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Advanced Macro  
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Homework #1, due October 2

1. Consider the Runge function discussed in class,  $f : [-5, 5] \rightarrow \mathbb{R}$ . Study the accuracy of two approximation methods. In the first, construct an  $n$ -point grid in the domain of  $f$  in which there is a fixed interval between the  $n$  grid points ('fixed interval method'). In the second, construct an  $n$ -point grid in the domain of  $f$  in which the grid points correspond to the zeros of a Chebyshev polynomial ('Chebyshev method'). For the Chebyshev method, it is useful to have a mapping from any set,  $[a, b]$ , into  $[-1, 1]$ . Following is one possibility, with  $b > a$ :

$$\begin{aligned}\varphi & : [a, b] \rightarrow [-1, 1] \\ \varphi(x) & = 2\frac{x-a}{b-a} - 1.\end{aligned}$$

Obviously,  $\varphi$  is invertible. Interpolate the Runge function using the fixed interval method and using the Chebyshev method setting  $n = 10$  and 12. Graph the interpolating functions over the domain of  $f$ ,  $[-5, 5]$ . Show that with the fixed interval method, the interpolating function is diverging from the Runge function, while it is getting closer in the case of the Chebyshev method.