1 Introduction

In class we stressed a particular timing assumption in developing a model of the business cycle fluctuations in unemployment. We assumed that as soon as a worker and firm meet, they bargain right away and upon the successful conclusion of bargaining, work begins immediately. This assumption is a convenient one for doing econometric work with quarterly data.\footnote{Although it is well known how to do econometrics with a model in which the time period is finer than the data sampling interval, it does involve some complications.}

In this exam, we explore what happens when we adopt the model timing assumption that is standard in the labor literature. Accordingly, we assume that when workers and firms meet, bargaining and work do not occur until the next ‘period’. Obviously, here we have in mind a time period that is considerably shorter than one quarter. This timing specification also permits us to suppose that offers and counteroffers in alternating offer bargaining occur over successive time periods in the model. This contrasts with the approach we took in class, in which bargaining occurs within subperiods of the model period. A consequence of the altered structure of bargaining considered here is that we do not need to place any upper bound on the number of periods that bargaining can in principle go on. In the approach described in class, the bargaining could only go on for the length of one quarter, which made the problem a little more complicated (and artificial) than the one we consider here. The assumption that alternating offer bargaining could in principle go on for an unbounded number of periods implies a stationarity property that is technically very convenient. A (slight) advantage of the model presented in class is that the solution to the alternating offer bargaining model involves a sharing rule that can nicely be compared to the Nash sharing rule. The solution to the alternating offer bargaining problem described here does not seem to have the same property (though, I’d love to be proved wrong on this!).
2 The Environment

The firm’s free entry condition is:

\[ c = q_t (E_t m_{t+1} J_{t+1} - \kappa). \]

If a firm decides in period \( t \) that it wants to meet a worker, then it must post a vacancy in period \( t \) at a cost, \( c \). The firm then meets a worker in period \( t \) with probability \( q_t \). Upon meeting a worker, the firm must also pay a fixed cost, \( \kappa \). That payment also occurs in period \( t \). The firm begins to bargain with the worker at the start of period \( t+1 \) and a property of the equilibrium is that with very high probability, bargaining results in a match. Work begins in period \( t+1 \) as soon as a match is formed. From the perspective of period \( t \), the worker brings value, \( E_t m_{t+1} J_{t+1} \), to the firm. Here, \( J_{t+1} \) is the value of a worker to the firm in period \( t+1 \) and \( m_{t+1} \) is the stochastic discount factor. The value of a worker to the firm when the wage rate is \( w_t \) is:

\[ J_t = \vartheta_t - w_t + \rho E_t m_{t+1} J_{t+1}, \]

where \( \vartheta_t \) is an exogenous productivity parameter (in the more elaborate type of model discussed in class, \( \vartheta_t \) is the endogenously determined price of the one unit of output that the worker produces). In the above expression, \( \rho \) is the probability that the worker-firm meeting persists into period \( t+1 \).

The value of an unemployed worker, \( U_t \), is

\[ U_t = D + E_t m_{t+1} [f_t V_{t+1} + (1 - f_t) U_{t+1}], \]

where \( f_t \) denotes the probability that a worker finds a job in period \( t \) and \( V_{t+1} \) is the value of that job to the worker in \( t+1 \). Similarly, \( 1 - f_t \) is the probability that the worker remains unemployed, and receives \( U_{t+1} \) in period \( t+1 \). The object, \( D \), is an unemployment benefit received by an unemployed worker (this is financed by a lump-sum tax levied on the representative household).

The value to a worker of a job that pays \( w_t \) in period \( t \) is

\[ V_t = w_t + E_t m_{t+1} [\rho V_{t+1} + (1 - \rho) U_{t+1}]. \]

Note that unlike in the case considered in class, there are no job-to-job transitions in this version of the model.

We denote the number of workers employed in period \( t \) by \( l_t \), and the law of motion for \( l_t \) is:

\[ l_{t+1} = (\rho + x_t) l_t, \]

where \( x_t l_t \) denotes the quantity of newly hired workers employed in period \( t+1 \) (\( x_t \) denotes the hiring rate). The quantity, \( x_t l_t \), is determined by the number of people searching in period \( t \), \( 1 - l_t \), and the number of vacancies posted in period \( t \), \( v_t l_t \), according to the matching function:

\[ x_t l_t = \sigma m (v_t l_t)^{1-\sigma} (1 - l_t)^{\sigma}, \sigma \in (0,1). \]
According the the matching function, the probability, \( f_t \), that a period \( t \) unemployed worker is employed in period \( t + 1 \) depends on market tightness, \( \Gamma_t \):

\[
f_t = \frac{x_t l_t}{1 - l_t} = \sigma_m \Gamma_t^{1 - \sigma}, \quad \Gamma_t \equiv \frac{v_t l_t}{1 - l_t}.
\]

The vacancy filling rate can also be deduced from the matching function:

\[
q_t = \frac{x_t l_t}{v_t l_t} = \sigma_m \Gamma_t^{-\sigma}.
\]

The resource constraint for this economy is:

\[
C_t + \left( \frac{c}{q_t} + \kappa \right) x_t l_t = \vartheta l_t,
\]

where \( C_t \) denotes consumption. Let \( v_t^c \) denote the share of vacancy costs in the total cost of meeting a worker:

\[
v_t^c \equiv \frac{c}{q_t + \kappa}.
\]

The stochastic discount factor, \( m_t \), is defined as follows:

\[
m_{t+1} = \beta \frac{C_t}{C_{t+1}},
\]

and the law of motion for \( \vartheta_t \) is given by

\[
\log \vartheta_t = \rho \log \vartheta_{t-1} + \epsilon^\theta_t.
\]

### 3 Wage Determination

Every firm and worker pair either met for the first time in period \( t - 1 \) or were already matched \( t - 1 \) and did not separate. All these worker-firm pairs bargain at the start of period \( t \). They bargain over the current flow wage rate, \( w_t \). Under *Nash bargaining*, the wage rate corresponds to the value of \( w_t \) which satisfies

\[
J_t = \frac{1 - \eta}{\eta} (V_t - U_t),
\]

Under *alternating offer bargaining*, the firm makes an opening offer to the worker.\(^2\) The worker is entitled to: (i) reject the offer and terminate bargaining; (ii) reject the offer and commit to return the next period to make a counteroffer; and (iii) accept the offer and start working right away. Under (i) the worker is unemployed and receives its outside option, \( U_t \). Under (ii) the worker is thrown to its outside option, \( U_t \), with probability \( \delta \), and with probability \( 1 - \delta \) the

\(^2\)For a simple illustration of alternating offer bargaining, see page 296 in your micro text.
worker returns the next day and makes a counteroffer to the firm. The utility
the worker receives by choosing (ii) is called its disagreement payoff. In practice,
it is unlikely that the worker’s outside option is better than its disagreement
payoff. The firm has the analogous three options when it responds to a wage
offer from a worker.

Let \( w_{f}^{t} \) denote the wage offer made by a firm that is negotiating in period
\( t \). This could be the firm’s opening offer or it could be a counteroffer to an
offer that made by the worker in the previous period and was rejected by the
firm. Similarly, let \( w_{w}^{t} \) denote the wage offer made by a worker to a firm in
case the worker has rejected that firm’s wage offer in the previous period. The
fact that \( w_{w}^{t} \) and \( w_{f}^{t} \) are not functions of the number of periods already spent
bargaining is the stationarity property that I alluded to above. This stationarity
property is obviously quite convenient. As discussed in class, if the number of
potential bargaining periods is \( M \), then the solution to the bargaining problem
is a sequence of \( M \) wage rates. With the stationarity property that we use here,
the solution to the bargaining problem is characterized by just two numbers,
\( w_{w}^{t} \) and \( w_{f}^{t} \):\(^3\)

From the point of view of the worker and firm, all variables apart from the
wage rate over which they are negotiating are taken as given time invariant
functions of the economy-wide aggregate state. This function is one of the
objects we solve for when we solve for the model equilibrium.

We now describe the bargaining in detail. The wage rate chosen by the firm,
\( w_{f}^{t} \), is as low as possible subject to not triggering a rejection by the worker:

\[
V_{f}^{t} = \max \{ U_{t}, \delta U_{t} + (1 - \delta) \left[ D + E_{t} m_{t+1} V_{w}^{t+1} \right] \}, \tag{3}
\]

where

\[
V_{f}^{t} = w_{f}^{t} + E_{t} m_{t+1} \left[ \rho V_{t+1} + (1 - \rho) U_{t+1} \right],
\]

\[
V_{w}^{t} = w_{w}^{t} + E_{t} m_{t+1} \left[ \rho V_{t+1} + (1 - \rho) U_{t+1} \right].
\]

The object on the right of (3) is the maximum over the workers’ outside option
and the worker’s disagreement payoff. In practice, it is assumed that the second
is higher than the first, though this is something that should in principle be
verified. Notice that the worker’s disagreement payoff is determined by the
worker’s period \( t + 1 \) counteroffer. That counteroffer is something the firm has
to figure out before it can determine the value of \( w_{f}^{t} \). Implicit in (4) is the
assumption that when a worker is indifferent between accepting an offer and
rejecting it, the worker accepts. Similarly for the firm.

The worker’s wage offer, \( w_{w}^{t} \), is as high as possible subject to not triggering
a rejection by the firm. That is,

\[
J_{w}^{t} = \max \left\{ 0, \delta \times 0 + (1 - \delta) \left[ -\gamma + E_{t} m_{t+1} J_{f}^{t+1} \right] \right\}, \tag{4}
\]

\(^3\)The stationarity property can presumably be demonstrated by considering an alternating
offer bargaining problem with \( M \) periods and letting \( M \to \infty \).
where
\[
J_f^t = \vartheta_t - w_f^t + \rho E_t m_{t+1} J_{t+1}
\]
\[
J_w^t = \vartheta_t - w_w^t + \rho E_t m_{t+1} J_{t+1}.
\]

The object on the right of the equality in (4) is the maximum of the firm’s outside option and its disagreement payoff. The former is simply zero. The firm’s disagreement payoff reflects that with probability \( \delta \) negotiations break down and the firm is thrown to its outside option. With the complementary probability, \( 1 - \delta \), the firm gets to make a counter offer at the start of the next period. The cost of making a counter offer is \(-\gamma\), and the benefit to the firm is that it will make a counteroffer at the start of \( t + 1 \), with payoff \( E_t m_{t+1} J_{t+1} \).

The latter is the value of employing the worker at the wage \( w_{t+1}^f \) in the next period when the firm makes its counteroffer. In contemplating the consequences of making a counteroffer in the next period, both the firm and worker know that it will be accepted. In practice it is assumed (though this should in principle be verified) that the disagreement payoff is higher than the outside option.

No one makes mistakes in the bargaining and each party knows all the relevant information about the other, so that the firm’s opening offer is accepted and
\[ w_t = w_f^t, \quad J_t = J_f^t, \quad V_t = V_f^t. \]

Note
\[
V_w^t = V_t + w_w^t - w_t = V_t + dw_t
\]
\[
J_w^t = J_t - (w_w^t - w_t) = J_t - dw_t,
\]
where
\[ dw_t = w_w^t - w_t. \]

Using this notation, the indifference condition that restricts the firm’s offer is:
\[ V_t = \delta U_t + (1 - \delta) [D + E_t m_{t+1} (V_{t+1} + dw_{t+1})] \]

Substituting our for \( dw_{t+1} \) in the latter condition from
\[ dw_t = J_t - (1 - \delta) [-\gamma + E_t m_{t+1} J_{t+1} + (1 - \delta) m_{t+2} J_{t+2}], \]
we obtain:
\[ V_t = \delta U_t + (1 - \delta) [D + E_t m_{t+1} (V_{t+1} + J_{t+1} + (1 - \delta) \gamma - (1 - \delta) m_{t+2} J_{t+2})]. \]

This is a dynamic version of the alternating offer sharing rule derived in class. Note that it is not static, like the one in class.
4 Equilibrium Conditions

We summarize the equilibrium conditions. There are 12 variables in the model:

\[ q_t, m_t, J_t, V_t, U_t, dw_t, w_t, \Gamma_t, \ell_t, f_t, x_t, C_t. \]  

Following are 12 equations:

\[
\begin{align*}
\frac{c}{q_t} + \kappa & = \ E_t m_{t+1} J_{t+1} \\
V_t & = \delta U_t + (1 - \delta) [D + E_t m_{t+1} (V_{t+1} + dw_{t+1})] \\
dw_t & = J_t - (1 - \delta) [-\gamma + E_t m_{t+1} J_{t+1}] \\
J_t & = \ell_t - w_t + \rho E_t m_{t+1} J_{t+1} \\
U_t & = D + E_t m_{t+1} \left[ f_t V_{t+1} + (1 - f_t) U_{t+1} \right] \\
V_t & = w_t + E_t m_{t+1} \left[ \rho V_{t+1} + (1 - \rho) U_{t+1} \right] \\
\ell_{t+1} & = (\rho + x_t) \ell_t \\
f_t & = \frac{x_t \ell_t}{1 - \ell_t} \\
f_t & = \sigma_m V_t^{1-\sigma} \\
q_t & = f_t / \Gamma_t \\
m_{t+1} & = \beta \frac{C_t}{C_{t+1}} \\
C_t + \left( \frac{c}{q_t} + \kappa \right) x_t \ell_t & = \vartheta \ell_t \\
\end{align*}
\]

plus the law of motion for technology:

\[
\log \vartheta_t = \rho \log \vartheta_{t-1} + \varepsilon_t^\vartheta.
\]

5 Model Steady State

There are two approaches to computing the model’s steady state, both of which must be pursued for this exam. The standard approach assigns numerical values to the model parameters and then solves for the steady state values of the 12
endogenous variables using the 12 steady state equilibrium conditions:

\[
\begin{align*}
\frac{c}{q} + \kappa &= \beta J \\
V &= \delta U + (1 - \delta) [D + \beta V + \beta dw] \\
dw &= J - (1 - \delta) [-\gamma + \beta J] \\
J &= \frac{\theta - w}{1 - \rho \beta} \\
U &= D + \beta [fV + (1 - f) U] \\
V &= w + \beta [\rho V + (1 - \rho) U] \\
l_1 &= \rho + x \\
f &= \frac{xl}{1 - l} \\
f &= \sigma_m \Gamma^{1 - \sigma} \\
q &= f / \Gamma \\
m &= \beta \\
C + \left(\frac{c}{q} + \kappa\right)xl &= \vartheta l
\end{align*}
\]

A second, calibration, approach to solving the steady state will also be necessary. We do calibration when we have a high level of confidence about the empirically plausible values of some endogenous variables of the model and we have very little confidence in the values of some of the parameters. Thus, we may move the steady state values of two variables, \(q_t\) and \(l_t\), from the list of endogenous variables to the list of endogenous variables. To do this, we must move two variables (say, \(c\) and \(\sigma_m\)) from the list of exogenous model parameters to the list of endogenous variables. Sometimes the algorithm for computing the steady state using calibration is simpler than computing the steady state in the standard way.

In what follows I describe algorithms for computing the steady states. You may be able to come up with simpler algorithms, and it is possible that I have inadvertently included some typos.

### 5.1 Model Calibration

Given \(l, q\), compute:

\[
f = \frac{(1 - \rho)l}{1 - l}, \quad \Gamma = f / q, \quad \sigma_m = f / \Gamma^{1 - \sigma}.
\]
Now, simplify the two worker value functions as follows:

\[
U = D + \beta [fV + (1 - f) U]
\]

\[
V = \frac{w + \beta (1 - \rho) U}{1 - \rho \beta}
\]

\[
U = D + \beta \frac{fu + f\beta (1 - \rho) U}{1 - \rho \beta} + (1 - \rho) U
\]

\[
= \frac{D + \beta fu}{1 - \beta (1 - f) - \beta f} + \beta (1 - \rho) U
\]

\[
= \frac{(1 - \beta) D + \beta f w}{(1 - \beta) (1 - (1 - f)) - f \beta^2 (1 - \rho)}
\]

\[
= a + bw;
\]

say, where

\[
a = \frac{(1 - \beta) D}{(1 - \beta) (1 - (1 - f)) - f \beta^2 (1 - \rho)}
\]

\[
b = \frac{\beta f}{(1 - \beta) (1 - (1 - f)) - f \beta^2 (1 - \rho)}
\]

Also,

\[
V = \frac{w + \beta (1 - \rho) U}{1 - \rho \beta} = d + gw,
\]

in obvious notation.

Now, substitute out for \(dw\) in indifference condition associated with the firm’s choice of \(w^f\):

\[
V = \delta U + (1 - \delta) \left[ D + \beta V + \beta (J + \delta^2 \gamma) J \right]
\]

\[
= \delta U + (1 - \delta) D + (1 - \delta) \beta V + (1 - \delta) \beta J + \beta (1 - \delta)^2 \gamma - \beta^2 (1 - \delta)^2 J
\]

\[
= \delta U + (1 - \delta) D + (1 - \delta) \beta V + \beta (1 - \delta)^2 \gamma + (1 - \delta) \beta [1 - \beta (1 - \delta)] J,
\]

so

\[
[1 - (1 - \delta) \beta] V = \delta U + (1 - \delta) D + \beta (1 - \delta)^2 \gamma + (1 - \delta) \beta [1 - \beta (1 - \delta)] J \ (6)
\]

Substitute out for \(J, V\) and \(U\) in (6):

\[
[1 - (1 - \delta) \beta] [d + gw] = \delta \left[ a + bw \right] + (1 - \delta) D + \beta (1 - \delta)^2 \gamma + (1 - \delta) \beta [1 - \beta (1 - \delta)] \frac{\beta - w}{1 - \rho \beta},
\]

which is one linear equation in \(w\). Solve it:

\[
[1 - (1 - \delta) \beta] d + [1 - (1 - \delta) \beta] w = \delta a + \delta bw + (1 - \delta) D + \beta (1 - \delta)^2 \gamma + (1 - \delta) \beta [1 - \beta (1 - \delta)] \frac{\beta - w}{1 - \rho \beta} - (1 - \delta) \beta [1 - \beta (1 - \delta)] \frac{w}{1 - \rho \beta}
\]
or,

\[ w = \frac{\delta a + (1 - \delta) \beta [1 - \beta (1 - \delta)] - \frac{\rho}{1 - \rho \beta} + (1 - \delta) D + \beta (1 - \delta)^2 \gamma - [1 - (1 - \delta) \beta] d}{[1 - (1 - \delta) \beta] - \delta b + (1 - \delta) \beta [1 - \beta (1 - \delta)] - \frac{1}{1 - \rho \beta}} \]

With \( w \) in hand, compute \( U \) and \( V \). We then obtain \( J \) from

\[ J = \frac{\vartheta - w}{1 - \rho \beta}. \]

Compute \( c \) as follows

\[ c = q(\beta J - \kappa). \]

Finally,

\[ dw = J - (1 - \delta)[-\gamma + \beta J] \] (7)

### 5.2 Standard Steady State

Now we compute a steady state for the endogenous variables of the model, for a given set of values of the parameter. Put a grid on the interval \([0, 1]\). Consider a value of \( l \) for each point on the grid. Then,

\[ f = \frac{(1 - \rho) l}{1 - l}, \quad \Gamma = \left( \frac{f}{\sigma_m} \right)^{\frac{1}{\lambda}}, \quad q = f/\Gamma, \quad J = \frac{1}{\beta} \left[ \frac{c}{q} + \kappa \right], \]

\[ w = \vartheta - (1 - \rho \beta) J, \quad U = a + bw, \quad V = d + gw. \]

Motivated by (6), evaluate \( Q \) in

\[ Q = -[1 - (1 - \delta) \beta] V + \delta U + (1 - \delta) \beta \delta J + (1 - \delta) D + \beta (1 - \delta)^2 \gamma, \]

for each \( l \) on the \([0, 1]\) grid. Verify that there is exactly one sign switch for \( Q \) on the grid. This defines a small interval in \([0, 1]\) where the value of \( l \) that sets \( Q = 0 \) lives. Provide that interval to fzero.m and it can quickly find the value of \( l \) desired. Once this is done, compute \( dw \) using (7).

### 6 Questions

1. Let the time period of the model be one day. Choose values for \( D, c \) and \( \sigma_m \) so that \( D/w = 0.4, \ l = 0.945 \) and the daily vacancy filling rate, \( q \), is 0.70 (i.e., 70 percent) at a quarterly rate. Choose a value for \( \rho \) so the probability of remaining matched, at a quarterly rate, is 90 percent. Explain how you map between quarterly and daily rates (this is necessarily a little crude). Also, set

\[ \delta = 0.005, \ \gamma = 0.01, \ \sigma = 1/2, \ \beta = 1.03^{-1/365}. \]

Adjust the value assigned to \( \kappa \) so that the calibrated steady state for the version of the model with alternating offer bargaining has a steady state
value of \(v^c\) in (1) roughly equal to 0.98. Report the values of the endogenous variables and of the model parameters. What are the values of \(w^w\) and \(w\)? Evaluate the assumption that the disagreement payoff exceeds the outside option for a worker and a firm doing alternating offer bargaining in steady state. Call the parameterization for the alternating offer model that you arrive at in this question the baseline parameterization of the alternating offer model. It is possible that there is no admissible interior equilibrium consistent with all the stated restrictions. If so, make adjustments so that you do get an admissible interior equilibrium. (By an admissible interior equilibrium, I mean that the finding rate must lie strictly inside the unit interval, \(c\) and \(\kappa\) must be positive, etc.).

2. Compute the calibrated steady state for the standard DMP model: by this I mean the version of the model with Nash bargaining and with \(\kappa = 0\). Set \(\eta = 1/2\), as long as this implies an admissible, interior equilibrium. Otherwise, choose a value for \(\eta\) that is nearby. Call the parameterization you arrive at the baseline parameterization of the standard DMP model.

3. Compute the objects, \(d \log \Gamma / d \log \vartheta\), \(d \log w / d \log \vartheta\) and \(d \log \Gamma / d D_w\), where \(D_w \equiv D/w\). The first two of these derivatives correspond to the impact on \(\Gamma\) and \(w\) of a change in \(\vartheta\) when the values of all other structural parameters are held fixed. Similarly, \(d \log \Gamma / d D_w\) corresponds to the effect on \(\Gamma\) of a change in \(D_w\) when the values of all other structural parameters are held fixed. To evaluate \(d \log \Gamma / d D_w\), replace \(D\) wherever it appears in the model with \(D_w w\) and treat \(D_w\) as a model parameter. The derivatives will naturally come out as expressions that involve endogenous model variables. There is no need to make the derivative formulas ‘pretty’ by, for example, arranging them in a format like the Ljungqvist-Sargent type derivative presented in class (i.e., the one involving \(\Upsilon\) and \(\tau_\kappa\) in the handout). The most reliable way to produce these formulas would be to use MATLAB’s symbolic logic. A much less reliable, but acceptable, approach would be to derive the formulas by hand. You could even compute the derivatives numerically. For example, if \(f(x)\) is a function, then its derivative can be approximated numerically on a computer like this:

\[
f'(x) \approx \frac{f(x + \varepsilon) - f(x)}{\varepsilon},
\]

for a small value of \(\varepsilon\). In any case, the derivatives involve the equations being solved in what I referred to as the standard approach to computing the steady state.

In working on this question, bear in mind that according to Shimer, the data suggest that \(d \log \Gamma / d \log \vartheta\) should lie somewhere between 20 and 30.

(a) Consider the standard DMP model. Graph \(d \log \Gamma / d \log \vartheta\), \(d \log w / d \log \vartheta\), \(d \log \Gamma / d D_w\) over a range of values of \(\eta\) in the unit interval where the model has an admissible interior steady state. When doing this, hold
all other model parameters at their baseline parameter values. Do you see a substantial effect on \(d \log \Gamma/d \log \vartheta\)? The difficulty that the standard DMP model has in producing an empirically relevant value of \(d \log \Gamma/d \log \vartheta\) using reasonable parameter values is what some people define as the Shimer puzzle.

(b) Redo the graphs in (a), allowing \(D_w\) to vary from 0.4 to 0.98. Hold the values of all other parameters at their baseline values. You should see that an increase in \(D_w\) can produce empirically realistic values for \(d \log \Gamma/d \log \vartheta\). Hagedorn and Manovskii argue that increases in \(D_w\) can solve the Shimer puzzle and that the resolution obtained in this way does not involve a substantial change in \(d \log w/d \log \vartheta\). Based on this, Hagedorn and Manovskii argue that the Shimer-Hall perspective on the Shimer puzzle (that solving the puzzle requires reducing the response of \(w\) to shocks) is misleading at best. Do your results support Hagedorn and Manovskii’s conclusion? Discuss. Costain and Reiter critized the Hagedorn and Manovskii solution to the Shimer puzzle on the grounds that while it does increase \(d \log \Gamma/d \log \vartheta\) into the empirically relevant range, it simultaneously makes \(d \log w/d D_w\) implausibly large. Based cross-country data, Costain and Reiter argue that \(d \log \Gamma/d D_w\) is small.4

(c) Pissarides argued that reducing \(\nu^c\) could help to resolve the Shimer puzzle.5 Redo the graphs in (a) for a range of values of \(\kappa\) greater than zero. For each value of \(\kappa\), solve for \(\sigma_m\) and \(c\) so that - holding the other parameters at their baseline values - the model is consistent with the calibration targets for \(l\) and \(q\). Report results for a range of \(\kappa\) greater than zero which are consistent with admissible, interior equilibria. It has been argued that this ‘solution’ to the Shimer puzzle moves things in the right direction, but not by enough. Are your results consistent with this conclusion? Explain.

(d) Hall and Milgrom suggested that alternating offer bargaining might solve the Shimer puzzle. Their intuition is that by ‘disconnecting’ the wage a little from outside economic conditions, alternating offer bargaining implies a smaller rise in the wage in an economic expansion, giving firms a larger share of profits, providing them with an incentive to post vacancies and expand employment. Redo the graphs in (a) for the alternating offer bargaining model, allowing a range of values for \(\kappa\) that fall from its calibrated value down as close as possible to zero, subject to the existence of an interior, admissible equilibrium. Hall and Milgrom’s make their suggestion in a standard DMP framework, that is, they set \(\kappa = 0\). Can Hall and Milgrom alone resolve the Shimer puzzle, or do you also need a little Pissarides? Redo the graphs with a lower value of \(\delta\). Do the results change your answer?

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4 Another criticism of Hagedorn and Manovskii is that their proposed solution to the Shimer puzzle requires an implausibly high value of \(D_w\).
5 See also Christiano-Trabandt-Walentin and Christiano-Eichenbaum-Trabandt.
Explain. Does the Hall and Milgrom solution to the Shimer puzzle involve a substantial drop in $d \log w/d \log \psi$? Do Hall and Milgrom and Pissarides help us to get around the Costain and Reiter critique? Explain, based on the behavior of $d \log \Gamma / d D_w$.

4. Now consider the dynamic properties of the alternating offer and standard DMP models, evaluated at their baseline parameterizations. Set $\rho_\theta = 0.95$, $var (\varepsilon_\theta) = 0.01^2$, the parameters favored in the RBC literature (see, e.g., Prescott’s classic 1986 Minneapolis Fed Quarterly Review paper).

(a) Enter the two dynamic models into Dynare and check each for determinacy of the steady state equilibrium. You should find determinacy in each case. In the case of the model presented in class, the steady state was indeterminate. A very simple argument was presented to establish indeterminacy (I did make the simplifying assumption, $m_{t+1} = \beta$). What part of that argument fails with the current models? Explain carefully.

(b) Compute impulse responses to a shock in $\psi_t$ for both models. Let $d \log \Gamma_t$ denote the deviation of $\log \Gamma_t$ from steady state at lag $t = 0, 1, 2, \ldots$ in the impulse response. Let $d \log \psi_t$ denote the analogous impulse response for $\log \psi_t$. Graph the ratio $d \log \Gamma_t / d \log \psi_t$ for $t = 0, 1, 2, \ldots, 20$ (don’t work with $t$ too large because $d \log \psi_t$ goes to zero for large $t$, so you’d be dividing by zero). This ratio is a dynamic analog of the steady state object, $d \log \Gamma / d \log \psi$. Graph that elasticity as a horizontal line along with your impulse response function. Do the impulse response function and horizontal line look similar for both models? If yes, it suggests a very simple way to obtain insight into model dynamics: just compute steady state responses, objects that can often be represented analytically, in easy-to-interpret formulas. Presumably, the steady state calculation is a bad approximation for the dynamic object when $\rho_\theta$ is very small.

(c) Graph the response of the wage and of employment for both models, to a one standard deviation shock in $\varepsilon_\theta$. Place the two wage and employment responses in the same graphs, for ease of comparison. Is the employment response in the alternating offer model much stronger than it is in the standard DMP model? Is this accomplished by a weaker wage response in the alternating offer model?