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FINC 520, Spring 2007
Homework 2, due Monday, April 16.

1. In class, we discussed the p^{th} order VAR:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p} + \varepsilon_t,$$

where ε_t is a white noise with variance-covariance matrix, Ω, and orthogonal with y_{t-s}, $s > 0$. Find a definition of ξ_t, so that the VAR(p) can be written in the following VAR(1) form:

$$\xi_t = F\xi_{t-1} + v_t.$$

2. Write the VAR(p) in transposed form:

$$y_t' = x_t'\Pi + \varepsilon_t',$$

where

$$\underset{(np+1)\times}{x_t} = \begin{pmatrix} 1 \\ y_{t-1} \\ \vdots \\ y_{t-p} \end{pmatrix}, \quad \underset{n\times(np+1)}{\Pi'} = \begin{bmatrix} c & \phi_1 & \cdots & \phi_p \end{bmatrix}.$$

Suppose we have data over the period $t = 1, ..., T$ and write:

$$Y = \begin{pmatrix} y_1' \\ y_2' \\ \vdots \\ y_T' \end{pmatrix}, \quad X = \begin{pmatrix} x_1' \\ x_2' \\ \vdots \\ x_T' \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon_1' \\ \varepsilon_2' \\ \vdots \\ \varepsilon_T' \end{pmatrix},$$

so that the VAR can be written as follows:

$$\underset{T\times n}{Y} = \underset{T\times(np+1)}{X} \underset{(np+1)\times n}{\Pi} + \underset{T\times n}{\varepsilon}.$$

Premultiply this by X' and then take expectations:

$$E[X'Y] = E[X'X]\Pi + E[X'\varepsilon].$$

Note that

$$EX'\varepsilon = E\sum_{t=1}^{T} x_t \varepsilon_t' = 0,$$

because of the assumed orthogonality properties of ε_t. Then,

$$\Pi = \{E[X'X]\}^{-1} E[X'Y].$$

Ergodicity (we established in class that ergodicity is satisfied) suggests using the following estimator for Π:

$$\hat{\Pi} = (X'X)^{-1} X'Y.$$

(a) Compute $\hat{\Pi}$ using the data in the MATLAB m file that has been provided, with $p = 4$. Let

$$y_t = \begin{pmatrix} R_t \\ \log \frac{GDP_t}{GDP_{t-1}} \\ \pi_t \end{pmatrix},$$

where R_t denotes the 3 month Tbill rate, π_t denotes the quarterly inflation rate and GDP_t denotes Gross Domestic Product in quarter t.

(b) Construct F, the matrix you constructed for question 1. Note how your way of handling the constant term causes there to be one eigenvalue equal to unity this matrix. Compute $\mu = Ey_t = [I - \phi_1 - \phi_2 - \phi_3 - \phi_4]^{-1} c$. Compare μ with:

$$P[y_{t+500} | y_t, y_{t-1}, ...],$$

Why should μ and this projection be so similar? Is the VAR(p) model you constructed covariance stationary and ergodic? Explain.

(c) Construct a 'predicted time series' on the long rate for the $T = 201$ quarters, 1955Q4-2005Q4. Graph it together with the actual 5 year rate, R_t^l. To do this, you'll have to do the calculations discussed in class. However, the geometric sum formula described there will not work if you do it with the version of F in question

1, because that has a unit root in it. You can simply add $I + F + ... + F^{19}$. How does the expectations hypothesis do? Do the same graph, only this time do it with

$$y_t = \begin{pmatrix} R_t \\ R_t^l \\ \log \frac{GDP_t}{GDP_{t-1}} \\ \pi_t \end{pmatrix}.$$

Do the results make any difference with this adjustment?

(d) Graph the spectrum, $(0, \pi)$, for the long rate, R_t^l, and the long rate as predicted by the model. (Hint: for this, note that each of these long rates can be expressed as $\gamma' y_t$, for a suitable choice of the column vector, γ. Then, the spectrum of $\gamma' y_t$ is just $\gamma' S_y \left(e^{-i\omega}\right) \gamma$.)

3. Calculate the variance, Γ_0, of VAR(p) process you estimated in question 2. Do so using the vectorization and the Riccatti equation methods discussed in class. Also compute Γ_0 using

$$\Gamma_0 = \int_{-\pi}^{\pi} S_y \left(e^{-i\omega}\right) d\omega,$$

using the Riemann approximation to the integral. How many approximation points do you have to use, in order for the integral approach to yield an accurate answer (say, to within 3 significant digits)? Do the same for

$$\Gamma_1 = \int_{-\pi}^{\pi} S_y \left(e^{-i\omega}\right) e^{i\omega} d\omega.$$