Economics 416.
Advanced Macroeconomics
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Homework #4, due October 23.

Consider the simple New Keynesian model discussed in the lecture. There are two time series representations for technology in this model. The 'difference stationary' or DS model is:

$$\Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t,\ E\varepsilon_t^2 = \sigma_\varepsilon^2,\ \Delta a_t \equiv a_t - a_{t-1}.$$

The 'trend stationary' or TS law of motion for technology is:

$$a_t = \rho a_{t-1} + \varepsilon_t,\ E\varepsilon_t^2 = \sigma_\varepsilon^2.$$

Let the parameters of the model be given by:

$$\begin{aligned}\beta &= 0.97,\ \phi_x = 0,\ \phi_\pi = 1.5,\ \rho = 0.2,\\ \varphi &= 1,\ \theta = 0.75,\ \sigma_\varepsilon = 0.02.\end{aligned}$$

You can use Dynare to answer the following questions.

1. Plot the response of log hours, log output and inflation to a shock in ε_t for the unit root model. Also plot the dynamic response of the real rate of interest, $r_t - E_t\pi_{t+1}$. On the same graphs, plot the 'natural' responses of n_t, y_t, π_t and $r_t - \pi_{t+1}$. Provide intuition on the reason for the difference between the actual and natural responses.

2. Do the same experiment as in (1), except with ϕ_π set to a bigger number. Explain why this makes the actual and first best responses closer. How big does ϕ_π have to be for the actual and first best allocations to be similar? Is raising ϕ_x another way to make the actual and first best responses more similar? Explain. Prove that it is mathematically impossible to select values of ϕ_π and ϕ_x to reproduce the Ramsey-optimal response to ε_t.

3. Redo (1) and (2) with the trend stationary representation. How do your results change, both in terms of the comparison of actual and first best responses and in terms of inference about the best interest rate rule parameters?

4. Consider the response of the variables to ε_t for $\phi_\pi = 1.001$ and $\phi_\pi = 0.99$ (keep $\phi_x = 0$). Note that Dynare crashes with a complaint about 'indeterminacy' in the latter case. This corresponds to the case where there are two A matrices that solve the relevant polynomial equation with eigenvalues all less than unity in absolute value.

   (a) develop an analytical proof that there are two stable A matrices when $\phi_\pi < 1$ and exactly one stable A matrix when $\phi_\pi > 1$. Hold the smoothing parameter in the Taylor rule fixed at zero for these calculations.

   (b) There is a very simple intuition for why there are multiple equilibria with $\phi_\pi < 1$. Describe it. (Hint: consider the steady state equilibrium as a baseline. Then, think intuitively about whether another equilibrium is possible when $\phi_\pi < 1$. The argument should be defeated when $\phi_\pi > 1$.)

5. Adopt the following time series representation for technology:

$$a_t = \rho a_{t-1} + \varepsilon_t + \xi_{t-1},$$

   where ε_t and ξ_{t-1} are mutually uncorrelated at all leads and lags and uncorrelated with a_{t-s}, $s > 0$. The statistical innovation in a_t is still an iid process. It's just that now the time t innovation is represented as $\varepsilon_t + \xi_{t-1}$. The point of this extra complication is that part of the period t innovation, namely ξ_{t-1}, is observed in period $t-1$ by agents.[1] Derive the expression for the natural rate and explain why the sign of the impact of ε_t and ξ_t on the natural rate of interest are opposite.

6. Most people take it for granted that a jump in news about the future (i.e., in ξ_t) will cause an expansion in the economy, and will lead to an immediate rise in inflation because the expansion is fundamentally

[1]The formal definition of the 'period t' innovation in a stochastic process, a_t, is the surprise in a_t relative to the past history on a_t :

$$a_t - P\left[a_t | a_{t-1}, a_{t-2}, ...\right],$$

which is $\varepsilon_t + \xi_{t-1}$ in this setting. The point is that agents in the model are assumed to observe a piece of the period t innovation in a_t one period in advance.

demand driven (remember, the signal is about *future* technology and involves no change in current technology).

(a) Compute the date t response of model variables to ξ_t. Show that there are parameter values under which π_t falls and y_t rises in the period of a jump in ξ_t. Provide the economic intuition for these responses. According to the model, what is the flaw in the conventional wisdom in terms of the response of inflation to good news about the future?

(b) What is the first best contemporaneous response in π_t and y_t to ξ_t? Why are the responses in the equilibrium with the interest rate rule so different? Can you explain this in terms of a deficiency in the interest rate rule?

7. The discussion in class suggested that if r_t is equated to the natural rate of interest, then the Ramsey equilibrium is realized.

(a) Replace the Taylor rule with $r_t = r_t^*$. Show that Dynare complains about indeterminacy in this case. Why?

(b) Use the results in question 4 to construct an interest rate rule that uniquely selects the Ramsey equilibrium allocations.

8. The HP filter ('Hodrick Prescott filter') takes as input a time series, y_t, and produces as output a smooth version of y_t denoted y_t^T, which solves the following problem:

$$\min_{\left\{y_t^T\right\}_{t=1}} \sum_{t=1}^{T} \left(y_t - y_t^T\right)^2 + \lambda \sum_{t=2}^{T-1} \left[\left(y_{t+1}^T - y_t^T\right) - \left(y_t^T - y_{t-1}^T\right)\right]^2$$

The parameter, λ, controls how 'smooth' y_t^T is. If $\lambda = 0$, then $y_t = y_t^T$. If $\lambda = \infty$, then y_t^T is a time trend (i.e., a line whose second derivative is zero). In business cycle analysis, it is customary to use $\lambda = 1600$ in studying quarterly data. The MATLAB m-file, [y_hp,y_hptrend]=HPFAST(y,lambda) takes y as input and outputs y_hp=$y_t - y_t^T$, y_hptrend=y_t^T. The object, is sometimes interpreted as an estimate of the 'output gap', or the HP-gap.

(a) Use the DS representation of technology with $\rho = 0.9$. Then, simulate 1,000 observations from this model using Dynare (use the stoch_simul command with periods=1000). Apply HPFAST to the level of GDP, y_t, from the model to obtain the HP-gap (you will have to simulate observations on $y_t - y_{t-1}$ and then apply the MATLAB program, cumsum, to obtain the level of GDP). Compare the HP-gap with the actual simulated gap. Graph the two and compute their correlation.

(b) Do the same as in a, except that the shock is generated by the TS representation.

(c) Note that the predicted relation between the HP-gap and the actual gap is very sensitive to whether the shocks emerge from the DS or the TS representations. Provide intuition for this result.