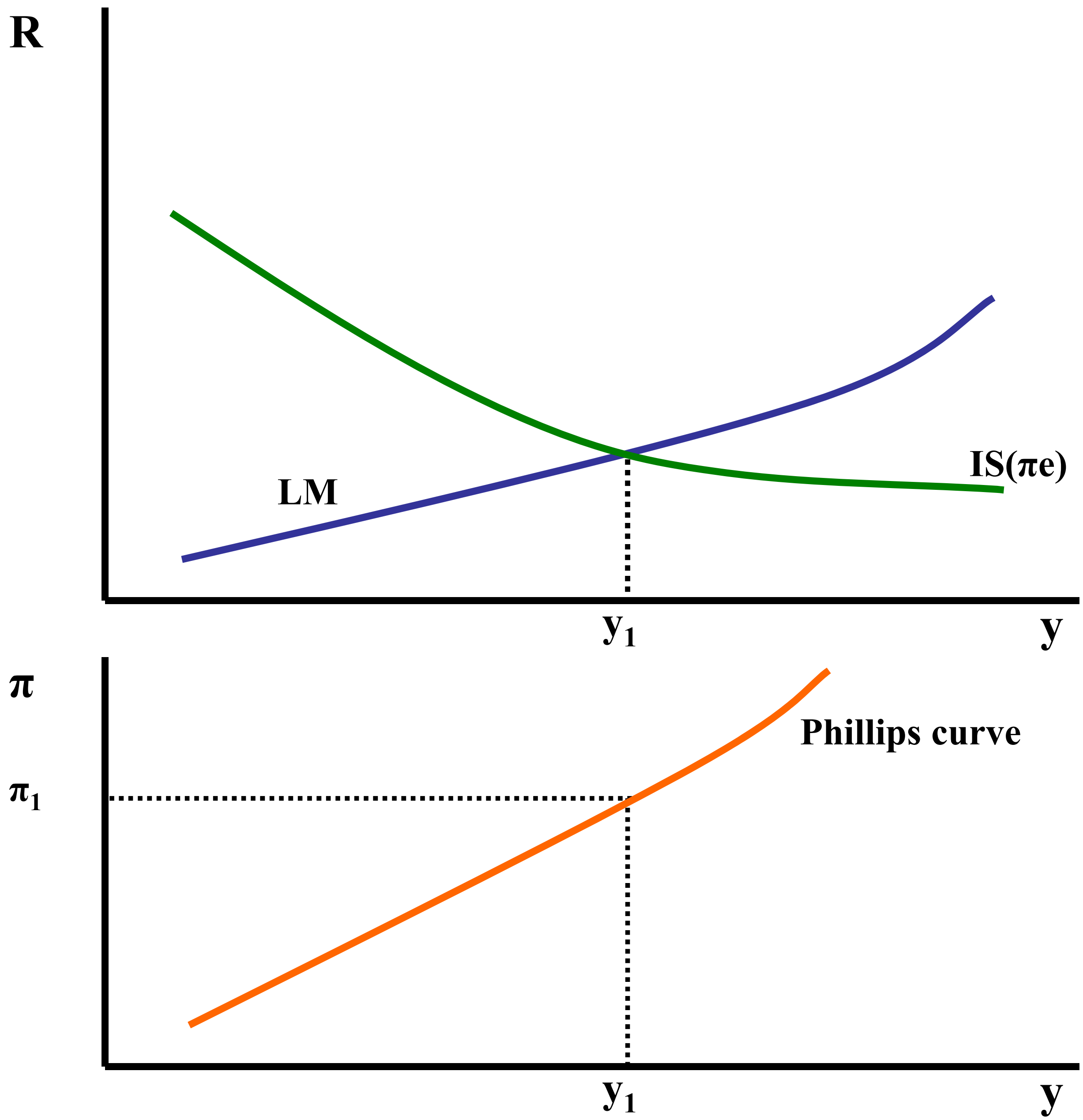
- Previous example illustrates importance of working capital channel ('lending channel')

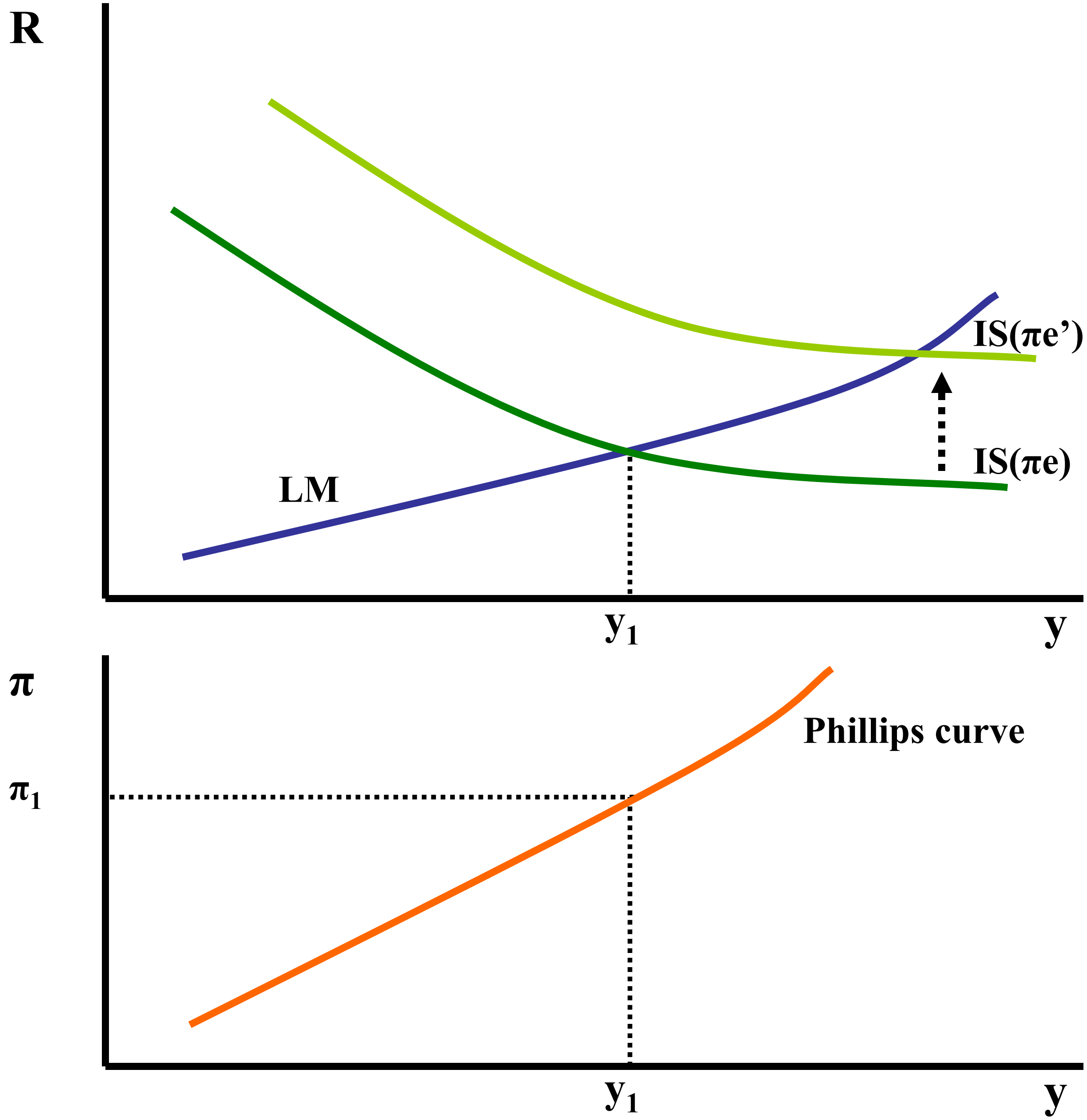
  – CCG model with Taylor rule illustrates this too

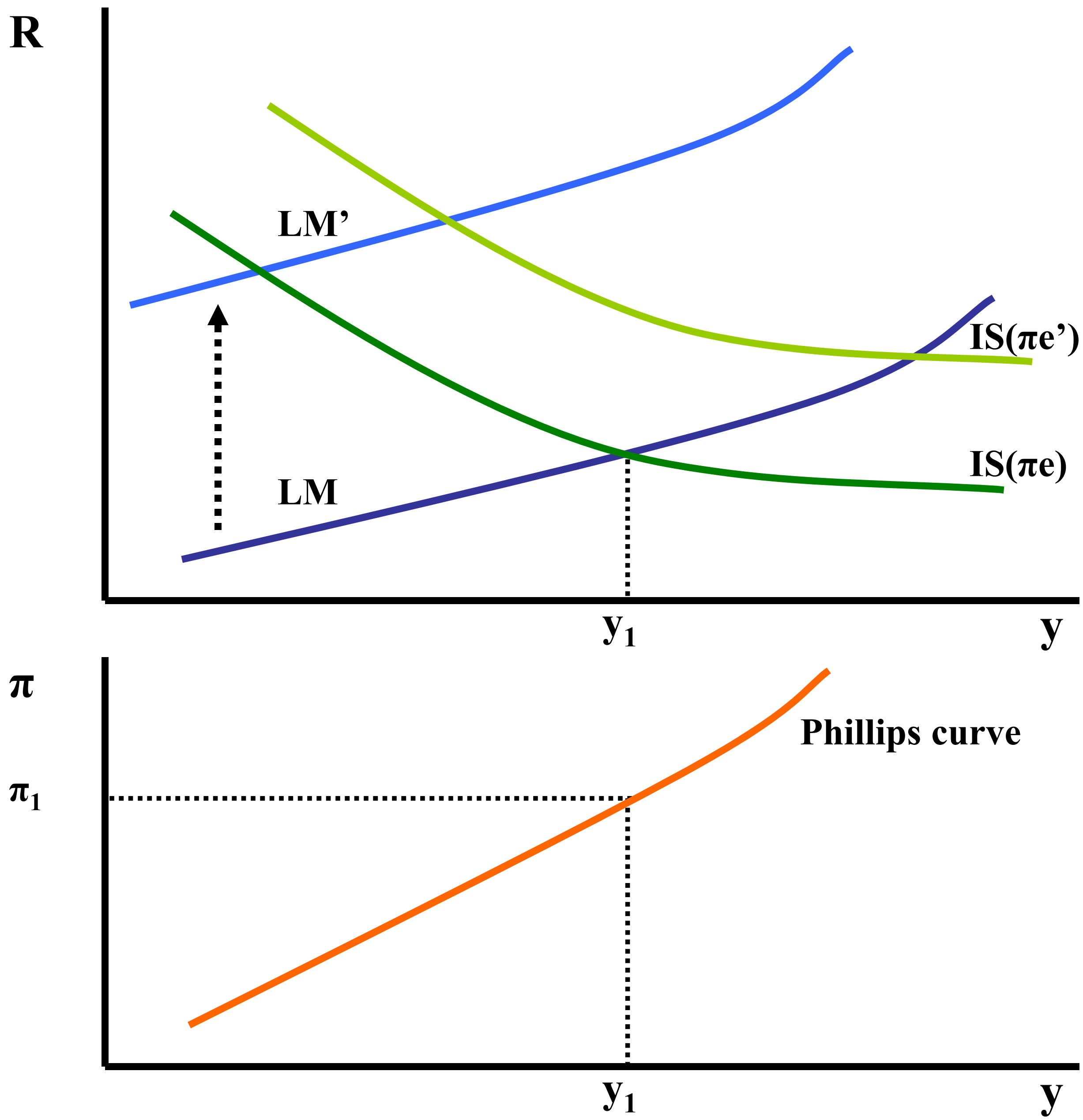
$$R_t = 1.5\pi^e_{t+1}$$

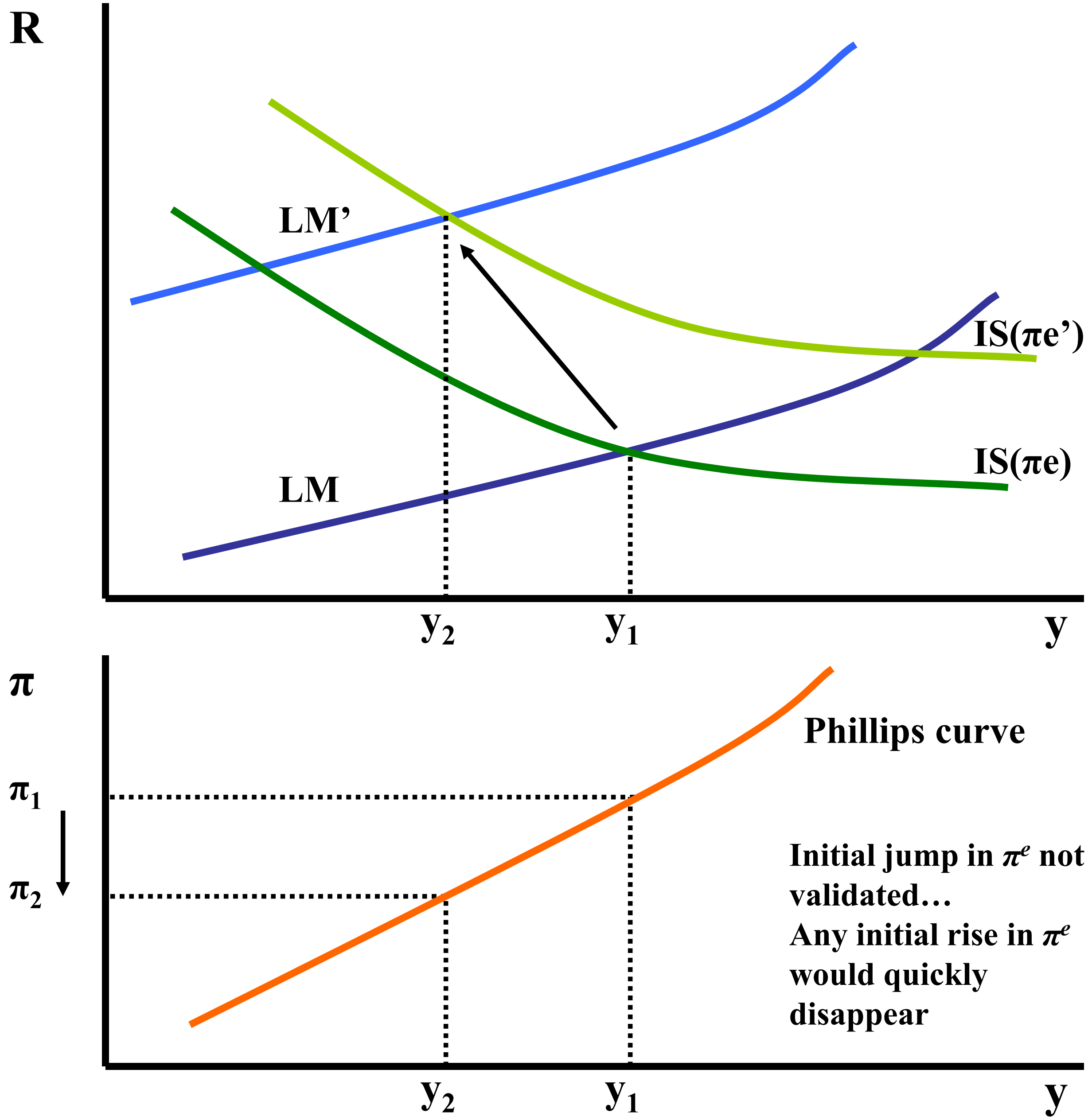
  – With $\psi = 0$, obtain standard result that inflation expectations cannot be self-fulfilling.

  – With $\psi > 0$, results turn upside down (see CMR, 'Two Reasons...')

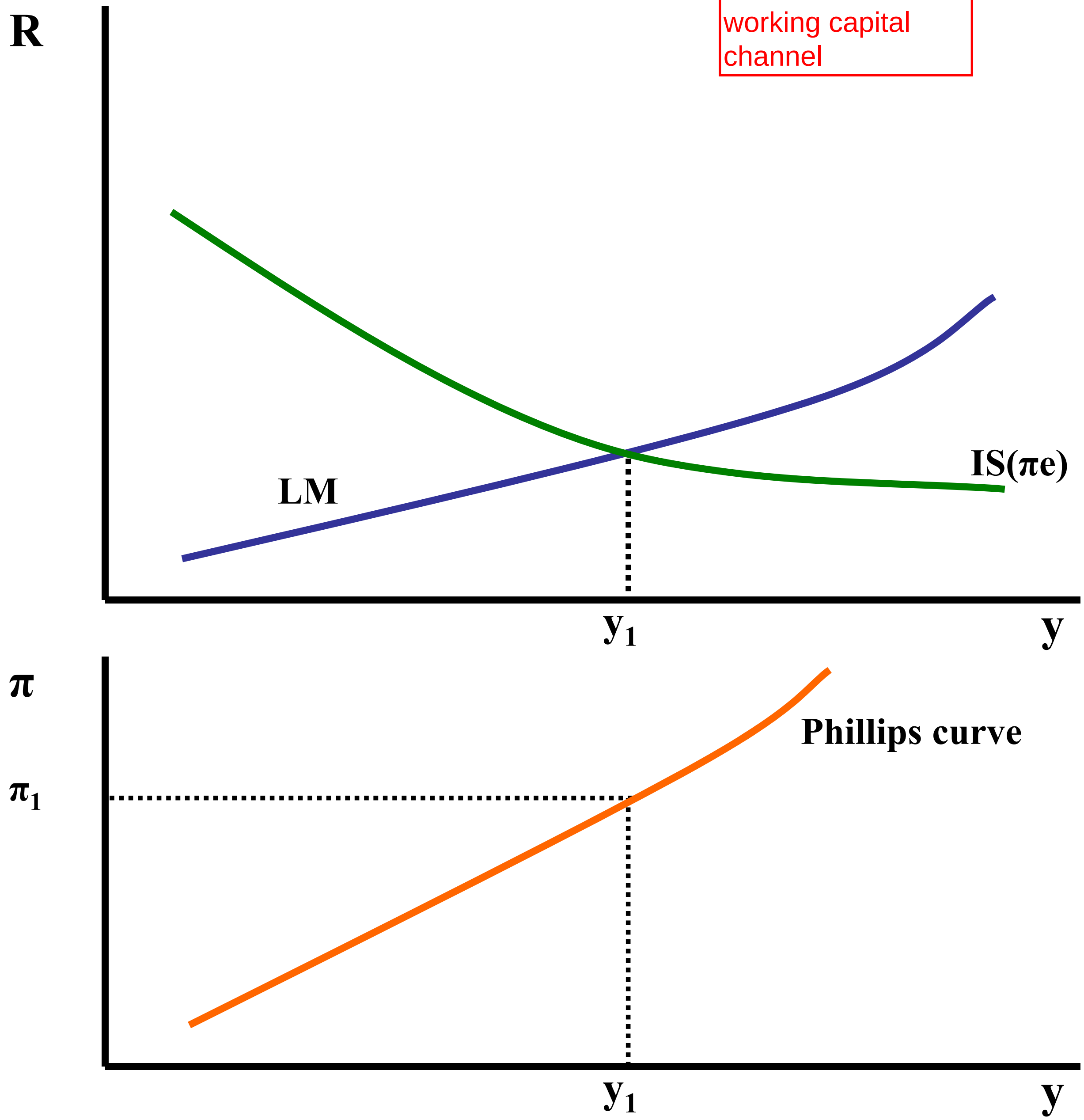

R
LM
IS(πe)
y1
y
π
Phillips curve
π1
y1
y

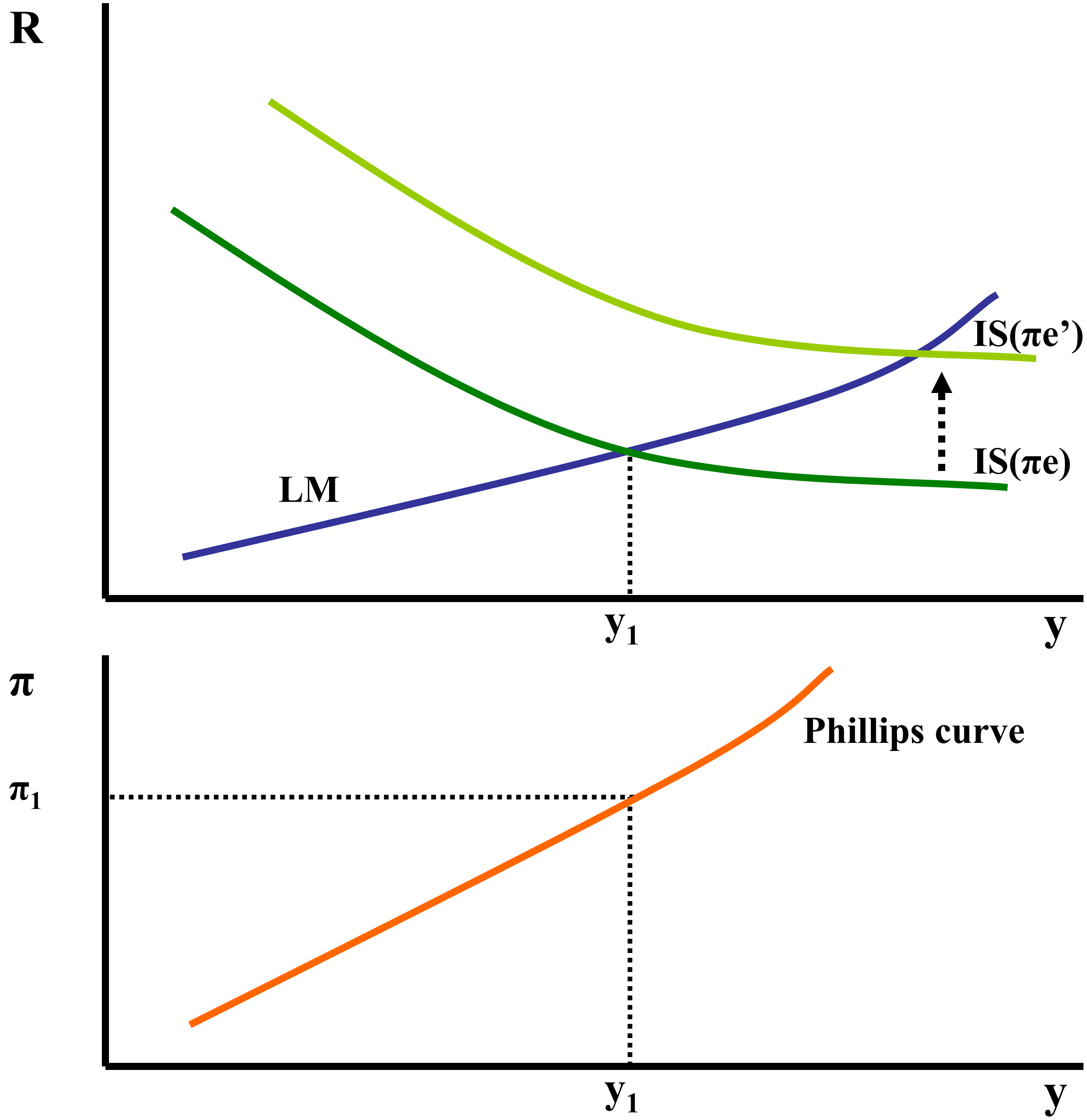

R
IS(πe')
IS(πe)
LM
y1
y
π
Phillips curve
π1
y1
y


R
LM'
LM
IS(πe')
IS(πe)
y1
y
π
π1
Phillips curve
y1
y

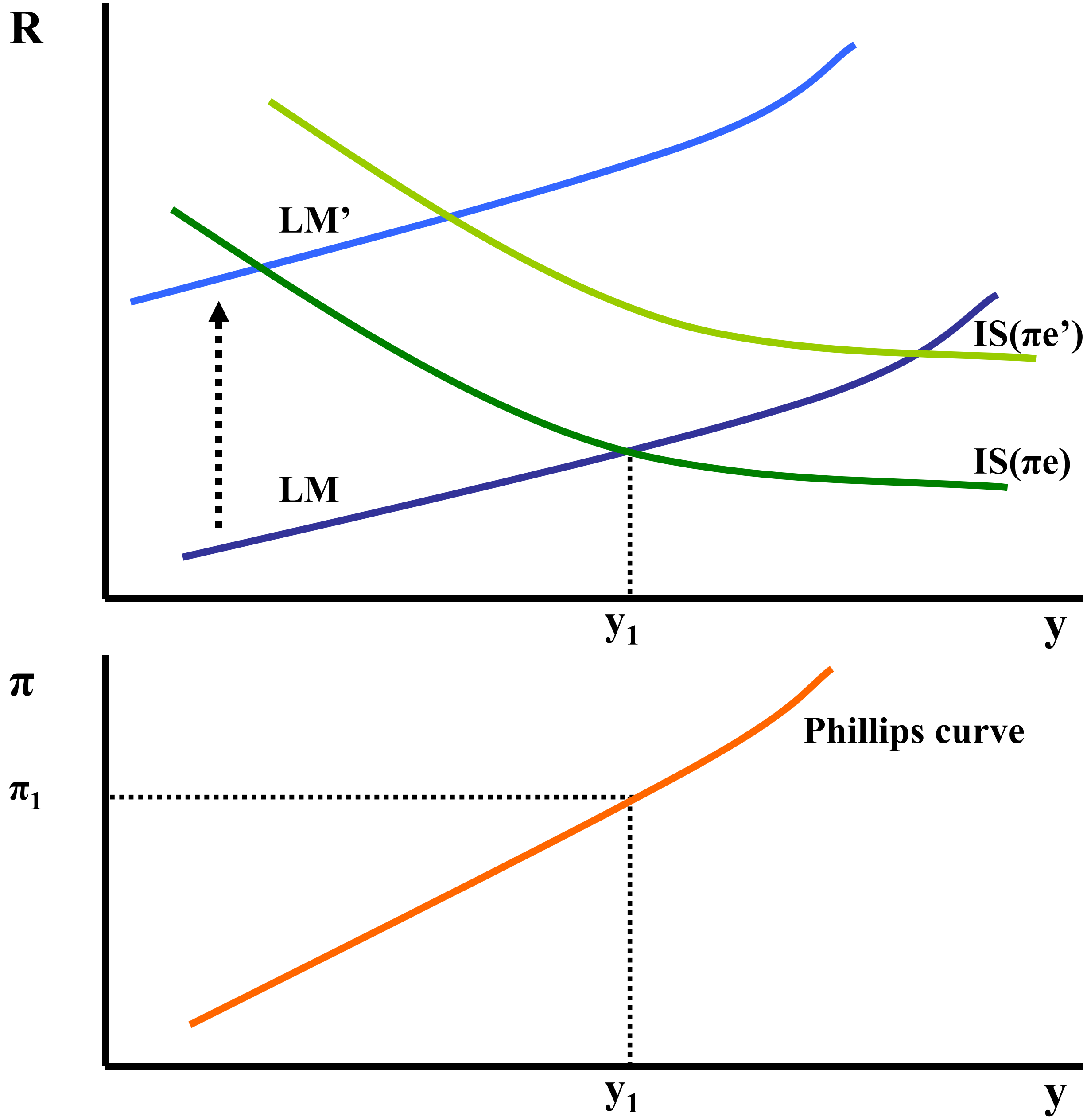

R
LM'
IS(πe')
IS(πe)
LM
y_2
y_1
y
π
Phillips curve
π_1
π_2
Initial jump in π^e not validated…
Any initial rise in π^e would quickly disappear
y_2
y_1
y

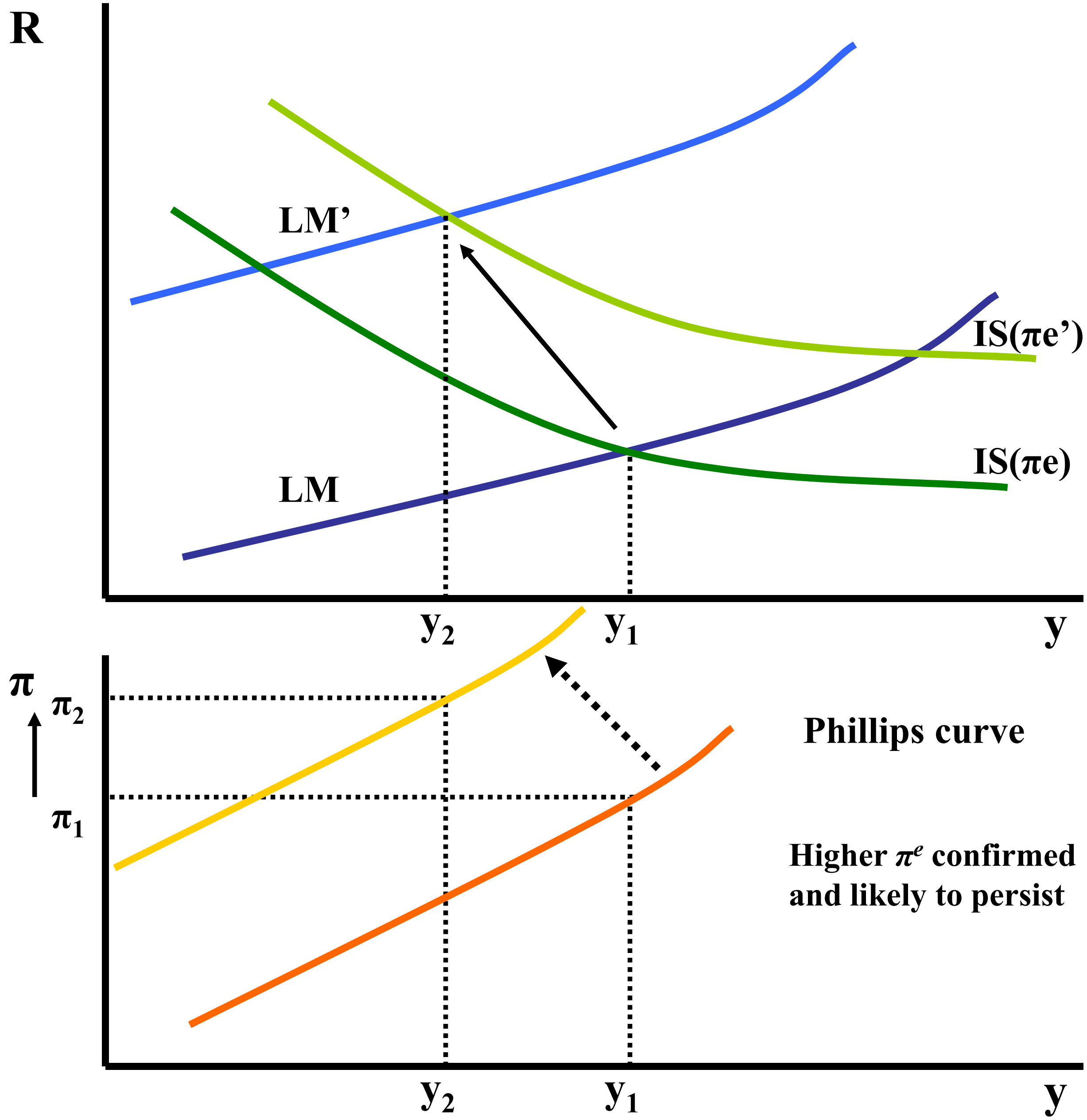
now consider the case with a working capital channel





R
IS(πe')
IS(πe)
LM
y1
y
π
Phillips curve
π1
y1
y


R
LM'
LM
IS(πe')
IS(πe)
y1
y
π
π1
Phillips curve
y1
y


R
LM'
IS(πe')
IS(πe)
LM
y2
y1
y
π
π2
π1
Phillips curve
Higher πe confirmed
and likely to persist
y2
y1
y